MSC 54 E15, 54 C10

ON UNIFORMLY PARACOMPACT SPACES AND MAPPINGS

[1]**Borubaev A.A.**, [2]**Kanetov B.E.**, [3]**Kurmanbek uulu M.**, [4]**Mukashova N.Sh.**
[1]*Institute of Mathematics of National Academy of Sciences of Kyrgyz republic,*
[2,3,4]*Jusup Balasagun Kyrgyz National University*

The problem of uniformization of paracompact spaces is an important and interesting problem in the theory of uniform spaces. Currently, there are different variants of uniform paracompactness of uniform spaces. In this work we show new approaches to the definition of a uniform analogue of paracompactness of spaces and mappings.

Key words: Uniformly locally finite uniform cover, finitely additive open cover, uniformly paracompact space.

Паракомпактуу мейкиндиктерди униформизациялоо маселеси бир калыптуу мейкиндиктердин маанилүү жана кызыктуу маселелеринен. Азыркы мезгилде бир калыптуу мейкиндиктердин бир калыптуу паракомпактуулугунун бир нече варианттары бар. Бул иште мейкиндиктердин жана чагылдыруулардын паракомпактуулугунун бир калыптуу аналогунун жаңы аныктамалары берилген.

Урунттуу сөздөр: Бир калыптуу локалдуу чектүү бир калыптуу жабдуу, чектүү аддитивдүү ачык жабдуу, бир калыптуу паракомпактуу мейкиндик.

Задача униформизация паракомпактных пространств является важной и интересной задачей теории равномерных пространств. В настоящее время существуют различные варианты равномерной паракомпактности равномерных пространств. В настоящей работе показывается новые подходы к определению равномерного аналога паракомпактности пространств и отображений.

Ключевые слова: Равномерно локально конечное равномерное покрытие, конечно аддитивное открытое покрытие, равномерно паракомпактное пространство.

For a family α of subsets of a set X, we put $St(x,\alpha)=\{A\in\alpha : x\in A\}$, $St(M,\alpha)=\{A\in\alpha : M\cap A\neq\varnothing\}$, $x\in X$, $M\subset X$. Then $\alpha(x)=\cup St(x,\alpha)$ and $\alpha(M)=\cup St(M,\alpha)$. For the infinite cardinal τ, $\alpha_\tau=\{\cup\beta : \beta\subset\alpha, |\beta|<\tau\}$. A family α is called τ-additive if $\alpha_\tau=\alpha$ and accordingly, a $\aleph_0$-additive family will be called finitely additive and denoted by $\alpha^{\angle}$.

Lemma 1. If α and β are uniformly locally finite uniform coverings, then $\alpha\wedge\beta$ is also a uniformly locally finite uniform covering.

Proof. Let α, β are uniformly locally finite uniform coverings. Then there exists $\mu\in U$ and $\eta\in U$ such that for any $M\in\mu$ and $N\in\eta$ it follows $M\subset\bigcup_{i=1}^{n}A_i$ and $N\subset\bigcup_{j=1}^{k}B_j$, where $A_i\in\alpha$, $B_j\in\beta$, $i=1,2,\dots,n$, $j=1,2,\dots,k$. Therefore,

$M \cap N \subset (\bigcup_{i=1}^{n} A_i) \cap (\bigcup_{j=1}^{k} B_k) \subset \bigcup_{i=1}^{n} \bigcup_{j=1}^{k} (A_i \cap B_j)$, i.e. the set $M \cap N \in \mu \wedge \eta$ intersects only with a finite number of elements of the uniform covering $\alpha \wedge \beta$. Therefore, $\alpha \wedge \beta$ is a uniformly locally finite uniform covering.

Lemma 2. Let $f:(X,U) \to (Y,V)$ be a uniformly continuous mapping. If β is a uniformly locally finite uniform covering of a uniform space (Y,V), then $f^{-1}\beta$ is also a uniformly locally finite uniform covering of a uniform space (X,U).

Proof. Let β be a uniformly locally finite uniform covering of a uniform space (Y,V). It is clear that the cover $f^{-1}\beta$ is a uniform covering of a uniform space (X,U). We show that $f^{-1}\beta$ is a uniformly locally finite covering. There is a uniform covering $\mu \in U$, each element of which intersects only with a finite number of elements of the uniform covering β, i.e. for any $M \in \mu$, there are $B_1, B_2, ..., B_n$ from β such that $M \subset \bigcup_{i=1}^{n} B_i$. Therefore $f^{-1}M \subset f^{-1}(\bigcup_{i=1}^{n} B_i) = \bigcup_{i=1}^{n} f^{-1}B_i$, where $f^{-1}B_i \in f^{-1}\beta$, $i = 1,2,...,n$ and $f^{-1}M \in f^{-1}\mu$. Since, $f^{-1}\mu \in U$, then $f^{-1}\mu$ is the desired uniform covering. Thus, $f^{-1}\beta$ is a uniformly locally finite uniform covering of the space (X,U).

Definition 1. A uniform space (X,U) is called uniformly paracompact if every finitely additive open covering has a uniformly locally finite uniform covering refinement.

Every uniformly paracompact space is a uniformly R-paracompact in the sense of M.D. Rice.

Proposition 1. If (X,U) is a uniformly paracompact space, then the topological space (X,τ_U) is paracompact. Conversely, if (X,τ) is paracompact, then the uniform space (X,U_X), where U_X is the universal uniformity, is uniformly paracompact.

Proof. Let α be an arbitrary open covering of the space (X,τ_U). Then for a finitely additive open covering $\alpha^{\angle}$ of the uniform space (X,U) there exists a locally finite uniform covering $\beta \in U$ which is a refinement of it. It is known that the interior $\langle\beta\rangle$ of the uniform covering β is a uniform covering. Put $\gamma=\langle\beta\rangle$. It is clear that γ is a locally finite open uniform covering of (X,U). For each $\Gamma\in\gamma$ choose $A_\Gamma\in\alpha^\angle$ such that $\Gamma\subset A_\Gamma$, where $A_\Gamma=\bigcup_{i=1}^{n}A_i$, $A_i\in\alpha$, $i=1,2,...,n$. Let $\alpha_0=\cup\{\alpha_\Gamma:\Gamma\in\gamma\}$, $\alpha_\Gamma=\{\Gamma\cap A_i:i=1,2,...,n\}$. Then α_0 is an open locally finite covering of the space (X,τ_U), and it is a refinement of the open covering α. So, the space (X,τ_U) is paracompact.

Conversely, let the space (X,τ) be paracompact. Then the set of all open coverings forms the base of the universal uniformity U_X of the space (X,τ). It is easy to see that a uniform space (X,U_X) is uniformly paracompact.

The following theorem gives a characteristic of uniform paracompactness in the spirit of Tamano.

Theorem 1. Let (X,U) be a uniform space and bX be a certain its compact Hausdorff extension. The uniform space (X,U) is uniformly paracompact, if and only if for each compactum $K\subset bX\setminus X$ there exists a locally finite uniform covering $\alpha\in U$ such that $[A]_{bX}\cap K=\varnothing$ for all $A\in\alpha$.

Proof. Necessity. Let (X,U) be a uniformly paracompact and $K\subset bX\setminus X$ be an arbitrary compactum. Then for each point $x\in X$ there is an open neighborhood O_x in bX such that $[O_x]_{bX}\cap K=\varnothing$. It is clear that $\gamma=\{O_x\cap X:x\in X\}$ is an open covering of the uniform space (X,U). We form an open covering $\gamma^\angle$ of (X,U), taking as elements of γ. Then $\gamma^\angle$ is a finite additive open covering of the space (X,U). According to the condition of the theorem, it is possible to refine a covering $\gamma^\angle$ by a locally finite uniform covering $\beta\in U$. Then $[B]_{bX}\subset[\cup_{i=1}^{n}(O_{x_i}\cap X)]_{bX}\subset\bigcup_{i=1}^{n}[O_{x_i}]_{bX}$. As $[O_{x_i}]_{bX}\cap K=\varnothing$ for any $i=1,2,...,n$, then $[B]_{bX}\cap K=\varnothing$ for any $B\in\beta$.

Sufficiency. Let the condition of the theorem be satisfied. Let α be an arbitrary finite additive open covering of a space (X,U). Then there is an open family β in bX such that $\beta \wedge \{X\} = \alpha$. Let $K = bX \setminus \cup\beta$. It follows that K is a compactum. Then, by the condition of the theorem, there exists a uniformly locally finite uniform covering $\gamma \in U$ such that $[\Gamma]_{bX} \cap K = \varnothing$ for any $\Gamma \in \gamma$. Since $[\Gamma]_{bX}$ is a compactum lying in bX there are $B_1, B_2, ..., B_n \in \beta$ such that $[\Gamma]_{bX} \subset \bigcup_{i=1}^{n} B_i$. Then $\Gamma \subset \bigcup_{i=1}^{n} A_i$, where $\bigcup_{i=1}^{n} A_i \in \alpha$. Consequently, (X,U) is a uniformly paracompact space.

A uniform space (X,U) is called strongly uniformly locally compact if there exists a uniformly locally finite uniform covering consisting of compact subsets.

The following theorem gives a connection between uniform paracompactness and strongly uniformly local compactness.

Theorem 2 Any strongly uniformly locally compact space is uniformly paracompact.

Proof. Let α be an arbitrary finitely additive open covering. Then there exists a uniformly locally finite uniform covering β consisting of compact subsets. It is easy to see that the covering β is a refinement of the additive open covering α.

Proposition 2. Any uniformly paracompact space is complete.

Proof. The proof follows easily from the fact that any uniformly R-paracompact space is complete.

The following two proposition show that uniform paracompactness is preserved when passing to a closed subspaces and any disjoint sum of uniform spaces.

Proposition 3. Any closed subspace M of a uniformly paracompact space (X,U) is uniformly paracompact.

Proof. Let γ be a finitely additive open covering of the space M. Denote by $\hat{\gamma}$ the open cover of the space (X,U) consisting of all elements of the cover γ and the set $X \setminus M$. It is clear that $\hat{\gamma}$ is a finite additive covering. By hypothesis, there exists a uniformly locally finite uniform covering $\beta \in U$ has in $\hat{\gamma}$ refinement. Let β_M be the trace of β on M. Then β_M is a uniformly locally finite uniform covering of the subspace M has in γ refinement. So, the subspace M is uniformly paracompact.

Proposition 4. The sum of any family of uniformly paracompact spaces is uniformly paracompact.

Proof. Let $\{(X_a,U_a): a\in M\}$ be an arbitrary family of uniformly paracompact spaces (X_a,U_a) and $(\coprod_{a\in M} X_a, \coprod_{a\in M} U_a)$ is the sum of uniform spaces. Consider an arbitrary finitely additive open covering α of the space $(\coprod_{a\in M} X_a, \coprod_{a\in M} U_a)$. It is easy to see that the family $\beta=\{X_a\cap A: a\in M, A\in\alpha\}$ is a refinement of the covering α again a finite additive open of the space $(\coprod_{a\in M} X_a, \coprod_{a\in M} U_a)$. For each $a_0\in M$ we put $\beta_{a_0}=\{X_{a_0}\cap A: a_0\in M, A\in\alpha\}$. It is clear that it is a finitely additive open covering of the space (X_{a_0},U_{a_0}). Then there exists a uniformly locally finite uniform covering $\gamma_{a_0}\in U_{a_0}$ has in β_{a_0} refinement. Let γ be the union of all uniformly locally finite families γ_a, $a\in M$. Then it is easy to see that the family γ is a refinement of the covering α of the uniform locally finite space $(\coprod_{a\in M} X_a, \coprod_{a\in M} U_a)$.

The following theorem shows that uniform paracompactness is preserved in the preimage direction by uniformly perfect mappings.

Theorem 3. Uniform paracompactness is preserved in the preimage direction by uniformly perfect mappings.

Proof. Let α be an arbitrary finitely additive open covering of a space (X,U). It is clear that the covering $\{f^{-1}y: y\in Y\}$ refines the covering α. Then $\beta=f^{\#}\alpha=\{f^{\#}A: A\in\alpha\}$, where $f^{\#}A=Y\setminus f(X\setminus A)$, is an open covering of the space (Y,V). Considering all possible finite unions of sets of β, we construct an open covering $\beta^{\angle}$. It is a finitely additive open covering. By the condition of the theorem, there is a uniformly locally finite uniform covering $\gamma\in V$ of it. It is easy to see that the covering $f^{-1}\beta^{\angle}$ is a refinement of the covering α. The $f^{-1}\gamma$ is a uniformly locally finite uniform covering of the space (X,U), and it is a refinement of α. So, the uniform space (X,U) is uniformly paracompact.

Recall (see, for example, [1]) that a continuous map $f:X\to Y$ of a topological space X to a topological space Y is called a ω-mapping, where ω is an open covering of X if for every point $y\in Y$ there exists a neighborhood O_y and $W\in\omega$, such that $f^{-1}(O_y)\subset W$.

The following theorem is a uniform analogue of the Dowker-Ponomarev theorem.

Theorem 4. A uniform space (X,U) is uniform paracompact if and only if for every finitely additive open covering ω of (X,U) there exists a uniformly continuous ω-mapping $f:(X,U)\to(Y,V)$ of the uniform space (X,U) onto a metrizable uniformly paracompact space (Y,V).

Proof. Necessity. Let (X,U) be a metrizable uniformly paracompact space and ω be an arbitrary finitely additive open covering. Then the identity map of a space (X,U) is the required uniformly continuous ω-mapping of (X,U) into a metrizable uniformly paracompact space.

Sufficiency. Let ω be an arbitrary finite additive open covering of the space (X,U). Then there exists a uniformly ω-continuous mapping $f:(X,U)\to(Y,V)$ of the uniform space (X,U) onto some metrizable uniformly paracompact space (Y,V). For each point $y\in Y$ there exists a neighborhood O_y whose preimage $f^{-1}O_y$ is contained in some element of the covering ω. Let $\beta=\{O_y : y\in Y\}$. We form an open covering $\beta^{\angle}$ consisting of all possible finite unions of elements of β. We refine a uniformly locally finite uniform covering $\gamma\in V$ in it. Then covering $f^{-1}\gamma$ is a refinement of the covering ω of the uniform space (X,U). Therefore, a uniform space (X,U) is uniformly paracompact.

Proposition 5. The product of a uniformly paracompact uniform space (X,U) to a compact uniform space (Y,V) is uniformly paracompact.

Proof. Let (X,U) be uniformly paracompact and (Y,V) is a compact uniform space. It is known [2] that the projection $\pi_X:(X,U)\times(Y,V)\to(X,U)$ is uniformly perfect. Then it is a ω-mapping of the product $(X,U)\times(Y,V)$ to a uniformly paracompact space (Y,V) for any finitely additive open cover ω of the space $(X,U)\times(Y,V)$. Therefore, according to Theorem 4, the uniform space $(X,U)\times(Y,V)$ is uniformly paracompact.

Definition 2. A uniformly continuous mapping $f:(X,U)\to(Y,V)$ of a uniform space (X,U) into a uniform space (Y,V) is called a uniformly paracompact, if for each finitely additive open covering α of a uniform space (X,U) there exists a finitely additive open covering β of a uniform space (Y,V) and a uniformly locally finite uniform covering $\gamma\in U$ such that $f^{-1}\beta\wedge\gamma\succ\alpha$.

Proposition 6. If $f:(X,U)\to(Y,V)$ is uniformly continuous and the uniform space (X,U) is uniformly paracompact, then the mapping f is uniformly paracompact.

Proof. Let α be an arbitrary finitely additive open covering of the uniform space (X,U). Then there exists a uniformly locally finite uniform covering $\gamma\in U$ such that $\gamma\succ\alpha$. Let β be some finitely additive open covering of the space (Y,V). Then $f^{-1}\beta\wedge\gamma\succ\alpha$. So, f is a uniformly paracompact mapping.

Proposition 7. If $f:(X,U)\to(Y,V)$ is a uniformly paracompact mapping and $M\subset X$ is a closed subset, then its restriction $f|_M:(M,U_M)\to(Y,V)$ is also a uniformly paracompact mapping.

Proof. Let α_M be an arbitrary finitely additive open covering of the space (M,U_M). Let λ be an open family of space (X,U) such that $\lambda\wedge\{M\}=\alpha_M$. It is clear that the family $\alpha=\{\lambda,X\setminus M\}$ is a finitely additive open covering of the space (X,U). Then there exist a finitely additive open covering β of the uniform space (Y,V) and a uniformly locally finite uniform covering $\gamma\in U$ such that $f^{-1}\beta\wedge\gamma\succ\alpha$. Easy to see that $(f|_M)^{-1}\beta\wedge\gamma_M\succ\alpha_M$. Consequently, the mapping $f|_M$ is uniformly paracompact.

Theorem 5. Let $f:(X,U)\to(Y,V)$ be a uniformly paracompact mapping of a uniform space (X,U) onto a uniformly paracompact space (Y,V). Then (X,U) is also uniformly paracompact.

Proof. Let f and (Y,V) are uniformly paracompact. Let α be an arbitrary finitely additive open covering of the uniform space (X,U). Then there exist a finitely additive open cover β of a uniform space (Y,V) and a uniformly locally finite uniform covering $\gamma \in U$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$. In the cover β refines in a uniformly locally finite uniform cover λ. Then, according to Lemmas 1 and 2, the covering $f^{-1}\lambda \wedge \gamma$ is refinement of the uniformly locally finite covering α. Therefore, the uniform space (X,U) is uniformly paracompact.

Proposition 8. The composition of two uniformly paracompact mappings is again a uniformly paracompact mapping.

Proof. Let $f:(X,U) \to (Y,V)$ and $g:(Y,V) \to (Z,W)$ are uniformly paracompact mappings. Let α be any finitely additive open covering of the uniform space (X,U). Then there exist a finitely additive open cover β of the space (Y,V) and a uniformly locally finite cover $\gamma \in U$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$. In turn, for a cover β of a space (Y,V) there exists a finitely additive open cover δ of the space (Z,W) and a uniformly locally finite cover $\lambda \in V$ such that $g^{-1}\delta \wedge \lambda \succ \beta$. Note that $(g \circ f)^{-1}\delta \wedge (f^{-1}\lambda \wedge \gamma) \succ f^{-1}\beta \wedge \gamma \succ \alpha$ i.e. $(g \circ f)^{-1}\delta \wedge \eta \succ \alpha$ and $\eta \in U$ where $\eta = f^{-1}\lambda \wedge \gamma$. According to Lemmas 1 and 2, the covering $\eta \in U$ is a uniformly locally finite covering. Therefore, composition $(g \circ f):(X,U) \to (Z,W)$ is a uniformly paracompact.

Theorem 6. Any uniformly paracompact mapping $f:(X,U) \to (Y,V)$ of a uniform space (X,U) onto a uniform space (Y,V) is a complete.

Proof. Let $f:(X,U) \to (Y,V)$ be a uniformly paracompact mapping. Let F be a Cauchy filter in a uniform space (X,U) such that fF converges to some point (Y,V). Suppose that F does not converge at any point x of the space (X,U). Then for each point $x \in X$ there exists a neighborhood O_x and Φ_x an element of F such that $\Phi_x \cap O_x = \varnothing$. Put $\alpha = \{O_x : x \in X\}$. Since the mapping f is uniformly paracompact, there exists a finitely additive open covering β of the space (Y,V) and a uniformly locally finite uniform cover $\gamma \in U$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$. Since

fF converges to some point $y \in Y$, then for any $B \in \beta$ such that $B \ni y$ $B \in fF$. It follows that $f^{-1}B \in f^{-1}\beta \cap F$. Note that there exists $\Gamma \in \gamma$ such that $\Gamma \in F$. Then $f^{-1}B \cap \Gamma \neq \varnothing$. Since $f^{-1}\beta \wedge \gamma \succ \alpha$, then there is $O = \bigcup_{i=1}^{n} O_{x_i} \in \alpha$ such that $f^{-1}B \cap \Gamma \subset \bigcup_{i=1}^{n} O_{x_i}$. Therefore, $\bigcup_{i=1}^{n} O_{x_i} \in F$ and $\bigcap_{i=1}^{n} \Phi_{x_i} \in F$, i.e. $(\bigcap_{i=1}^{n} \Phi_{x_i}) \cap (\bigcup_{i=1}^{n} O_{x_i}) \neq \varnothing$. Then there exists a number $i_0 \leq n$ such that $\Phi_{x_{i_0}} \cap O_{x_{i_0}} \neq \varnothing$. Contradiction. Therefore, f is a complete.

REFERENCES

1. P.S. Aleksandrov. Introduction to the theory of sets and general topology. Nauka, Moscow, 1977.
2. A.A. Borubaev. Uniform Spaces and Uniformly Continuous Mappings. Frunze: Ilim, 1990.
3. B.E. Kanetov. Some classes of uniform spaces and uniformly continuous mappings, Bishkek, 2013.
4. D.K. Musaev. Uniformly superparacompact, completely paracompact and strongly paracompact uniform spaces // Dokl. RUz. - 2004. - № 4. P. 3–9.
5. V.I. Ponomarev. On paracompact and finally compact spaces // Dokl. USSR Academy of Sciences. - 1961. - T.141, № 3. - P.56–563.
6. H. Corson. The determination of paracompactness by uniformities // Amer. J. Math. – 1958. – Vol. 80. – P. 185–190.
7. C.H. Dowker. On countable paracompact spaces // Canad. J. of Math. – 1951. – Vol. 3. – P. 219–224.
8. J. Isbell. Uniform space. – Providence, 1964. – 175 p.
9. Z. Frolik. On paracompact uniform spaces // Czech. Math. J. – 1983. – Vol. 33. – P. 476–484.
10. B.A. Pasynkov, D. Buhagiar. On uniform paracompactness // Czech. Math. J. – 1996. – V.46 (121). – P. 577– 586.
11. M.D. Rice. A note on uniform paracompactness // Proc. Amer. Math. Soc. – 1977. – Vol. 62, № 2. – P. 359–362.

12. H. Tamano. On paracompactness // Pacific. J. Math. – 1960. – Vol. 10. – P. 1043–1047.

MSC 54 E15, 54 C10

ON THE THEORY OF COMPACTIFICATION OF MAPPINGS

[1]Kanetov B.E., [2]Baidzhuranova A.M., [3]Almazbekova B.A.
[1,3]Jusup Balasagun Kyrgyz National University, [2]Institute of Mathematics of National Academy of Sciences of Kyrgyz republic

In this article we introduces and studies a strongly τ-finally paracompact and a τ-finally superparacompact mappings by means of compactification of mappings. In particular, the conservation of a strongly τ-finally paracompactness (respectively, a τ-finally superparacompactness) under proofs of a strongly τ-finally paracompact (respectively, a τ-finally superparacompact) mappings towards the inverse image is proved.
Key words: Strongly τ-finally paracompact mappings, τ-finally superparacompact mappings, compactification of mappings.

Макалада күчтүү τ-финалдуу паракомпактуу жана τ-финалдуу суперпаракомпактуу чагылдыруулар компактификация аркылуу аныкталат жана изилденилет. Күчтүү τ-финалдуу паракомпактуулук (τ-финалдуу суперпаракомпактуулук) күчтүү τ-финалдуу паракомпактуу (τ-финалдуу суперпаракомпактуулук) чагылдырууда прообраз тарабына сакталуусу далилденет.
Урунттуу сөздөр: Күчтүү τ-финалдуу паракомпактуу чагылдыруу, τ-финалдуу суперпаракомпактуу чагылдыруу, чагылдыруулардын компактификациясы.

В статье вводятся и изучаются сильно τ-финально паракомпактные и τ-финально суперпаракомпактные отображения посредством компактификации отображений. В частности доказываются сохранения сильно τ-финально паракомпактности (соответственно, τ-финально суперпаракомпактности) при сильно τ-финально паракомпактных (соответственно, τ-финально суперпаракомпактных) отображениях в сторону прообраза.
Ключевые слова: Сильно τ-финально паракомпактное отображение, τ-финально суперпаракомпактное отображение, компактификация отображений.

Recently, the theory of continuous mappings has been intensively developing in modern topology. This theory is devoted, first of all, to the extension to mappings of basic concepts and statements concerning topological and uniform spaces. According to the results of research on the theory of continuous mappings, a number of important analogues of spaces and mappings were revealed.

It is known that any topological space can be considered as a special case of continuous mapping, identifying this space with mapping it to a point. The properties of mappings are defined so that no properties are imposed on the spaces

X and Y, i.e. the properties of continuous mappings $f: X \to Y$ are studied independently of the topological properties of the spaces X and Y.

The article introduces and studies a strongly τ - finally paracompact and a τ - finally superparacompact mappings by means of compactification of mappings.

By space is meant a topological space, by mapping a continuous mapping of spaces.

The mapping $bf: \hat{X} \to Y$ is called the compactification of the mapping $f: X \to Y$, if

1. $X \subset \hat{X}$,
2. $[X] = \hat{X}$,
3. bf is perfect mapping [3].

For two compactifications $b_1 f: X_1 \to Y$ and $b_2 f: X_2 \to Y$ of the mapping $f: X \to Y$, $b_2 f \geq b_1 f$ is considered if there exists a mapping $\varphi: X_2 \to X_1$ such that $b_2 f = b_1 f \cdot \varphi$ and on X the mapping φ is identical. It is known that any Tychonoff mapping f of the space X onto the space Y has at least one Tychonoff compactification and that among all Tychonov compactifications of the mapping f there exists its maximal compactification $\beta f: \beta_f X \to Y$ [2].

Recall, that a topological space X is called strongly τ-finally paracompact if every open cover [1] has a locally finite of cardinality $\leq \tau$ open refinement; a topological space X is called τ-finally superparacompact if every open cover has a finite component of cardinality $\leq \tau$ open refinement.

For compactification $bf: \hat{X} \to Y$ of a mapping $f: X \to Y$ with a remainder, we will call the subspace $\hat{X} \setminus X$.

Let $f: X \to Y$ be an arbitrary mapping.

Definition 1. A mapping $f: X \to Y$ is called a strongly τ-finally paracompact if for any closed set $F \subset \hat{X} \setminus X$ there exists a star finite open covering α of space X of cardinality $\leq \tau$ such that $F \cup \cap [\alpha] = \varnothing$.

Proposition 1. Every perfect mapping $f: X \to Y$ be a strongly τ-finally paracompact.

Proof. Let $f: X \to Y$ be a perfect mapping. Then the reminder $\hat{X} \setminus X$ is empty, therefore, for any star finite open cover α of the space X of cardinality $\leq \tau$, the condition $F \cup \cap[\alpha] = \varnothing$ is always satisfied. Consequently, the mapping $f: X \to Y$ is a strongly τ-finally paracompact.

Definition 2. Every perfect mapping $f: X \to Y$ is a τ-finally superparacompact, if for any closed set $F \subset \hat{X} \setminus X$ there exists a finite component open covering α of the cardinality $\leq \tau$ space X such that $F \cup \cap[\alpha] = \varnothing$.

Proposition 2. Every perfect mapping $f: X \to Y$ is a τ-finally superparacompact.

Proof. Let $f: X \to Y$ be a perfect mapping. Then the reminder of $\hat{X} \setminus X$ is empty. Therefore, for any open cover, including for a finite component open cover α of the space X of cardinality $\leq \tau$, the condition $F \cup \cap[\alpha] = \varnothing$ is satisfied. A τ-finally superparacompactness of the mapping f is proved.

Proposition 3. Any a τ-finally superparacompact mapping $f: X \to Y$ is a strongly τ- finally paracompact.

Proof. The proof follows from the fact that every finite component covering is a star finite covering.

It is known that under perfect mappings, many topological properties of spaces pass from the inverse image to the image and vice versa. For mappings, one can obtain similar statements.

Proposition 4. Let Tychonoff mappings $f: X \to Y$ and $g: Y \to Z$ and a perfect mapping $\lambda: X \to Z$ be given, such that $\lambda = g \cdot f$. If the mapping g is a strongly τ-finally paracompact, then the mapping f is also a τ-finally paracompact.

Proof. The proof follows from the existence of an extension $\hat{h}: \beta_f X \to \beta_f Y$ such that $\beta f = (\beta g)\hat{h}$ [2].

Proposition 5. Let Tychonoff mappings $f:X\to Y$ and $g:Y\to Z$ and a perfect mapping $\lambda:X\to Z$ be given, such that $\lambda=g\cdot f$. If the mapping g is a strongly τ-finally superparacompact, then the mapping f is also a τ-finally superparacompact.

Proof. The proof, with slight modifications, is similar to the proof of Proposition 1.

Proposition 6. Let $f:X\to Y$ be a mapping. If the mapping f is a strongly τ-finally paracompact and $Y=\{y\}$ then the space X is a strongly τ-finally paracompact.

Proof. Let f be a strongly τ-finally paracompact, Y - be a one-point space, and $\lambda=\{O_s : s\in S\}$ is an arbitrary open cover of the space X. Then $F=\hat{X}\setminus\cup\alpha$, where $\hat{O}_s=\hat{X}\setminus[X\setminus O_s]_{\hat{X}}$ is a closed set. By the strongly τ-finally paracompactness of the map f there exists a star finite open covering $\alpha=\{A_s : s\in S\}$ of the space X of cardinality $\leq\tau$ such that $F\cup\cap[\alpha]_{\hat{X}}=\varnothing$. Since the mapping $\hat{f}:B_s=[B_s]_{\hat{X}}\to Y$ is perfect, then B_s is compact. In the covering $\hat{\lambda}=\{\hat{O}_s : s\in S\}$ of the set $\hat{A}_s$ we write the final open $\hat{A}_s$ into the covering $\hat{\beta}$. Then the covering $\beta=\hat{\beta}\wedge A_s$ of the set B_s is finite open in the system X inscribed in λ. The system β' of all elements of all such systems β will be a star finite open coverage of cardinality $\leq\tau$, the space X refined in λ. Therefore, space X is a strongly τ-finally paracompact.

Proposition 7. Let $f:X\to Y$ be a mapping. If the mapping f is a τ-finally superparacompact and $Y=\{y\}$ then the space X is also a τ-finally superparacompact.

Proof. Let the condition of the proposition be satisfied and let $\lambda=\{O_z : s\in S\}$ be an arbitrary open covering of space X. Then the set $F=\hat{X}\setminus\cup\alpha$, where $\hat{O}_s=\hat{X}\setminus[X\setminus O_s]_{\hat{X}}$ is a closed set. Since the mapping f is a τ-finally superparacompact, there exists such a finite component open covering $\alpha=\{A_s : s\in S\}$ space X, of cardinality $\leq\tau$, such that $F\cup\cap[\alpha]_{\hat{X}}=\varnothing$. Since the mapping $\hat{f}: B_s = [B_s]_{\hat{X}} \to Y$ is perfect, the compactness of the set B_s follows. In the covering $\hat{\lambda} = \{\hat{O}_s : s \in S\}$ of the set $\hat{A}_s$ we write the final open $\hat{A}_z$ into the covering $\hat{\beta}$. Then the covering $\beta = \hat{\beta} \wedge A_s$ of the set B_s is finite open in the system X refined in λ. The system β' of all elements of all such systems β will be a finite-component open covering of cardinality $\le \tau$, the space X refined in λ. Therefore, space X is a τ-finally superparacompact.

Theorem 1. Let $f: X \to Y$ be a mapping. If space Y and the mapping f is a strongly τ-finally paracompact, then space X also has the same property.

Proof. Let the space Y and the mapping f be a strongly τ-finally paracompact. Let $\alpha = \{A_s : s \in S\}$ be an arbitrary open covering of space X. Put $\hat{A}_s = \hat{X} \setminus [X \setminus A_s]_{\hat{X}}$ and $F = \hat{X} \setminus \cup\alpha$. From a τ-finally paracompactness, mapping f there exists such a star finite open covering $\beta = \{B_s : s \in S\}$ of space X , of cardinality $\le \tau$ that $F \cap \cup[\beta]_{\hat{X}} = \varnothing$. Since the mapping $\hat{f}: B_s = [B_s]_{\hat{X}} \to Y$ is perfect, then B_s is a strongly τ-finally paracompact. In the covering $\hat{\alpha} = \{\hat{A}_s : s \in S\}$ of the set $\overline{B}_s$, we write the locally finite open covering $\hat{\lambda}$ of the cardinality $\le \tau$ in $\overline{B}_s$. Then the covering $\lambda = \lambda \wedge B_s$ of the set B_s is locally finite open in X of cardinality $\le \tau$ system refined in α. The system λ' of all elements of all such systems λ will be a stellar finite open covering of cardinality $\le \tau$, the space X refined in α. Thus, a strongly τ-finally paracompactness X is proved.

Theorem 2. Let $f: X \to Y$ be a mapping. If the space Y is regular, and the space X is a strongly τ-finally paracompact, then the mapping f also has the same property.

Proof. Let space X be a strongly τ-finally paracompact, and Y be regularly. Then the space $\hat{X}$ is regular. Consider a set $F \subset \hat{X} \setminus X$ closed in $\hat{X}$. For each point $x \in X$ there exists a neighborhood O_x such that $[O_x]_{\hat{X}} \cap F = \varnothing$. According to the hypothesis of the theorem, in the cover $\alpha = \{O_x : x \in X\}$ we write a star finite open cover β of cardinality $\leq\tau$ in the space X. It's clear that $F\cap\cup[\beta]_{\hat{X}}=\varnothing$. Therefore, f is a strongly τ-finally paracompact.

Theorem 3. Let $f:X\to Y$ be a mapping. If space Y and the mapping f is a τ-finally superparacompact, then space X also has the same property.

Proof. Let Y, f be a strongly τ-finally superparacompact and an arbitrary open covering $\alpha=\{A_z:s\in S\}$ of space X. Let $\hat{A}_s=\hat{X}[X\setminus A_s]_{\hat{X}}$ and $F=\hat{X}\setminus\cup\alpha$. Then there exists a finite component open covering $\beta=\{B_s:s\in S\}$ of cardinality $\leq\tau$ space X such that $F\cap\cup[\beta]_{\hat{X}}=\varnothing$. Since the mapping $\hat{f}:B_s=[B_s]_{\hat{X}}\to Y$ is perfect, the set B_s is strongly τ-finally superparacompact. Therefore, there is such a covering $\hat{\lambda}$ of cardinality $\leq\tau$ that is refined in the covering $\hat{\alpha}=\{\hat{A}_s:s\in S\}$ of sets $\overline{B}_s$. Then the covering $\lambda=\hat{\lambda}\wedge B_s$ of the set B_s is a finite component open system in X of the cardinality $\leq\tau$ refined in α. The system α' of all elements of all such systems λ will be a finite component open covering of the cardinality $\leq\tau$, the space X refined in α. Thus, a τ-finally the superparacompactness of space X is proved.

Theorem 4. If space Y is regular, and space X is a τ-finally superparacompact, then the mapping f also has the same property.

Proof. Suppose that the hypothesis of the theorem is satisfied and that a set $F\subset\hat{X}\setminus X$ is closed in $\hat{X}$. For each point $x\in X$ there exists a neighborhood O_x such that $[O_x]_{\hat{X}}$. According to the hypothesis of the theorem, a finite component open covering $\alpha=\{O_x:x\in X\}$ of the cardinality $\leq\tau$ of space X can be refined the cover β. It's clear that $F\cap\cup[\beta_{\hat{X}}]=\varnothing$. Therefore, f is a τ-finally superparacompact map.

REFERENCES

1. Borubaev A.A. On uniformly - topological properties. Research on topology and geometry: Sat. scientific. - Frunze, 1985. - P. 18 -27.

2. Pasynkov B. A. On the extension to maps of some concepts and statements concerning spaces // Mappings and functors. - M., 1984. - P. 72 - 102.

3. Whyburn G. T. Compactifications of mappings // Math. Ann. 1966. – Vol. 166, № 1. – P. 168 – 174.

MSC 54 E15, 54 C10

SOME PROPERTIES OF WEIGHT AND PSEUDO-WEIGHT UNIFORMLY CONTINUOUS MAPPINGS

[1]Kanetov B.E., [2]Litviakova K.V.
[1,2]Jusup Balasagun Kyrgyz National University,

In the theory of uniform spaces and uniformly continuous mappings, the most important cardinal invariants are weight, pseudo-weight, and τ - boundedness. In this article, we study some properties of the weight and pseudo-weight of uniformly continuous mappings.
Key words: weight, pseudo-weight, τ - boundedness.

Бир калыптуу мейкиндиктер жана бир калыптуу үзгүлтүксүз чагылдыруулар теориясынын маанилүү кардиналдык инварианттары болуп салмак, псевдосалмак жана τ -чектелгендик саналат. Бул макалада бир калыптуу үзгүлтүксүз чагылдыруулардын салмагынын жана псевдосалмагынын айрым касиеттери изилденет.
Урунттуу сөздөр: Вес, псевдовес, τ - чектелгендик.

В теории равномерных пространств и равномерно непрерывных отображений важнейшими кардинальными инвариантами являются вес, псевдовес и τ -ограниченность. В настоящей работе исследуются некоторые свойства веса и псевдовеса равномерно непрерывных отображений.
Ключевые слова: Вес, псевдовес, τ - ограниченность.

Let (X,U) be a uniform space. The least cardinal number, which is the cardinality of some base of uniformity U, is called the weight of uniformity U and is denoted by $w(U)$ [1].

A system $B \subset U$ is called a pseudo-base of uniform space (X,U) or uniformity U if $\cap\{\beta(x): \beta \in B\} = \{x\}$ for any point $x \in X$ [1].

Let $f:(X,U) \to (Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) onto a uniform space (Y,V). A pseudo-uniformity $U_f \subset U$ is called a base of uniformly continuous mapping f, if for any $\alpha \in U$ there exist $\gamma \in U_f$ and $\beta \in V$ such that $f^{-1}(\beta) \wedge \gamma \succ \alpha$ [1]. The least a cardinal number τ, which is the cardinality of some base U_f, is called the weight of a uniformly continuous mapping f and is denoted by $w(f)$ [2].

A covering system $U_f \subset U$ is called a pseudo-base of a uniformly continuous mapping f if $\cap\{\{f^{-1}\beta \wedge \gamma)(x) : \beta \in V, \gamma \in U_f\} = \{x\}$ for any $x \in X$ [2].

The least cardinal number τ, which is the cardinality of a pseudo-base U_f, is called a pseudo-weight of a uniformly continuous mapping f and is denoted by $pw(f)$ [2].

Proposition 1. Let $f:(X,U) \to (Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) onto a uniform space (Y,V) and a uniformly continuous mapping $g:(Y,V) \to (Z,W)$ of a uniform space (Y,V) onto a uniform space (Z,W). If $w(f) \le \tau$ and $w(g) \le \tau$ then $w(g \circ f) \le \tau$.

Proof. Let $f:(X,U) \to (Y,V)$, $g:(Y,V) \to (Z,W)$ be a uniformly continuous mapping and $w(f) \le \tau$, $w(g) \le \tau$. Let $\alpha \in U$ be an arbitrary uniform covering. Then there exist $\gamma \in U_f$ and $\beta \in V$ such that $f^{-1}(\beta) \wedge \gamma \succ \alpha$. In turn, there are such $\eta \in V_s$ and $\beta \in V$ that $g^{-1}(\mu) \wedge \eta \succ \beta$. Then $f^{-1}(g^{-1}\mu \wedge \eta) \wedge \gamma \succ f^{-1}\beta \wedge \gamma$. It follows that, $(g \circ f)^{-1}\mu \wedge (f^{-1}\eta \wedge \gamma) \succ \alpha$. Put $U_{g \circ f} = \{f^{-1}\eta \wedge \gamma : \eta \in V_s, y \in U_f\}$. Because the $|U_f| = \tau$ and $|V_g| = \tau$, то $|U_{g \circ f}| = \tau$. So, $w(g \circ f) \le \tau$.

Proposition 2. If a uniformly continuous mapping $f:(X,U) \to (Y,V)$ has weight $\le \tau$, then for any uniform subspace (M,U_M) of a uniform space (X,U) and any uniform space (N,V_N) of a uniform space (Y,V) of restriction $f|_M : (M,U_M) \to (fM,V_{fM})$ and $f_N : (f^{-1}N,U_{f^{-1}N}) \to (N,V_N)$ have a weight $\le \tau$.

Proof. Let $\alpha_M \in U_M$ be an arbitrary uniform covering. Then there is a covering $\alpha \in U$ such that $\alpha \wedge \{M\} = \alpha_M$. Since the mapping $f:(X,U) \to (Y,V)$ has weight $\le \tau$, there is uniform cover $\beta \in V$ and uniform cover $\gamma \in U_f$ such that $f^{-1}(\beta) \wedge \gamma \succ \alpha$ where $|U_f| = \tau$. It is easy to see that $f^{-1}|_M \beta_M \wedge \gamma_M \succ \alpha_M$, where $\beta_M = \beta \wedge \{M\}$, $\gamma_M = \gamma \wedge \{M\}$, $\gamma_M \in U_M^f$. It's clear that $|U_M^f| = \tau$. Hence, $w(f|_M) \le \tau$. Further, it is similarly proved that $w(f_N) \le \tau$.

Theorem 1. Let $f=\prod_{a\in M} f_a$ be the Cartesian product, where $f_a:(X_a,U_a)\to(Y_a,V_a)$, $a\in M$, M is a countable set. A $w(f)\le\tau$ if and only if $w(f_a)\le\tau$ for any $a\in M$.

Proof. Necessity. Let the Cartesian product $f=\prod_{a\in M} f_a$ has a weight $\le\tau$, where $f_a:(X_a,U_a)\to(Y_a,V_a)$. Then all mappings $f_a:(X_a,U_a)\to(Y_a,V_a)$ have weight $\le\tau$.

Sufficiency. Let all mappings $f_a:(X_a,U_a)\to(Y_a,V_a)$ have weight $\le\tau$. We show that the product $f=\prod_{a\in M} f_a$ has weight $\le\tau$. Let U_{f_0} has cardinality $\le\tau$ and is the basis for $f_a:(X_a,U_a)\to(Y_a,V_a)$. Then uniformity $U_f=\prod_{a\in M} U_{f_0}$ has cardinality $\le\tau$. We show that $U_f=\prod_{a\in M} U_{f_0}$ is the base of the mapping $f=\prod_{a\in M} f_a$. Let $\alpha\in\prod_{a\in M} U_a$. It is clear that coverings $\overset{n}{\underset{i=1}{\wedge}}\pi_{a_i}^{-1}\alpha_{a_i}$, $\alpha_{a_i}\in U_{a_i}$, $i=1,2,\ldots,n$ form the base of uniformity $\prod_{a\in M} U_a$, so we can assume that $\alpha=\overset{n}{\underset{i=1}{\wedge}}\pi_{a_i}^{-1}\alpha_{a_i}$. Since U_{f_0} has cardinality τ and is the base of the mapping $f_a:(X_a,U_a)\to(Y_a,V_a)$, then there is a uniform covering β_{a_0} of V_{a_0} and a uniform covering γ_{a_0} of U_{f_0} such that $f_{a_0}^{-1}(\beta_{a_0})\wedge\gamma_{a_0}\succ\alpha_{a_0}$, $i=1,2,..n$. Put $\gamma=\overset{n}{\underset{i=1}{\wedge}}\pi_{a_i}^{-1}\gamma_{a_i}$, $\beta=\overset{n}{\underset{i=1}{\wedge}}P_{a_i}^{-1}\beta_{a_i}$, $\pi_{a_i}:\prod_{a\in M} X_a\to X_{a_i}$, $P_{a_i}:\prod_{a\in M} Y_a\to Y_{a_i}$. By definition of uniformity $\beta\in\prod_{a\in M} V_a$ and $\gamma\in\prod_{a\in M} U_{f_0}$. Therefore, $f^{-1}(\beta)\wedge\gamma\succ\alpha$. So the product $f=\prod_{a\in M} f_a$ has weight $\le\tau$.

Proposition 3. Let $\{f_a\}_{a\in M}$ be a system of uniformly continuous mappings $f_a:(X,U)\overset{на}{\to}(Y_a,V_a)$. If there exists $a_0\in M$ such that the mapping $f_{a_i}:(X,U)\overset{на}{\to}(Y_{a_i},V_{a_i})$ has weight $\le\tau$, then the diagonal product $f=\Delta\{f_a:a\in M\}$ has weight $\le\tau$.

Proof. Let the mapping $f_{a_i}:(X,U)\overset{на}{\rightarrow}(Y_{a_i},V_{a_i})$ has weight $\leq\tau$ and $\pi_{a_i}:\prod_{a\in M}(Y_a,V_a)\rightarrow(Y_{a_i},V_{a_i})$ is the projection. Then $f_{a_i}=\pi_{a_i}\circ f$, where $f=\underset{a\in M}{\Delta} f_a$. Therefore, the mapping f has weight $\leq\tau$.

Theorem 2. For a uniformly continuous mapping, the following conditions are equivalent:

1) $f:(X,U)\rightarrow(Y,V)$ has weight $\leq\tau$.

2) The completion $\hat{f}:(\hat{X},\hat{U})\rightarrow(Y,V)$ has weight $\leq\tau$.

Proof. 1) $\Rightarrow$2). Let a uniformly continuous mapping $f:(X,U)\rightarrow(Y,V)$ has weight $\leq\tau$ and $\hat{\alpha}\in\hat{U}$ is an arbitrary uniform cover. Put $\alpha=\hat{\alpha}\wedge\{X\}$. Due to the fact that the mapping $f:(X,U)\rightarrow(Y,V)$ has weight $\leq\tau$, there is a uniform covering $\beta\in V$ and uniform covering $\gamma\in U_f$ such that $f^{-1}(\beta)\wedge\gamma\succ\alpha$. Then there is such a uniform cover $\hat{\gamma}\in\hat{U}$ that $\gamma=\hat{\gamma}\wedge\{X\}$. Easy to see that $\hat{f}^{-1}(\beta)\wedge\hat{\gamma}\succ\hat{\alpha}$. Therefore, $w(\hat{f})\leq\tau$.

2)$\Rightarrow$1). Obviously.

Theorem 3. Every uniformly continuous mapping $f:(X,U)\rightarrow(Y,V)$ of a uniform space (X,U) onto a uniform space (Y,V) has a unique completion $\hat{f}:(\hat{X},\hat{U})\rightarrow(Y,V)$ up to a uniform isomorphism and $w(\hat{f})=w(f)$.

Proof. The proof follows from Theorem 2 and Proposition 2.3.17 [see 1., P. 104].

Corollary 1. Every uniform space (X,U) has a unique completion $(\tilde{X},\tilde{U})$ up to a uniform isomorphism and $w(\tilde{U})=w(U)$.

Theorem 4. Let $f:(X,U)\rightarrow(Y,V)$ be a uniformly continuous mapping. If $pw(X)\leq\tau$, then $pw(f)\leq\tau$. Conversely, if $pw(f)\leq\tau$ and $pw(V)\leq\tau$ then $pw(U)\leq\tau$.

Proof. Let $f:(X,U)\rightarrow(Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) onto a uniform space (Y,V) and $pw(U)\leq\tau$. Let $\alpha\in U$ be an arbitrary uniform covering. Then, due to the fact that space (X,U) has weight $pw(U)\leq\tau$, then there is a uniform covering $\gamma\in B$ refined in $\alpha\in U$. Due to the uniform continuity of the mapping $f:(X,U)\to(Y,V)$, the inverse image $f^{-1}\beta$ of any uniform covering $\beta\in V$ is a uniform covering and $f^{-1}(\beta)\wedge\gamma\succ\alpha$. Put $B_f=\{f^{-1}\beta\wedge\gamma:\beta\in V,\gamma\in B\}$. It's clear that $|B_f|=\tau$. Therefore, $pw(f)\le\tau$. The converse, let $pw(f)\le\tau$ and $pw(V)\le\tau$. Let $\alpha\in U$ be an arbitrary uniform covering. Then there are such uniform cover $\beta\in V$ and uniform cover $\gamma\in U$ such that $f^{-1}(\beta)\wedge\gamma\succ\alpha$. Since, $pw(V)\le\tau$, then there is $\beta_0\in V$ such that $\beta_0\succ\beta$. Therefore, a uniform cover $f^{-1}\beta_0\wedge\gamma$ will be refined in a uniform cover $\alpha\in U$. So, the family of cover of the form $B_x=\{f^{-1}\lambda\wedge\gamma\}$ where $\lambda\in B_Y$, $\gamma\in U_f$ forms the base of space (X,U). Since both $|B_Y|=\tau$ and $|U_f|=\tau$, then $|B_x|=\tau$. Hence, $pw(U)\le\tau$.

Proposition 4. If (Y,V) is a one-point uniform space and $pw(f)\le\tau$. Then $pw(U)\le\tau$.

Proof. Let $\cap\{(f^{-1}\beta\wedge\gamma)(x):\beta\in V,\gamma\in B\}=\{x\}$ be for any $x\in X$. Because $Y=\{y\}$, that $f^{-1}(\beta)\wedge\gamma\succ\gamma$, where $|B|=\tau$. Therefore, $\cap\{\gamma(x):\gamma\in B\}=\{x\}$ for any $x\in X$, i.e. $pw(U)\le\tau$.

REFERENCES

1. Borubaev A.A. Uniform spaces and uniformly continuous mappings. - Frunze: Ilim, 1990, - 176p.

2. Kasymova T.D. Cand. diss. On complete and uniformly perfect mappings and their inverse spectra. - Bishkek, 1998.

MSC 54 E15, 54 C10

ON INDEX BOUNDED REMAINDERS AND COMPLETENESS OF UNIFORM SPACES

[1]**Kanetov B.E.**, [2]**Joldubaev M.M.**, [3]**Sultanmamytova A.N.**
[1,2,3]Jusup Balasagun Kyrgyz National University

In this work we study index bounded remainders and completeness of uniform spaces.
Key words**:** Remainder, co-covering, co-τ -bounded, sequentially complete.

Илимий макалада бир калыптуу мейкиндиктердин өсүндүлөрүнүн чектелгендик индекси жана бир калыптуу мейкиндиктердин толуктугу изилденет.
Урунттуу сөздөр: Өсүндү, ко-жабдуу, ко-τ -чектелгендик, секвенциалдуу толуктуулук.

В настоящей работе исследуются индекс ограниченности наростов равномерных пространств и полнота равномерных пространств.
Ключевые слова: Нарост, ко-покрытие, ко-τ -ограниченность, секвенциальная полнота.

It is interesting to compare properties of the uniform spaces (X,U) and its remainders $(\widetilde{X}\setminus X,\widetilde{U}_{\widetilde{X}\setminus X})$. Naturally the problem arises of characterizing properties of the space $(\widetilde{X}\setminus X,\widetilde{U}_{\widetilde{X}\setminus X})$ by properties of the dual space (X,U).

The least cardinal number τ is said to be an index of boundedness of the uniform space (X,U) is the uniformity U has a base consisting of coverings of cardinality $\leq\tau$. A uniform space (X,U) is called totally bounded, if for each $\alpha\in U$ there exists a finite set $M\subset X$ such that $\alpha(M)=X$. A uniform space (X,U) is called τ-bounded, if for each $\alpha\in U$ there exists a set $M\subset X$, $|M|\leq\tau$ such that $\alpha(M)=X$ [1]. A uniform space (X,U) is called P-compact if the uniformity U has a base consisting of coverings with property P [1]. Property P can be: a finite uniform covering, a uniform covering of cardinality $\leq\tau$, a locally finite uniform covering, a uniform covering of multiplicity $\leq n$, a point wise finite uniform covering, a star finite uniform covering. It is said that the dimension of the uniform space (X,U) is not greater than n and it is written as $\dim U\leq n$ if the uniformity U has a base consisting of coverings of multiplicity $\leq n+1$[1]. A uniform space is called uniformly u-paracompact if every of its uniformly cover has a locally finite uniform refinement [2]. A uniform space is called strongly uniformly u-paracompact if every of its uniformly cover has a star finite uniformly refinement [2].

A Cauchy filter F in (X,U) is called free if $\cap\{[N]_X : N\in F\}=\varnothing$. A family α of the space (X,U) is called co-covering of the space (X,U) if $\alpha\cap F\neq\varnothing$, for any free Cauchy filter F in (X,U) [2].

For coverings α and β of the set X, the symbol $\alpha \succ \beta$ means that the covering α is a refinement of the covering β, i.e. for any $A \in \alpha$ there exists $B \in \beta$ such that $A \subset B$.

Let (X,U) be a uniform space.

Definition 1. A uniform space (X,U) is called co-τ-bounded, if for any $\alpha \in U$ there exists a family μ of cardinality $\leq \tau$ of free Cauchy filters such that the subfamily $\alpha_0 = \{A : A \in \alpha \cap F, F \in \mu\}$ is a co-covering of (X,U).

Any τ-bounded space (X,U) is a co-τ-totally bounded. Indeed, let $\alpha \in U$ be an arbitrary uniform covering of the space (X,U). Then $\tilde{\alpha} = \{\tilde{A} : A \in \alpha\}$, where $\tilde{A} = \tilde{X} \setminus [X \setminus A]_{\tilde{X}}$ is a uniform covering of the completion of $(\tilde{X},\tilde{U})$ and the family $\hat{\alpha} = \{\hat{A} : A \in \alpha\}$ where $\hat{A} = \tilde{A} \cap \tilde{X} \setminus X$ is a uniform covering of the remainder $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$. Since the space (X,U) is co-τ-bounded, the cover α contains a subcover α_0 of cardinality $\leq \tau$. Then $\hat{\alpha}_0 = \{\hat{A} : A \in \alpha_0\}$ is a uniform covering of cardinality $\leq \tau$ of the space $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$. Selecting one element $\hat{x}_{\hat{A}}$ from each $\hat{A}$, we have a set $\{\hat{x}_i : i = 1,2,\ldots,n\}$ of cardinality $\leq \tau$. Let $\tilde{B}(\hat{x}_{\hat{A}})$ be a filter of neighborhoods of points of $\hat{x}_{\hat{A}}$ in $(\tilde{X},\tilde{U})$. Put $F_{\hat{x}_{\hat{A}}} = \tilde{B}(\hat{x}_{\hat{A}}) \wedge \{X\}$. Then every element of the family $\{F_{\hat{x}_{\hat{A}}} : A \in \alpha_0\}$ is a free Cauchy filter. Since α_0 is a uniform covering, then the family $\{A : A \in \alpha \cap F_{\hat{x}_{\hat{A}}}, A \in \alpha_0\}$ is a co-covering of the space (X,U). Thus, the space (X,U) is a co-τ-bounded.

Theorem 1. The remainder $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$ of a space (X,U) is τ-bounded if and only if the space (X,U) is co-τ-bounded.

Proof. Necessity. Let the remainder $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$ of space (X,U) is τ-bounded and α is an arbitrary uniform covering of space (X,U). Then $\tilde{\alpha} = \{\tilde{A} : A \in \alpha\}$, where $\tilde{A} = \tilde{X} \setminus [X \setminus A]_{\tilde{X}}$ is a uniform covering of space $(\tilde{X},\tilde{U})$. It follows that the family $\hat{\alpha} = \{\hat{A} : A \in \alpha\}$, where $\hat{A} = \tilde{A} \cap \tilde{X} \setminus X$ is a uniform covering of the $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$. Then there exists a set M of cardinality $\leq \tau$ such that $\hat{\alpha}(M) = \tilde{X} \setminus X$. Let $\tilde{B}(\hat{x})$ be a filter of neighborhoods of points $\hat{x}$ in $(\tilde{X},\tilde{U})$. Then for trace $F_{\hat{x}}$ filter of the neighborhood $\tilde{B}(\hat{x})$ of point $\hat{x}$ is a free Cauchy filter in (X,U). Put $\alpha_0 = \{A : A \in \alpha \cap F\}$. We show that α_0 is a co-covering of the space (X,U). Let F be an arbitrary free Cauchy filter in (X,U). Then it converges to some point $\hat{x} \in \tilde{X} \setminus X$. Since $\hat{\alpha}(M) = \cup\{\hat{\alpha}(\hat{x}) : \hat{x} \in M\} = \tilde{X} \setminus X$, then $\hat{x} \in \hat{\alpha}(\hat{x}')$ for some $\hat{x}' \in M$, i.e. exists $\hat{A} \in \hat{\alpha}$ such that $\hat{A} \ni \hat{x}'$ in $\hat{A} \ni \hat{x}$. Let $\tilde{B}(\hat{x})$ be a filter of neighborhoods of point $\hat{x}$ in $(\tilde{X},\tilde{U})$. Notice, that $F \supset \hat{B}(\hat{x}') \wedge \{X\}$. Easy to see that $A \in \alpha_0 \cap F$. Therefore, the space (X,U) is a co-τ-bounded.

Sufficiency. Let $\hat{\alpha}$ be an arbitrary uniform covering of the space $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$. Then there exists a uniform covering α of space (X,U) such that $\tilde{\alpha} \wedge \{\tilde{X} \setminus X\} = \hat{\alpha}$ where $\tilde{\alpha} = \{\tilde{A} : A \in \alpha\}$, $\tilde{A} = \tilde{X} \setminus [X \setminus A]_{\tilde{X}}$. According to the hypothesis of the theorem there exists a family $\{F\}$ of cardinality $\leq \tau$ of free Cauchy filters such that the subfamily $\alpha_0 = \{A : A \in \alpha \cap F, F \in \{F\}\}$ is a co-covering of the space (X,U). Every free Cauchy filter F converges to some point $\hat{x}_F \in \tilde{X} \setminus X$. Then $M = \{\hat{x}_F : F \in \{F\}\}$ is a subset of cardinality $\leq \tau$ of space $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$. Since the subfamily α_0 is a co-covering of the space (X,U), the family $\hat{\alpha}_0$ is covering of the space $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$. Show that $\hat{\alpha}_0(M) = \tilde{X} \setminus X$. Let $\hat{x} \in \tilde{X} \setminus X$ be an arbitrary point. By $\tilde{B}(\hat{x})$ we denote filter of neighborhoods of the point $\hat{x}$ in $(\tilde{X},\tilde{U})$. The trace F filter of neighborhoods $\tilde{B}(\hat{x})$ on X is free Cauchy filter in (X,U). Therefore, $\alpha_0 \cap F \neq \varnothing$, i.e. there is $A \in \alpha_0$ such that $A \in F$. By hypothesis there exists $F' \in \{F\}$ such that $A \in F'$. Since F' converges to the point $\hat{x}_{F'}$, then $\hat{x} \in \hat{A} \subset \hat{\alpha}_0(\hat{x}_{F'}) \subset \hat{\alpha}_0(M)$.

Therefore, that $\hat{\alpha}_0(M) = \tilde{X} \setminus X$. Therefore, the space $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$ is a τ-bounded.

If τ is a finite cardinal, then a co-τ-bounded space of (X,U) is called co-totally bounded.

Theorem 2. The remainder $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ of a space (X,U) is compact if and only if the space (X,U) is co-totally bounded and open subspace in the completion of $(\tilde{X},\tilde{U})$.

Proof. Necessity. Let the remainder $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ be compact, i.e. totally bounded and complete. According to the Theorem 1, the space (X,U) is a co-totally bounded. Since any complete subspace of a uniform space is closed, then the remainder $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ is closed. Then the uniform space (X,U) is an open subspace in the completion of $(\tilde{X},\tilde{U})$.

Sufficiency. Let (X,U) be co-totally bounded and open. According to the Theorem 1, the remainder $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ is a totally bounded. Since the space (X,U) is an open subspace of the completion of $(\tilde{X},\tilde{U})$, then the remainder $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ is closed. Further, from the fact that every closed subspace of a complete space is complete, the completeness of the remainder of $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ follows. Therefore, the remainder $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ is a compactum.

Theorem 3. For the remainder $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ of the space (X,U) is a P-compact sufficiency if every uniform covering $\alpha\in U$ contains a subfamily α_0 with property P which is a co-cover of uniform space (X,U).

Proof. Let $\hat{\alpha}$ be an arbitrary uniform covering of the space $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$. Then there exists a uniform covering α of the space (X,U) such that $\tilde{\alpha}\wedge\{\tilde{X}\setminus X\}=\hat{\alpha}$, where $\tilde{\alpha}=\{\tilde{A}:A\in\alpha\}$, $\tilde{A}=\tilde{X}\setminus[X\setminus A]_{\tilde{X}}$. According to the hypothesis of theorem α it contains a subfamily α_0 with property P, which is co-covering of the uniform space (X,U). It is easy to see that the covering $\hat{\alpha}_0$ has property P. Therefore, the space $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ is a P-compact.

Among the many corollaries of Theorem 3 we note the following.

Corollary 1. A remainder $(\tilde{X}\setminus X,\tilde{U}_{\tilde{X}\setminus X})$ of a space (X,U) is $\dim\tilde{U}_{\tilde{X}\setminus X}\leq n$ if each uniform covering $\alpha \in U$ contains subfamily α_0 of a multiplicity $\leq n+1$, that is a co-covering of the uniform space (X,U).

Corollary 2. A remainder $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$ of a space (X,U) is uniformly u-paracompact if each uniform covering $\alpha \in U$ contains locally finite subfamily α_0, that is a co-covering of the uniform space (X,U).

Corollary 3. A remainder $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X}\setminus X})$ of a space (X,U) is strongly uniformly u-paracompact if and only if each uniform covering $\alpha \in U$ contains star finite subfamily α_0, that is a co-covering of the uniform space (X,U).

Filter F called is Cauchy H-filter, if $\alpha \cap F \neq \varnothing$ every $\alpha \in H$ [1].

Uniform space (X,U) called is H-sequentially complete, if every Cauchy H-filter F of countable base has at least one adherent point.

The least cardinal number η called index of sequentially of the uniform space (X,U), if there exists such system $H \subset U$, that $|H| = \eta$ and (X,U) is a H-sequentially complete.

An index of sequentially completeness of a uniform space (X,U) is denoted as $ic_{\aleph_0}(U)$.

Proposition 1. Let (X,U) be a uniform space. If exist covering $\alpha \in U$ such that every subspace (A,U_A), $A \in \alpha$ of (X,U) is sequentially complete, then the space (X,U) is sequentially complete and $ic_{\aleph_0}(U) \leq \max\{\sup\{ic_{\aleph_0}(U_A) : A \in \alpha\}, |\alpha|\}$.

Proposition 2. Let $(X,U) = \prod_{s \in S}(X_s, U_s)$. Then we have

$$ic_{\aleph_0}(U) \leq \max\{\sup\{ic_{\aleph_0}(U_s) : s \in S\}, |S|\}.$$

Theorem 4. For a uniform space (X,U) the following conditions are equivalent:

1. $ic_{\aleph_0}(U) \leq \eta$;
2. Uniform space (X,U) can be mapped onto any sequentially complete (Y,V) of weight $w(V) \leq \eta$ by means of sequentially complete mappings.

REFERENCES

1. A.A. Borubaev. Uniform Spaces and Uniformly Continuous Mappings. Ilim, Frunze, 1990.

2. B.E. Kanetov. Some classes of uniform spaces and uniformly continuous mappings. Bishkek, 2013.

MSC 54 E15, 54 C10

ON ONE UNIFORM ANALOGUE A τ-FINALLY PARACOMPACT SPACES

[1]**Kanetov B.E.**, [2]**Baigazieva N.A.**, [3]**Taalaibekova A.T.**
[1,2,3]*Jusup Balasagun Kyrgyz National University*

As is known, in the theory of uniform spaces there are various approaches to the definition of uniform paracompactness of uniform spaces. In the article a uniformly τ-finally paracompact spaces are introduced and investigated.

Key words: Finitely additive open cover, uniformly τ-finally paracompactness, conservative uniform cover of cardinality $\leq \tau$.

Бир калыптуу мейкиндиктердин теориясында бир калыптуу паракомпактуулуктун түрдүү берилүү жолдору бар экендиги белгилүү. Макалада бир калыптуу τ-финалдуу паракомпактуу мейкиндиктер киргизилет жана изилденет.

Урунттуу сөздөр: Чектүү аддитивдүү ачык жабдуу, бир калыптуу τ-финалдуу паракомпактуулук, $\leq \tau$ кубаттуулуктагы консервативдүү бир калыптуу жабдуу.

Как известно, в теории равномерных пространств существуют различные подходы к определению равномерной паракомпактности равномерных пространств. В настоящей работе вводится и исследуются равномерно τ-финально паракомпактные пространства.

Ключевые слова: Конечно аддитивное открытое покрытие, равномерно τ-финальная паракомпактность, консервативное равномерное покрытие мощности $\leq \tau$.

A τ-finally paracompact spaces of topological spaces were introduced and studied in [1].

Let (X,U) be a uniform space.

A uniform space (X,U) is called a uniformly τ-finally paracompact, if every finitely additive open cover has a conservative uniform cover of cardinality $\leq \tau$ refinement.

Proposition 1. If (X,U) is a uniformly τ-finally paracompact, then the Tychonoff space (X,τ_U) is a τ-finally paracompact. Conversely, if (X,τ) is a paracompact, then the uniform space (X,U_X) is a uniformly τ-finally paracompact, where U_X is the universal uniformity.

Proof. Let γ be an arbitrary open covering of space (X,τ_U). In the finite additive open covering $\gamma^{\angle}$ of uniform space (X,U), we refined a conservative uniform covering β of cardinality $\leq\tau$. For each $B\in\beta$, there is such $\Gamma_B\in\gamma^{\angle}$ that $B\subset\Gamma_B$, where $\Gamma_B=\bigcup_{i=1}^{n}\Gamma_i$, $\Gamma_i\in\alpha$, $i=1,2,...,n.$. Let $\gamma'=\{\gamma_B : B\in\beta\}$, $\gamma_B=\{B\cap\Gamma_i : i=1,2,...,n\}$. It is easy to see that γ' is a conservative covering of the space (X,τ_U) of cardinality $\leq\tau$ refined in an open cover γ. Therefore, the space (X,τ_U) is a τ-finally paracompact.

Conversely, let a Tychonoff space (X,τ) be a τ- finally paracompact and γ is an arbitrary finitely additive open covering of space (X,U_X). By virtue of the τ-finally paracompactness of the space (X,τ), a conservative open covering β of cardinality $\leq\tau$ can be refined into the covering γ. As is known, the family of all open coverings of a given Tychonoff space (X,τ) forms the base of the universal uniformity of U_X. Then β is a conservative uniform cover of cardinality $\leq\tau$.

The following theorem is a uniform analogue of the in the spirit of Tamano's theorem.

Theorem 1. Let (X,U) be the uniform space, (sX,sU) is the Samuel's compactification of the space (X,U). The uniform space (X,U) is a uniformly τ-finally paracompact if and only if for any compactum $K\subset sX\setminus X$ there is a conservative uniform covering γ of cardinality $\leq\tau$ such that $K\cap[\Gamma]_{sX}=\varnothing$ for any $\Gamma\in\gamma$.

Proof. Necessity. Let (X,U) be a uniformly τ-finally paracompact and $K\subset sX\setminus X$. For each point $x\in X$ there exists a neighborhood O_x open in sX such that $[O_x]_{sX}\cap K=\varnothing$. We denote by β the traces of the neighborhood of O_x, the point $x \in X$ on X. It is easy to see that β is an open covering of the space (X,U). In the finite additive open covering $\beta^{\angle}$ we write a conservative uniform covering γ of cardinality $\le \tau$. Then the set $[\Gamma]_{sX}$ is contained in $[(\bigcup_{i=1}^{n} U_{X_i} \cap X)]_{sX}$, and the latter is contained in $\bigcup_{i=1}^{n}[O_{x_i}]_{sX}$. Where $K \cap [\Gamma]_{sX} = \varnothing$ for any $\Gamma \in \gamma$.

Sufficiency. Let λ be any finitely additive open covering of the space (X,U). Then there exists an open family γ in (sX, sU) such that the trace of the covering γ on X coincides with the covering λ. Let K be the complement of the set $\cup\gamma$ to Samuel's compactification (sX, sU). The K is compactum. Then there exists a conservative uniform covering β of cardinality $\le \tau$, such that the set $K \cap [B]_{sX} = \varnothing$ for any $B \in \beta$. Since $[B]_{sX}$ is contained in sX, then there exist $\Gamma_i, i = 1,2,...,n$ such sets from γ that $[B]_{sX} \subset \bigcup_{i=1}^{n}\Gamma_i$. Then $B \subset \bigcup_{i=1}^{n} L_i$, where $\bigcup_{i=1}^{n} L_i \in \lambda$. Therefore, the space (X,U) is a uniformly τ-finally paracompact.

A uniform space (X,U) is called a strongly uniformly τ-locally compact if there is a conservative uniform covering of cardinality $\le \tau$ consisting of compact subsets.

Proposition 2. Any strongly uniformly τ-locally compact space is a uniformly τ-finally paracompact.

Proof. Let λ be an arbitrary finitely additive open covering. Then there exists a conservative uniform covering β of cardinality $\le \tau$ consisting of compact subsets. It is easy to see that a conservative uniform covering β of cardinality $\le \tau$ is refined in a finite additive open covering λ.

Proposition 3. Any uniformly τ-finally paracompact space is complete.

Proof. Let (X,U) be a uniformly τ-finally paracompact space and F is a Cauchy filter in it. Suppose that the Cauchy filter F does not converge at any point in the space (X,U). Then for each point $x \in X$ there is a neighborhood of O_x and N_x from F such that $N_x \cap O_x = \varnothing$. By λ we denote the families of such neighborhoods O_x of a point $x \in X$. Then, for a finite additive open covering $\lambda^{\angle}$

there is a conservative uniform covering β of cardinality $\leq \tau$ refined in it. It follows that the cover $\lambda^{\angle}$ and the Cauchy filter F have a common element i.e. there exist O_{x_i} from $i=1,2,...,n$ such that the set $\overset{n}{\underset{i=1}{\cup}} O_{x_i}$ is contained in $\lambda^{\angle} \cap F$. Then the intersection of the sets $\overset{m}{\underset{i=1}{\cap}} F_{x_i}$ and $\overset{n}{\underset{i=1}{\cup}} O_{x_i}$ are not empty. Consequently, $F_{x_{i_s}} \in F$ and $O_{x_{i_s}} \in F$.

Contradictions. There fore, the uniform space (X,U) is a uniformly τ-finally paracompact.

Proposition 4. A closed subspace H is a uniformly τ-finally paracompact space (X,U) is a uniformly τ-finally paracompact.

Proof. Let γ_H be a finitely additive open cover of a closed subspace of H. Put $\gamma = \{\gamma_H, X \setminus H\}$. It is clear that γ is a finite additive open covering. Then there exists a conservative uniform covering α of cardinality $\leq \tau$ refined in γ. Put $\alpha_H = \alpha \wedge \{H\}$. Then it is easy to see that α_H is a conservative uniform covering of cardinality $\leq \tau$ of the subspace H refined in γ.

Theorem 2. Let ω be an arbitrary finitely additive open covering. If $f:(X,U) \to (Y,V)$ is a uniformly continuous ω-mapping of a uniform space (X,U) onto a uniformly τ-finally paracompact space (Y,V), then a uniform space (X,U) is a uniformly τ-finally paracompact.

Proof. Let ω be an arbitrary finitely additive open covering of the space (X,U) and $f:(X,U) \to (Y,V)$ is ω the mapping of the uniform space (X,U) onto a uniformly τ-finally paracompact space (Y,V). Then for each point $y \in Y$ there exists a neighborhood O_y such that $f^{-1}O_y$ is contained in some element of the cover ω. By α we denote the set of all such neighborhoods of O_y, points of $y \in Y$. Put $\alpha^{\angle} = \{\cup \alpha_0 \subset \alpha - \text{finite}\}$. Into the covering $\alpha^{\angle}$ we write a conservative uniform covering β of cardinality $\leq \tau$. Then $f^{-1}\beta$ is a conservative uniform covering of cardinality $\leq \tau$ of space (X,U) refined in ω. Therefore, the uniform space (X,U) is a uniformly τ-finally paracompact.

Since any uniformly perfect mapping is a τ-mapping, then Theorem 2 implies:

Corollary 1. The uniform τ-finally paracompactness is preserved towards the inverse image of uniform perfect mappings.

Theorem 3. The product of uniformly τ-finally paracompact space (X,U) onto a compact uniform space (Y,V) is a uniformly τ-finally paracompact.

Proof. Let (X,U) be a uniformly τ- finally paracompact space, and (Y,V) is a compact uniform space. Let $\lambda=\{S_{(x,y)}\times T_{(x,y)}:(x,y)\in X\times Y)$ be an arbitrary finitely additive open covering of the space $(X,U)\times(Y,V)$. For each point $(x,y)\in X\times Y$ of the set $S_{(x,y)}$, $T_{(x,y)}$ there are neighborhoods of the points x and y, respectively. Let $x'\in X$. Then the family $\{T_{(x',y)}:y\in Y\}$ forms an open covering of the space (Y,V). Due to the compactness of the latter, we select a finite subcover $\{T_{(x',y_1)},T_{(x',y_2)},...,T_{(x',y_n\}}\}$ from an open covering $\{T_{(x',y)}:y\in Y\}$. Let $S_{x'}=\bigcap_{i=1}^{n}T_{(x',y_i)}$. Then the family $\eta=\{T_{x'}:x'\in X\}$ is a finitely additive open covering, into which, by virtue of the uniform τ-finally paracompactness of the space (X,U), we can refined a conservative uniform covering $\mu=\{T_{x'}:x'\in X\}$ of cardinality $\leq\tau$. Put $\beta=\{S_{x'}\times T_{(x',y_i)}:x'\in X, i=1,2,...,n\}$. The latter is a uniform covering, since covering μ and $\{T_{(x',y_1)},T_{(x',y_2)},...,T_{(x',y_n\}}\}$ are uniform. Easy to see that $\beta\succ\lambda$. It remains to show that β is a conservative covering of cardinality $\leq\tau$. Since μ is a conservative covering of cardinality $\leq\tau$ in place (X,U), then $[(\bigcup_{j\in J_0}S_{x'_j})]=\bigcup_{j\in J_0}[S_{x'_j}]$ is for each $J_0\subset J$. Therefore $[(\bigcup_{j\in J_0}(S_{x'_j}\times T_{(x'_j,y_i)})]=\bigcup_{j\in J_0}[(S_{x'_j}\times T_{(x'_j,y_i)})]$ for each $J_0\subset J$ and $i=1,2,...,n$.

A uniform space (X,U) is called a strongly uniformly τ-finally paracompact if each finitely additive open cover has a conservative uniform cover of cardinality $\leq\tau$ refinement.

Any strongly uniformly τ-finally paracompact space is a uniformly τ-finally paracompact.

Theorem 4. For a uniform space (X,U), the following conditions are equivalent:

1) The uniform space (X,U) is a strongly uniform τ-finally paracompact;
2) The uniform space (X,U) is a uniformly τ-finally paracompact and the topological space (X,τ_U) is a strongly τ- finally paracompact.

Proof. 1) $\Rightarrow$2) It is obvious.

2) $\Rightarrow$1). Let λ be any finitely additive open covering of a uniform space (X,U). The cover λ has a star finite open cover β of cardinality $\leq\tau$ refinement. It is easy to see that the cover $\beta^{\angle}=\{\cup\beta_0:\beta_0\subset\beta-\text{ finite}\}$ is of finitely additive and star finite covering. By the hypothesis of the theorem, we refined in a covering $\beta^{\angle}$ a conservative uniform covering γ of cardinality $\leq\tau$. It is easy to see that the star-finite uniform covering $\beta^{\angle}$ is refined in the finitely additive open covering λ. Therefore, the uniform space (X,U) is a strongly uniformly τ-finally paracompact.

Corollary 2. Any uniformly τ-finally paracompact space (X,U) whose topological space is locally compact, is a strongly uniformly τ-finally paracompact.

Theorem 5. A locally compact uniform space (X,U) is a uniformly τ-finally paracompact if and only if the uniform space (X,U) is a strongly uniformly τ-locally compact.

Proof. Necessity. Let a uniform space (X,U) be uniformly τ-finally paracompact and a topological space (X,τ_U) is a locally compact. Then for every point x there exists a neighborhood O_x whose closure $[O_x]$ is compact. It is easy to see that the family λ consisting of the neighborhoods O_x of the point $x\in X$ is an open covering of the space (X,U). Put $\lambda^{\angle}=\{\cup\lambda_0:\lambda_0\subset\lambda-\text{ finite}\}$. Conservative uniform covering β of cardinality $\leq\tau$ we refined in $\lambda^{\angle}$. For each $B\in\beta$ there exists a set $\bigcup_{i=1}^{n}O_{x_i}$ containing B. Closure $[B]$ is contained in $[(\bigcup_{i=1}^{n}U_{x_i})]$. The set $[B]$ is compactum, since $[(\bigcup_{i=1}^{n}U_{x_i})]=\bigcup_{i=1}^{n}[U_{x_i}]$. So, the closed covering $[\beta]=\{[B]:B\in\beta\}$ is a conservative uniform covering of cardinality $\leq \tau$ consisting of compact subsets. Therefore, (X,U) is a strongly uniformly locally compact.

Sufficiency. Let α be an arbitrary conservative uniform covering consisting of compact subsets of cardinality $\leq \tau$ and λ is an arbitrary finite additive open covering. It is easy to see that α is refined in cover λ. Therefore, the uniform space (X,U) is a uniformly τ-finally paracompact.

REFERENCES

1. Borubaev A.A. On uniformly-topological properties. Research on topology and geometry: Sat. scientific. - Frunze, 1985. - P. 18 -27.

MSC 34K06, 35G05

ON BOUNDEDNESS OF SOLUTIONS OF THIRD ORDER LINEAR DIFFERENTIAL EQUATIONS WITH FUNCTIONAL ON HALF-AXIS

Iskandarov Samandar
Institute of Mathematics of NAS of Kyrgyz Republic

Established the sufficient conditions of boundedness on half-axis all solutions and their first, second derivatives of third-order linear differential equations with functional. For this aim developed the non-standard method of reduce of equations to system. The illustrative example is given.

Key words: linear differential equation of third order, functional, non-standard method of reduce to system, boundedness.

Функционалы бар үчүнчү тартиптеги сызыктуу дифференциалдык теңдеменин бардык чыгарылыштарынын жана алардын биринчи, экинчи туундарыларынын жарым октогу чектелгендигинин жетиштүү шарттары табылат. Бул үчүн теңдемелерди системага стандарттык эмес келтирүү методу өнүктүрүлөт. Иллюстравдик мисал тургузулат.

Ачкыч сөздөр: Үчүнчү тартипетеги сызыктуу дифференциалдык теңдеме, функционал, теңдемени системага стандарттык эмес келтирүү методу, чектелгендик.

Устанавливаются достаточные признаки ограниченности на полуоси всех решений и их первых, вторых производных линейного дифференциального уравнения третьего порядка с функционалом. Для этого развивается нестандартный метод сведения к системе. Строится иллюстративный пример.

Ключевые слова: Линейное дифференциальное уравнение третьего порядка, функционал, нестандартный метод сведения к системе, ограниченность.

All functions and their derivatives in this paper are continuous and the relations are true at $t \geq t_0; J = [t_0, \infty)$; DE-differential equation; DEF-DE with functional.

The problem. To establish sufficient conditions of boundedness on J all solutions and their first, second derivatives of third-order DEF:

$$x'''(t) + a_2(t)x''(t) + a_1(t)x'(t) + a_0(t)x(t) = f(t) +$$
$$+F(t; x), \qquad t \geq t_0, \qquad (1)$$

where $F(t; x)$ is continuous at $t \geq t_0$ functional is described in [1, p.280].

The main result. In DEF (1) we made the following non-standard replacement:

$$x'''(t) + p(t)x'(t) + q(t)x(t) = W(t)y(t), \qquad (2)$$

where $p(t), q(t), W(t)$ are some weighting functions, y(t) is new unknown function. Then from (2) by differentiations we have:

$$x'''(t) = -p(t)x''(t) - p'(t)x'(t) - q(t)x'(t) - q'(t)x(t) + W(t)y'(t) + W'(t)y(t) =$$
$$= -p(t)[-p(t)x'(t) - q(t)x(t) + W(t)y(t)] - [p'(t) + q(t)]x'(t) -$$
$$-q'(t)x(t) + W(t)y'(t) + W'(t)y(t) = [p^2(t) - p'(t) - q(t)]x'(t) +[$$
$$p(t)q(t) - q'(t)]x(t)$$
$$+[W'(t) - p(t)W(t)]y(t) + W(t)y'(t). \qquad (3)$$

Now by using (2), (3) we reduce the DEF (1) to following equivalent system:

$$\begin{cases} x''(t) + p(t)x'(t) + q(t)x(t) = W(t)y(t), \\ y'(t) + b_2(t)y(t) + b_1(t)x'(t) + b_0(t)x(t) = W(t)^{-1}[f(t) + \\ +F(t; x), t \geq t_0, \end{cases} \qquad (4)$$

where

$$b_2(t) \equiv a_2(t) - p(t) + W'(t)\left(W(t)\right)^{-1},$$

$$b_1(t) \equiv [a_1(t) - a_2(t)p(t) + p^2(t) - p'(t) - q(t)]\left(W(t)\right)^{-1},$$

$$b_0(t) \equiv [a_0(t) - a_2(t)q(t) + p(t)q(t) - q'(t)]\left(W(t)\right)^{-1}.$$

The replacement (2) under $p(t) \equiv const > 0, q(t) = const > 0$ is used in paper [2].

For any solution $(x(t), y(t))$ the first of equation from system (4) multiplied

$x'(t)$, second equation - to $y(t)$, after integrated on $[t_0,t]$, added the obtained expressions. Then can be obtained the following energetic identity:

$$V(t) \equiv \left(x'(t)\right)^2 + 2\int_{t_0}^{t} p(s)\left(x'(s)\right)^2 ds + q(t)\left(x(t)\right)^2 + \left(y(t)\right)^2 +$$
$$2\int_{t_0}^{t} b_2(s)(y(s))^2 ds \equiv V(t_0) + \int_{t_0}^{t}\Big\{q'(s)\left(x(s)\right)^2 + 2W(s)y(s)x'(s) +$$
$$2\left(W(s)\right)^{-1} y(s)[f(s)+F(s;x)] -$$
$$2y(s)[b_1(s)x'(s) + b_0(s)x(s)]\Big\} ds. \qquad (5)$$

By using the integral inequality [4] is provided the following

Theorem. Let 1) $p(t) \geq 0, W(t) > 0;$

2) $q(t) > 0, q'(t) \geq 0;$ 3) $b_2(t) \geq 0;$

4) $W(t) + \left(W(s)\right)^{-1}[|f(s)| + |F(s;x)|] + |b_1(s)| + +|b_0(s)|\left(q(s)\right)^{-\frac{1}{2}} \epsilon$ $L^1(J, R_+ \backslash \{0\}).$

Then any solution $x(t)$ of DEF (1) is bounded on J and is true the following properties:

$$(x'(t))^2 = q(t)O(1),$$
$$p(t)\left(x'(t)\right)^2 \epsilon L^1(J, R_+),$$
$$b_2(t)\left(y(t)\right)^2 \epsilon L^1(J, R_+),$$

In addition, let 5) $q(t) = O(1),$ then $\forall x'(t) = O(1);$

6) $p(t) = O(1), q(t) = O(1), |F(t;x)| = O(1),$ then $\forall\, x''(t) = O(1).$

Example. For DEF:

$$x'''(t) + \left[e^{t\sqrt[3]{\sin(t)}} + \frac{2t+3}{t+2}\right]x''(t) + [a_2(t)p(t) - e^{-2t}]x'(t)$$
$$+ \left[a_2(t)q(t) + q'(t) - p(t)q(t) + \frac{e^{-2t}}{t^2+1}\right]x(t)$$
$$= \sin e^{-3t} -$$
$$-\frac{e^{-2t}}{1 + \left|\int_0^t x(s)ds\right|}, \qquad t \geq 1,$$

where $p(t) \equiv \frac{t+1}{t+2}, q(t) \equiv p^2(t) - \frac{1}{(t+2)^2},$ all conditions of theorem are satisfied under $W(t) \equiv e^{-t},$ in here

$t_0 = 1, b_2(t) \equiv e^{t\sqrt[3]{\sin t}}, b_1(t) \equiv -e^{-t}, b_0(t) \equiv \frac{1}{t^2+1}, \ f(t) \equiv \sin e^{-3t}$,

$|F(t;x)| \le e^{-2t}$.

We denote, that the result of this theorem is new in case $F(t;x) \equiv 0$ (see, for example, [5]).

REFERENCES

1. El'sgol'ts L.E. Differntial equations and variation calculus. – Moscow: Nauka, 1969. – 424 p (in Russian).
2. Iskandarov S. On new variant of non-standard method of the reduce to system for linear Volterra third-order integro-differential equation // Investigations on integro-differential equations.- Bishkek: Ilim, 2007. – Issue 37. – P.24-29 (in Russian).
3. Iskandarov S. The method of weighting and cutting functions and asymptotical properties of Volterra type integro-differntial and integral solutions. – Bishkek: Ilim, 2002.-216 p. (in Russian).
4. Ved' Ju A., Pahyrov Z. The sufficient conditions of boundedness of solutions of linear integro-differential equations // Investigations on integro-differentail equations in Kirgizii. Frunze: Ilim, 1973. – Issue 9. – P.68-103.
5. Kiguradze I.T., Chanturiya T.A. Asymptotic properties of solutions of nonautonous ordinary differential equations. –Moscow: Nauka, 1990.-432 p. (in Russian).

MSC 45A05, 45B 05

UNIQUENESS AND STABILITY OF SOLUTIONS OF STIELTJES LINEAR INTEGRAL EQUATIONS OF THE FIRST KIND WITH TWO VARIABLES

Avyt Asanov[1], Zuurakan A.Kadenova[2]

[1]*Department of Mathematics , Kyrgyz-Turkish Manas University,*

[2]*Institute of Mathematics of NAS of the Kyrgyz Republic*

In this work dedicated to the research of the problem of uniqueness and stability for linear integral equationss of the first kind with two variables. Here the operator generated by the kernels is not the compact operator.

Key words and phrases: linear, inteqral equations, first kind, two variables, solution, uniqueness.

Бул макала биринчи түрдөгү эки өзгөрүлмөлүү сызыктуу интегралдык теңдемелердин чечимдеринин жалгыздыгын жана туруктуулугун изилдөөгө арналган. Мында, ядрону аныктоочу оператор компактуу оператор.

Урунттуу сөздөр: сызыктуу интегралдык теңдеме, биринчи түрдөгү, эки өзгөрүлмөлүү, жалгыздыгы.

Данная работа посвящена исследованию единственности и устойчивости решений линейных интегральных уравнений первого рода с двумя переменными. Здесь оператор, генерируемый ядрами, не является компактным оператором.

Ключевые слова: линейные, интегральные уравнения, первого рода, с двумя независимыми переменными, единственность.

1. Introduction

We consider the integral equation

$$Ku = f(t,x), (t,x) \in G = \{(t,x) \in R^2 : t_0 \le t \le T, a \le x \le b\}, \tag{1}$$

where

$$Ku \equiv \int_a^b K(t,x,y)u(t,y)d\varphi(y) + \int_{t_0}^t Q(t,x,s)u(s,x)d\psi(s) +$$

$$+\int_{t_0}^T \int_a^b C(t,x,s,y)u(s,y)d\varphi(y)d\psi(s),$$

$K(t,x,y), H(t,x,s), C(t,x,s,y)-$ are given functions, $\varphi(x)$ is the given strictly increasing continuous function in $[a,b], \psi(t)$ is the given strictly increasing continuous function in $[t_0,T], u(t,x)$ and $f(t,x)$ are the desired and given functions respectively, $(t,x) \in G.$

Here

$$K(t,x,y) = \begin{cases} A(t,x,y), t_0 \le t \le T, a \le y \le x \le b, \\ B(t,x,y), t_0 \le t \le T, a \le x \le y \le b, \end{cases}$$

$$P(t,x,y) = A(t,x,y) + B(t,y,x), (t,x,y) \in G_1, \tag{2}$$

where

$$G_1 = \{(t,x,y): t_0 \le t \le T, \ a \le y \le x \le b\};$$
$$G_2 = \{(t,x,y): t_0 \le t \le T, \ a \le x \le y \le b\};$$
$$G_3 = \{(t,x,s): t_0 \le s \le t \le T, \ a \le x \le b\}, \quad G^2 = G \times G.$$

Various issues concerning of integral equations of the first kind were studied in [1-6]. Some practical and theoretical investigations were made in paper [1] for nonclassical Volterra integral equations of the first kind. In [2, 3] for system of Volterra integral equations of the first kind were constructed the Volterra regularized operators. In [4] for the systems of Volterra integral equations of the first kind with two independent variables were investigated the problems of regularization and uniqueness. More specifically, fundamental results for Fredholm integral equations of the first kind were obtained in [5], where regularizing operators in the sense of M.M.Lavrent'ev vere constructed for solutions of linear Fredholm integral equations of the first kind.

In this work for the investigation of the integral equation (1) we it is based on the notion of the derivative of function with respect to the strictly increasing function [7]. Then we need the concept of the derivative defined in the work [7]. Apparently the first notion of the derivative, with respect to the strictly increasing function, was introduced in [7].

Definition. The derivative of a function $f(x)$ with respect to $\varphi(x)$ is the function $f_{\varphi}^{'}(x)$ whose value at $x \in (a,b)$ is the number:

$$f_{\varphi}^{'}(x) = \lim_{\Delta \to 0} \frac{f(x+\Delta) - f(x)}{\varphi(x+\Delta) - \varphi(x)}, \qquad (3)$$

where $\varphi(x)$ is the given strictly increasing continuous function in (a,b).

If the limit in equation (3) exists, we say that $f(x)$ has a derivative (is differentiable) with respect to $\varphi(x)$. The first derivative $f_{\varphi}^{'}(x)$ may also be differentiable function with respect to $\varphi(x)$ at every point $x \in (a,b)$. If so, its derivative

$$f_{\varphi}^{''}(x) = (f_{\varphi}^{'}(x))_{\varphi}^{'},$$

is called the second derivative of $f(x)$ with respect to $\varphi(x)$. The names continue as you imagine they would, with.

$$f_{\varphi}^{(n)}(x) = (f_{\varphi}^{(n-1)}(x))_{\varphi}^{'}$$

denoting the $n-th$ derivative of $f(x)$ with respect to $\varphi(x)$.

Similarly, we define partial derivatives of functions of many variables.

In this work, on the basis of the consepts of the partial derivative of functions $P(t,x,y), H(t,x,s)$ with respect to the increasing continuous functions $\varphi(x)$ and $\psi(t)$ and methods of functional analysis, the uniqueness theorems are proved and estimates of the stability of solutions for the integral equation (1) are obtained.

We asumethat only if

$$u(t,x)\in L^2_{\varphi,\psi}(G), \quad \int_{t_0}^{T}\int_{a}^{b}\left|u(t,x)\right|^2 d\varphi(x)d\psi(t)<\infty.$$

2. Uniqueness and Stability of solutions of integral equations

Assume that the following conditions are satisfied:

$$(i).P(t,b,a)\geq 0 \quad t\in[t_0,T], P(t,b,a)\in C[t_0,T],\ P'_{\varphi(y)}(t,y,a)\leq 0,$$
$$(t,y)\in G, P'_{\varphi(z)}(t,z,a)\in C(G),\ P'_{\varphi(z)}(s,b,z)\geq 0,\ (s,z)\in G,$$

$$P'_{\varphi(z)}(s,b,z)\in C(G),\ P''_{\varphi(z)\varphi(y)}(s,y,z)\leq 0,\ (s,y,z)\in G_1,$$
$$P''_{\varphi(z)\varphi(y)}(s,y,z)\in C(G_1).$$

$$(ii).Q(T,y,t_0)\geq 0,\ y\in[a,b],\ Q(T,y,t_0)\in C[a,b],\ Q'_{\varphi(s)}(s,y,t_0)\leq 0$$
$$(s,y)\in G, Q'_{\varphi(s)}(s,y,t_0)\in C(G), Q'_{\psi(\tau)}(T,y,\tau)\geq 0 \quad (y,\tau)\in G,$$

$$Q'_{\psi(\tau)}(T,y,\tau)\in C(G),\ Q''_{\psi(\tau)\psi(s)}(s,y,\tau)\leq 0,$$
$$(s,y,\tau)\in G_2, Q''_{\psi(\tau)\psi(s)}(s,y,\tau)\in C(G_2).$$

(iii). At least one of the following conditions holds:

$$(a)P'_{\varphi(y)}(s,y,a)<0 \ \textit{for almost all} \ (s,y)\in G;$$
$$(b)P'_{\varphi(z)}(s,b,z)>0 \ \textit{for almost all} \ (s,z)\in G;$$

$$(c)Q'_{\psi(s)}(s,y,t_0)<0 \ \textit{for almost all} \ (s,y)\in G;$$
$$(d)Q'_{\psi(\tau)}(T,y,\tau)>0 \ \textit{for almost all} \ (y,\tau)\in G;$$

$(e)\ P''_{\varphi(z)\varphi(y)}(s,y,z) < 0$ *for almost all* $(s,y,z) \in G_1$;

$(f)\ Q''_{\psi(\tau)\psi(s)}(s,y,\tau) < 0$ *for almost all* $(s,y,\tau) \in G_2$.

For $v(t,x) \in L^2_{\phi,\psi}(G),\ \int_a^x A(t,x,y)v(t,y)d\phi(y),\ \int_x^b B(t,x,y)v(t,y)d\phi(y) \in L^2_{\phi,\psi}(G),$

here

$C[t_0,T],\ C(G), C(G_1)$ and $C(G_3)$-the space of all continuous functions, respectively $[t_0,T],\ G,\ G_1$ and G_3.

(iv). $C(t,x,s,y) \in L_2(G^2)$ and

$$C(t,x,s,y) = \sum_{i=1}^{m} \lambda_i \varphi_i(t,x)\varphi_i(s,y),\ m \le \infty,\ 0 \le \lambda_i,\ i = 1,2,\ldots,m \tag{4}$$

where $|\lambda_1| \ge |\lambda_2| \ge \ldots$ and $\varphi_1(t,x),\ \varphi_2(t,x),\ldots$ is an orthonormal sequence of eigen functions from $L^2_{\phi,\psi}(G)$ and $\lambda_1, \lambda_2, \ldots$ is the sequence of corresponding nonzero eigenvalues of the Fredholm integral operator C generated by the kernel $C(t,x,s,y)$, with the elements $\lambda_1, \lambda_2, \ldots$ arranged in decreasing order of their absolute values.

Theorem 1. Let conditions (i)-(iv) be satisfied. Then the solution of the equation (1) is unique in $L^2_{\varphi,\psi}(G)$.

Proof. Taking the multiplication of both sides of the equation (1) with u(t, x), integrating the results on G, and integrating by parts and using the Dirichlet formula we obtain

$$\int_a^b\int_{t_0}^T\int_a^y A(s,y,z)u(s,z)u(s,y)d\psi(z)d\psi(s)d\varphi(y)+$$

$$+\int_a^b\int_{t_0}^T\int_a^y B(s,y,z)u(s,z)u(s,y)d\psi(z)d\psi(s)d\varphi(y)+$$

$$+\int_a^b\int_{t_0}^T\int_{t_0}^s Q(s,y,\tau)u(\tau,z)u(s,y)d\psi(\tau)d\psi(s)d\varphi(y)+$$

$$+\int_a^b\int_{t_0}^T\int_{t_0}^s\int_a^b C(s,y,\tau,z)u(\tau,z)u(s,y)d\varphi(z)d\psi(\tau)d\psi(s)d\varphi(y)=$$

$$=\int_a^b\int_{t_0}^T f(s,y)u(s,y)d\psi(s)d\varphi(y)$$

$$\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)u(s,z)u(s,y)d\varphi(z)d\varphi(y)d\psi(s)+$$

$$+\int_{t_0}^T\int_a^b\int_{t_0}^s Q(s,y,\tau)u(\tau,y)u(s,y)d\psi(\tau)d\varphi(y)d\psi(s)+$$

$$+\int_a^b\int_{t_0}^T\int_{t_0}^s\int_a^b C(s,y,\tau,z)u(\tau,z)u(s,y)d\varphi(z)d\psi(\tau)d\varphi(s)d\varphi(y)=$$

$$=\int_a^b\int_{t_0}^T f(s,y)u(s,y)d\psi(s)d\varphi(y). \tag{5}$$

$$\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)u(s,z)u(s,y)d\phi(z)d\phi(y)d\psi(s)=$$

$$=-\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)\frac{\partial}{\partial z}\left(\int_z^y u(s,\nu)d\phi(\nu)\right)dz\;u(s,y)d\phi(y)d\psi(s)=$$

$$=-\int_{t_0}^T\int_a^b\left\{P(s,y,z)\left(\int_z^y u(s,\nu)d\phi(\nu)\right)\Bigg|_a^y-\right.$$

$$\left.-\int_a^y P'_{\phi(z)}(s,y,z)\left(\int_z^y u(s,\nu)d\phi(\nu)\right)d\phi(z)\right\}u(s,y)d\phi(y)d\psi(s)=$$

$$=\frac{1}{2}\int_{t_0}^T\int_a^b P(s,y,a)\left[\frac{\partial}{\partial y}\left(\int_a^y u(s,\nu)d\phi(\nu)\right)^2\right]d\phi(y)d\psi(s)+$$

$$+\frac{1}{2}\int_{t_0}^T\int_a^b\int_z^b P'_{\phi(z)}(s,y,z)\left[\frac{\partial}{\partial y}\left(\int_z^y u(s,\nu)d\phi(\nu)\right)^2\right]d\phi(y)d\phi(z)d\psi(s)=$$

$$+\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}\left[P'_{\varphi(z)}(s,y,z)\left(\int_{z}^{y}u(s,\nu)d\varphi(\nu)\right)^2\right]\Big|_z^b d\varphi(z)d\psi(s)-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}\int_{z}^{b}P''_{\varphi(z)\varphi(y)}(s,y,z)\left(\int_{z}^{y}u(s,\nu)d\varphi(\nu)\right)^2\varphi(y)d\varphi(z)d\psi(s)=$$

$$=\frac{1}{2}\int_{t_0}^{T}P(s,b,a)\left(\int_{a}^{b}u(s,\nu)d\phi(\nu)\right)^2 d\psi(s)-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}P'_{\phi(y)}(s,y,a)\left(\int_{a}^{y}u(s,\nu)d\phi(\nu)\right)^2 d\phi(y)d\psi(s)+$$

$$+\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}P'_{\varphi(z)}(s,b,z)\left(\int_{z}^{b}u(s,\nu)d\varphi(\nu)\right)^2 d\varphi(z)d\psi(s)-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}\int_{a}^{y}P''_{\varphi(z)\varphi(y)}(s,y,z)\left(\int_{z}^{y}u(s,\nu)d\varphi(\nu)\right)^2 d\varphi(z)d\varphi(y)d\psi(s). \qquad (6)$$

Similarly integrating by parts and using the Dirichlet formula analogically we have

$$\int_{a}^{b}\int_{t_0}^{T}\int_{t_0}^{s}Q(s,y,\tau)u(\tau,y)u(s,y)d\psi(\tau)d\psi(s)d\varphi(y)=$$

$$=\frac{1}{2}\int_{a}^{b}Q(T,y,t_0)\left(\int_{t_0}^{T}u(\xi,y)d\psi(\xi)\right)^2 d\phi(y)-$$

$$-\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}Q'_{\psi(s)}(s,y,t_0)\left(\int_{t_0}^{s}u(\xi,y)d\psi(\xi)\right)^2 d\psi(s)d\phi(y)+$$

$$+\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}Q'_{\psi(\tau)}(T,y,\tau)\left(\int_{\tau}^{T}u(\xi,y)d\psi(\xi)\right)^2 d\psi(\tau)d\phi(y)-$$

$$-\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}\int_{t_0}^{s}Q''_{\psi(\tau)\psi(s)}(s,y,\tau)\left(\int_{\tau}^{s}u(\xi,y)d\psi(\xi)\right)^2 d\psi(\tau)d\psi(s)d\phi(y). \qquad (7)$$

Taking into account (6), (7) and (4) from (5) we obtain

$$\frac{1}{2}\int_{t_0}^{T} P(s,b,a)\left(\int_a^b u(s,\nu)d\phi(\nu)\right)^2 d\psi(s)-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_a^b P'_{\phi(y)}(s,y,a)\left(\int_a^y u(s,\nu)d\phi(\nu)\right)^2 d\phi(y)d\psi(s)+$$

$$+\frac{1}{2}\int_{t_0}^{T}\int_a^b P'_{\varphi(z)}(s,b,z)\left(\int_z^b u(s,\nu)d\varphi(\nu)\right)^2 d\varphi(z)d\psi(s)-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_a^b\int_a^y P''_{\varphi(z)\varphi(y)}(s,y,z)\left(\int_z^y u(s,\nu)d\varphi(\nu)\right)^2 d\varphi(z)d\varphi(y)d\psi(s)+$$

$$+\frac{1}{2}\int_a^b Q(T,y,t_0)\left(\int_{t_0}^{T} u(\xi,y)d\psi(\xi)\right)^2 d\phi(y)-$$

$$-\frac{1}{2}\int_a^b\int_{t_0}^{T} Q'_{\psi(s)}(s,y,t_0)\left(\int_{t_0}^{s} u(\xi,y)d\psi(\xi)\right)^2 d\psi(s)d\phi(y)+$$

$$+\frac{1}{2}\int_a^b\int_{t_0}^{T} Q'_{\psi(\tau)}(T,y,\tau)\left(\int_{\tau}^{T} u(\xi,y)d\psi(\xi)\right)^2 d\psi(\tau)d\varphi(y)-$$

$$-\frac{1}{2}\int_a^b\int_{t_0}^{T}\int_{t_0}^{s} Q''_{\psi(\tau)\psi(s)}(s,y,\tau)\left(\int_{\tau}^{s} u(\xi,y)d\psi(\xi)\right)^2 d\psi(\tau)d\psi(s)d\varphi(y)+$$

$$+\sum_{i=1}^{m}\lambda_i\left(\int_a^b\int_{t_0}^{T}\varphi_i(s,y)u(s,y)d\psi(s)d\varphi(y)\right)^2 = \int_a^b\int_{t_0}^{T} f(s,y)u(s,y)d\psi(s)d\varphi(y). \qquad (8)$$

Let $f(t,x)\equiv 0,\ (t,x)\in G.$ Then by virtue of conditions (i)-(iv), from (8) we have $u(t,x)=0,$ for almost all $(t,x)\in[t_0,T]\times[a,b]$. The theorem 1 is proved.

v).The Fredholm operator C generated by the kernel $C(t,x,s,y),$ defined by (4) is positive, i.e. all the eigenvalues $\{\lambda_i\}$ of $C(t,x,s,y),$ are positive (i = 1, 2, ..., m, m = ∞) and $\varphi(x)\in C^1[a,b],\ \psi(t)\in C^1[t_0,T]$.

The family of well-posedness depending on the parameter α is defined as

$$M_\alpha=\left\{u(t,x)\in L_2(G):\sum_{\nu=1}^{\infty}\lambda_i^{-\alpha}\left|u^{(\nu)}\right|^2\le c\right\},$$

were $c>0,\quad 0<\alpha<\infty,$

$$u^{(\nu)}=\int_{t_0}^{T}\int_{a}^{b}u(t,x)\varphi_\nu(t,x)\varphi'(x)\psi'(t)\,dxdt,\ \nu=1,2,...,\infty.$$

Theorem 2. Let conditions (i)-(ii)and (v) be satisfied. Then the solution $u(t,x)$of the equation (1) is unuque in $L^2_{\varphi,\psi}(G)$. Moreover, on the set $K(M_\alpha)\subset L_2(G)$ is the image of M_α under the action of the operator K defined by formula (1)), the inverse $K^{-1}$of operator K is uniformly continuous with the Holder exponent $\frac{\alpha}{2+\alpha}$, i.e.

$$\|u(t,x)\|_{L_2}\le c^{\frac{1}{2+\alpha}}\|f(t,x)\|_{L_2}^{\frac{\alpha}{2+\alpha}},\ 0<\alpha<\infty. \tag{9}$$

were

$$\|u(t,x)\|_{L_2}=\int_{t_0}^{T}\int_{a}^{b}\|u(t,x)\|^2\varphi'(x)\psi'(t)\,dxdt.$$

Proof. a) In this case, the orthonormal sequence of eigenfunctions $u(t,x)\in M_\alpha$ is complete in $L^2_{\varphi,\psi}(G)$.There fore (8) implies the uniqueness of the solution to equation (1) in $L^2_{\varphi,\psi}(G)$. Let $f(t,x)\in K(M_\alpha)$. Then the equation (1) has a solution $u(t,x)\in M_\alpha$ and it follows from (8) that

$$\sum_{\nu=1}^{\infty}\lambda_\nu\left|u^{(\nu)}\right|^2\le\|f(t,x)\|_{L_2}\|u(t,x)\|_{L_2}. \tag{10}$$

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|^2\le\left[\sum_{\nu=1}^{\infty}\frac{\left|u^{(\nu)}\right|^2}{\lambda_\nu^{-1}}\right]^{\frac{\alpha}{2+\alpha}}\cdot\left[\sum_{\nu=1}^{\infty}\lambda_\nu^{-\alpha}\left|u^{(\nu)}\right|^2\right]^{\frac{1}{\alpha+1}}, \tag{11}$$

On the other hand,

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|^2\le\left[\|f(t,x)\|\|u(t,x)\|\right]^{\frac{\alpha}{1+\alpha}}c^{\frac{1}{1+\alpha}} \tag{12}$$

Combining the last two inequalities gives estimate (9). The theorem 2 is proved.

REFERENCES

1. A.S. Aparstyn, Nonclassical linear Volterra Equations of the First Kind. VSP, Utrecht, p.168, (2003).
2. A.Asanov, Regularization, Uniqueness and Existence of Solutions of Volterra Equations of the First Kind. VSP, Utrecht, p.272. (1998).
3. M.I.Imanaliev and A.Asanov, On solutions of systems of Volterra nonlinear integral equations of the first kind, Dokl. Akad. Nauk SSSR, 309(5), 1052-1055, (1989).
4.M.I.Imanaliev and A.Asanov, On solutions of systems of nonlinear two-dimentional Volterra integral equations of the first kind, Dokl. Akad. Nauk SSSR, 317(1), (1991).
5.M.M.Lavrentiev, On-integral equations of the first kind, Dokl. Akad. Nauk SSSR, 127(1), 31-33, (1959).
6.M.I.Imanaliev, A.Asanov and Z.A.Kadenova, A Class of linear integral equations of the first kind with two independent variables, Doklady Mathematucs , 89(1), 98-102 (2014).
7.A.Asanov, The derivative of a function by means of a increasing function, Fen Bilimleri Dergisi, Kyrgyz-Turkish Manas University, 1, 18-64 (2001).

MSC: 34M60, 34E10, 34A12

ASYMPTOTIC ANALYSIS OF SOLUTIONS OF SYSTEMS OF THREE SINGULARALLY PERTURBED FIRST-ORDER EQUATIONS

[1]Alybaev K.S., [2]Narymbetov T.K.
[1]Jalal-Abad State University, [2]Medical and Social Research Institute

The article considers a system of three singularly perturbed first-order equations. A degenerate system of this system has several solutions. For solutions of a singularly perturbed system satisfying given initial conditions, the existence of attraction regions in the complex plane to individual solutions of a degenerate system is proved.
Keywords: singular perturbation, degenerate system, analytic function, level lines, sequential approximation, uniform convergence.

Макалада биринчи тартиптеги үч сингулярдык дүүлүккөн теңдемеден турган система каралган. Системага тиешелеш болгон кубулган система бир нече чечимге ээ болот. Баштапкы шарттарды шарттарды канааттандырган сингулярдык дүүлүккөн системанын чечиминин

комплекстик тегиздикте кубулган системанын айрым чечимдерине тартылуу областарынын жашашы далилденди.

Урунттуу сөздөр: сингулярдык дүүлүгүү, кубулган система, аналитикалык функция, деңгээл сызыктары, удаалаш жакындаштыруу, бир калыпта жыйналуу.

В статье рассматривается система из трех сингулярно возмущенных уравнении первого порядка. Вырожденная система данной системы имеет несколько решений. Для решений сингулярно возмущенной системы удовлетворяющие заданным начальным условиям, доказано существование областей притяжений в комплексной плоскости к отдельным решениям вырожденной системы.

Ключевые слова: сингулярное возмущение, вырожденная система, аналитическая функция, линии уровня, последовательное приближение, равномерная сходимость.

Formulation of the problem

Let the system be considered

$$\varepsilon z'(t,\varepsilon) = A(t)z(t,\varepsilon) + z^2(t,\varepsilon) + \varepsilon f\big(t, z(t,\varepsilon)\big) \quad (1)$$

with initial condition $z(0,\varepsilon) = z^0(\varepsilon)$ (2)

where t∈ $\Pi \subset C$ is set of complex numbers,

$\Pi = \big\{t \in C \,\big|\, 0 \le t_1 \le \alpha, -\beta \le t_1 \le \beta,\ \alpha, \beta \in R\big\}$,

$z(t,\varepsilon) = colon\big(z_1(t,\varepsilon), z_2(t,\varepsilon), z_3(t,\varepsilon)\big), A(t) = diag\big(a_1(t), a_2(t), a_2(t)\big)$,

$z^2(t,\varepsilon) = colon\big(z_1^2(t,\varepsilon), z_2^2(t,\varepsilon), z_3^2(t,\varepsilon)\big)$,

$f(t,\varepsilon) = colon\big(f_1(t,\varepsilon), f_2(t,\varepsilon), f_3(t,\varepsilon)\big), z^0(\varepsilon) = colon\big(z_{10}(\varepsilon), z_{20}(\varepsilon), z_{30}(\varepsilon)\big)$.

From system (1), flat ε = 0, we obtain a degenerate (unperturbed) system

$$A(t)\xi(t) + \xi^2(t) = 0. \quad (3)$$

The system (3) has eight solutions

$\xi_1(t) = colon(0;0;0),\ \xi_2(t) = colon(0; -a_2(t); 0)$,

$\xi_3(t) = colon(-a_1(t); 0; 0), \xi_4(t) = colon(-a_1(t); -a_2(t); 0)$,

$$\xi_5(t) = colon\big(0; 0; -a_3(t)\big),\ \xi_6(t) = colon\big(0; -a_2(t); -a_3(t)\big), \quad (4)$$

$\xi_7(t) = colon\big(-a_1(t); 0; -a_3(t)\big), \xi_7(t) = colon\big(-a_1(t); -a_2(t); -a_3(t)\big)$.

Definition. If: 1. There exists a domain $\Pi_0 \subseteq \Pi$ and $z(t,\varepsilon)$ is a solution to problem (1) - (2) defined in Π_0.

2. ∀t∈ Π_0 (z (t, ε) → $\xi_j(t)$ in ε), then the domain Π_0 is called the attraction domain of the solution $\xi_j(t)$.

For solutions (4), we prove the existence of attraction regions.

The solution of the problem

To solve this problem, we use the level line method [1].

Similar problems were investigated in [2,3,4].

Let the conditions be satisfied:

У1. $\forall t \in \Pi\big(\alpha_j(t) \in Q(\Pi)\ and\ Im\alpha_j(t) > 0, j = 1,2,3\big), Q(\Pi)$ is space of analytic functions in Π.

У2. $f(t,z) \in Q(D)$ и $\forall((t,\tilde{z}),(t,\tilde{\tilde{z}}) \in \Delta(\|f(t,\tilde{z}) - f(t,\tilde{\tilde{z}})\| \le M_1\|\tilde{z} - \tilde{\tilde{z}}\|,$ where Λ is a certain set of variables (t, z). Further, all constants will be denoted by the letters $M_k(k = 1,2,\ldots)$.

We construct the functions $A_j(t) = \int_0^t \alpha_j(\tau)d\tau\ (j = 1,2,3)$ and introduce the level lines $(p_{j1}) = \{t \in \Pi \mid ReA_j(t) = 0\}, j = 1,2,3.$

У3. The level lines (p_{j1}) in domain Π have only one common point $t = 0$.

According to this condition, the level lines in the domain Π arranged in a certain order. We assume (clockwise) that (p_{11}), comes first, then (p_{21}) and then (p_{31}).

The lines (p_{j1}) the domain Π are divided into four parts, which we denote by Π_k (k = 1,2,3,4).

1) For $t \in \Pi_1$, we assume $\|z_0\| \le M_0\varepsilon$. (5)

2) If $t \in \Pi_2$, then suppose $\|z_0 - \xi_3(0)\| \le M_0\varepsilon$ (6)

3) For $t \in \Pi_3$, we assume $\|z_0 - \xi_4(0)\| \le M_0\varepsilon$ (7)

4) For $t \in \Pi_4$ we take $\|z_0 - \xi_8(0)\| \le M_0\varepsilon$ (8)

The following theorems are correct.

Theorem 1. Suppose that conditions У.1, У.2, У.5 $(r = 1)$, У.6 $(r = 1)$, and $t \in \Pi_1$. are satisfied. Then: 1. There exists a domain $\Pi_{11} \subseteq \Pi_1$ and a solution to problem (1), (5) defined in Π_{11}.

2. The domain Π_{11} is the domain of attraction of the solution $\xi_1(t)$.

Theorem 2. Suppose that conditions У.1-У.3, and У.4 $(r = 2)$, У.6 $(r = 2)$, and $t \in \Pi_1$ are satisfied. Then: 1. There exists a domain $\Pi_{21} \subseteq \Pi_2$ and a solution to problem (1), (6) defined in Π_{21}.

2. The domain Π_{21} is the domain of attraction of the solution $\xi_3(t)$.

Theorem 3. Suppose that conditions У.1-У.3, and У.4 $(r=2)$ are satisfied. Then: 1. There exists a domain $\Pi_{31} \subseteq \Pi_3$ and a solution to problem (1), (7) defined in Π_{31}.

2. The domain Π_{31} is the domain of attraction of the solution $\xi_4(t)$.

Theorem 4. Suppose that conditions У.1-У.3, and У.4 $(r=3)$, У.5 $(r=3)$ are satisfied. Then: 1. There exists a domain $\Pi_{41} \subseteq \Pi_4$ and a solution to problem (1), (8) defined in Π_{41}.

2. The domain Π_{41} is the domain of attraction of the solution $\xi_8(t)$.

The proof of the formulated theorems is carried out using the method of successive approximations.

Let us prove Theorem 1. The domains Π_1 and Π_2 have a common boundary (p_{11}). Further, when considering the domain Π_1, we assume that (p_1) is not included in Π_1 and Π_2.

The problem (1), (1) is replaced by the following

$$z(t,\varepsilon) = E(t,0,\varepsilon)z_0 + \frac{1}{\varepsilon}\int_0^t E(t,\tau,\varepsilon)\left[z^2(\tau,\varepsilon) + \varepsilon f\big(\tau, z(\tau,\varepsilon)\big)\right]d\tau, \quad (9)$$

where

$$E(t,\tau,\varepsilon) = diag\left(exp\frac{1}{\varepsilon}\big(A_1(t) - A_1(\tau)\big),\ exp\frac{1}{\varepsilon}\big(A_2(t) - A_2(\tau)\big),\right.$$
$$\left. exp\frac{1}{\varepsilon}\big(A_3(t) - A_3(\tau)\big)\right) \text{ moreover } A_j(0) \equiv 0.$$

We will choose integration paths. The path, for all components of the vector z, consists of the part $(p_{11})[0,\tilde{t}]$ connecting the points and a straight line segment of the connecting point $(\tilde{t}, t,\ \tilde{t} = \tilde{t}_1 + i\tilde{t}_2, t \in \Pi_1)$. In (9), we carry out the following transformations, taking into account the chosen integration paths:

$$z(t,\varepsilon) = E(t,\tilde{t},\varepsilon)[E(\tilde{t},0,\varepsilon)z^0 + \frac{1}{\varepsilon}\int_0^{\tilde{t}} E(\tilde{t},\tau,\varepsilon)\big[z^2(\tau,\varepsilon) +$$
$$\varepsilon f\big(\tau, z(\tau,\varepsilon)\big)d\tau\big] +$$
$$+\frac{1}{\varepsilon}\int_{\tilde{t}}^t E(t,\tau,\varepsilon)\left[z^2(\tau,\varepsilon) + \varepsilon f\big(\tau, z(\tau,\varepsilon)\big)\right]d\tau. \quad (10)$$

In (10), the expression contained in [...] gives the value of the function $z(t,\varepsilon)$ for $t = \tilde{t}$. Then (10) can be represented as

$$z(t,\varepsilon) = E(t,\tilde{t},\varepsilon)\, z(\tilde{t},\varepsilon) + \frac{1}{\varepsilon}\int_{\tilde{t}}^{t} E(t,\tau,\varepsilon)\left[z^2(\tau,\varepsilon) + \varepsilon f\left(\tau, z(\tau,\varepsilon)\right)\right]d\tau. \quad (11)$$

Thus, to solve the problem posed, we first need to study the function $z(\tilde{t},\varepsilon)$, then the function $z(t,\varepsilon)$ i.e. we first consider the case $t-\tilde{t} \in (p_{11})$ then $t \in \Pi_1$.

Consider the function $z(\tilde{t},\varepsilon)$. We have

$$z(\tilde{t},\varepsilon) = E(\tilde{t},0,\varepsilon)\, z_0 + \frac{1}{\varepsilon}\int_{0}^{\tilde{t}} E(\tilde{t},\tau,\varepsilon)\left[z^2(\tau,\varepsilon) + \varepsilon f\left(\tau, z(\tau,\varepsilon)\right)\right]d\tau. \quad (12)$$

We apply the method of successive approximations to (12). We define successive approximations as follows (for the convenience of records, we omit the arguments of an unknown function):

$$z^m = E(\tilde{t},0,\varepsilon) z_0 + \frac{1}{\varepsilon}\int_0^{\tilde{t}} E(\tilde{t},\tau,\varepsilon)[(z^{m-1})^2 + \varepsilon f(\tau, z^{m-1})]d\tau,$$
$$z^0(t,\varepsilon) \equiv 0,\ m = 1,2,\dots\ . \quad (13)$$

Given the chosen integration paths (13), we represent in the form

$$z^m = E(\tilde{t},0,\varepsilon) z_0 + \frac{1}{\varepsilon}\int_0^{\tilde{t}} E(\tilde{t},\tau,\varepsilon)[(z^{m-1})^2 + \varepsilon f(\tau, z^{m-1})]\,(1 + i\varphi'_1(\tau_1))d\tau_1$$
$$z^0(t,\varepsilon) \equiv 0,\ m = 1,2,\dots\ , \quad (14)$$

where $\tilde{t} = \tilde{t}_1 + i\varphi_1(\tilde{t}_1),\ \tau = \tau_1 + i\varphi_1(\tau_1)$.

From (14), flat *m*=1, we obtain the first approximations

$$z^1 = E(\tilde{t},0,\varepsilon) z_0 + \int_0^{\tilde{t}} E(\tilde{t},\tau,\varepsilon) f(\tau,0)\,(1 + i\varphi'_1(\tau_1))d\tau_1. \quad (15)$$

In (14) each component of the vector function $z^1 = colon(z_{11}, z_{21}, z_{31})$ is estimated separately. Take z_{11}.

$$z_{11} = z_1 exp\frac{A_1(\tilde{t})}{\varepsilon} + \int_0^{\tilde{t}_1} exp\frac{1}{\varepsilon}(A_1(\tilde{t}) - A_1(\tau)) f_1(\tau,0)(1 + i\varphi'_1(\tau_1))d\tau_1 \quad (16)$$

In (16) we introduce the notation $f_{11}(\tau) \equiv f_1(\tau,0)(1 + i\varphi'_1(\tau_1))$ and we apply integration by parts to the integral on the right-hand side. We get

$$z_{11} = z_{10} exp\frac{A_1(\tilde{t})}{\varepsilon} - \varepsilon[\frac{f_{11}(\tilde{t}_1)}{a_1(\tilde{t}_1)} - \frac{f_{11}(0)}{a_1(0)} exp\frac{1}{\varepsilon}A_1(\tilde{t}) -$$
$$- \int_0^{\tilde{t}_1} \frac{f_{11}(\tau)}{a_1(\tau)} exp\frac{1}{\varepsilon}\left(A_1(\tilde{t}) - A_1(\tau)\right)d\tau_1].$$

From here, passing to the module and considering the limitations

$\left|\frac{f_{11}(\tilde{t}_1)}{a_1(\tilde{t}_1)}\right|$, $\left|\left(\frac{f_{11}(\tau)}{a_1(\tau)}\right)'\right|$ (according to conditions У.1, У.2) and $A_1(\tilde{t}) = i\Im m A_1(\tilde{t})$, $|z_{10}| \leq M_0\varepsilon$ we obtain $|z_{11}| \leq M_1\varepsilon$.

Now take z_{21} and z_{31}. Since the assessment procedures z_{21} and z_{31} do not significantly differ, their assessment will be carried out jointly.

$$|z_{s1}| \leq |z_{s0}| exp\frac{ReA_s(\tilde{t})}{\varepsilon} + M_1 \left|\int_0^{\tilde{t}_1} exp\frac{1}{\varepsilon}(ReA_s(\tilde{t}) - ReA_s(\tau))d\tau_1\right|, \qquad (17)$$

$$s = 2,3.$$

According to the conditions У.5 ($r = 1$), У.6 ($r = 1$) we have $ReA_s(\tilde{t}) = ReA_{s1}(\tilde{t}_1 + i\varphi_1(\tilde{t}_1)) \equiv A_{s1}(\tilde{t}_1) \leq 0$ and $A'_{s1}(\tilde{t}_1) < 0$ i.e. $A_{s1}(\tilde{t}_1)$ decreases with $0 \leq \tilde{t}_1 \leq \alpha_0 < \alpha$, which implies a limited expression $exp\frac{1}{\varepsilon}(ReA_s(\tilde{s}) - ReA_s(\tau))$.

Integrating the integral on the right-hand side of (17) by parts and taking into account what has been said, we obtain $|z_{11}| \leq M_1\varepsilon$. As a result, we obtain $\|z^1\| \leq M_1\varepsilon$.

Further

$$\|z^2\| \leq \|z^1\| + \frac{1}{\varepsilon}\left|\int_0^{\tilde{t}_1} \|E(\tilde{t},\tau,\varepsilon)\| + [\|z^1\|^2 + \varepsilon\|f(\tau,z^1) - f(\tau,0)\|] \times\right.$$

$$\left.\times \left|1 + i\varphi'_1(\tau_1)\right| d\tau_1\right|.$$

Hence, given the condition У.2, $\|E(\tilde{t},\tau,\varepsilon)\| \leq 1$,

$|1 + i\varphi'_1(\tau_1)| \leq M_{11}$ and an estimate for $\|z^1\|$ we obtain

$\|z^2\| \leq M_1\varepsilon + (\varepsilon(M_1^2 + M_1 M)M_{11})\tilde{t}_1 \leq \varepsilon(M_1 + (M_1^2 + M_1 M)M_{11}\alpha_0)$,

where $0 < \alpha_0 \leq \alpha$

Let α_0 satisfy the inequality $\alpha_0 \leq (M_2 - M_1)/M_2$,

where $M_2 = (M_1^2 + M_1 M)M_{11}$ and believe that $M_2 > M_1$.

Then we have $\|z^2\| \leq M_2\varepsilon$.

For the third approximation, we obtain

$$\|z^3\| \leq \|z^1\| + \frac{1}{\varepsilon}\left|\int_0^{\tilde{t}_1} \|E(\tilde{t},\tau,\varepsilon)\| + [\|z^2\|^2 + \varepsilon\|f(\tau,z^2) - f(\tau,z^1)\|] \times\right.$$

$$\times\left|1+i\varphi'_1(\tau_1)\right|d\tau_1\right| \leq M_1\varepsilon+\varepsilon(M_2^2+M\cdot M_2)M_{11}\alpha_0=$$
$$=\varepsilon(M_1+(M_2^2+M\cdot M_2)M_{11}\alpha_0.$$

Let $\alpha_0 \leq M_2 - M_1/(M_2^2+M\cdot M_2)M_{11},$ (18)

Then $\|z^3\| \leq M_2\varepsilon$. Continuing the process, we get

$$\|z^m\| \leq M_2\varepsilon,\ \ m=1,2,\dots\ . \tag{19}$$

(19) is valid under constraint (18). Now we prove the convergence of successive approximations $\{z^m(t,\varepsilon)\}$. To do this, it suffices to prove the uniform convergence of the series $\sum_{m=1}^{\infty}(z^m - z^{m-1})$.

Because $\|\sum_{m=1}^{\infty}(z^m - z^{m-1})\| \leq \sum_{m=1}^{\infty}\|z^m - z^{m-1}\|$,

then we will evaluate $\|z^m - z^{m-1}\|$ using the series comparison feature.

Given the chosen integration paths and the estimation of successive approximations, we have.

$$\|z^m - z^{m-1}\| \leq \varepsilon\,\frac{M_2^2+M\cdot M_2}{2M_2+M}\cdot\frac{(M_{11}\cdot(2M_2+M)\tilde{t}_1)^{m-1}}{(m-1)!}\ \ m=2,3,\dots\ . \tag{20}$$

The uniform convergence of the series follows from (20)

$$\|\textstyle\sum_{m=1}^{\infty}(z^m - z^{m-1})\| \text{ for values } \tilde{t} \leq \alpha_0.$$

Therefore, the sequence $\{z^m(t,\varepsilon)\}$ uniformly converges to some function $z(t,\varepsilon)$, which is a solution to system (12) in the part $(p_{10}) \subseteq (p_{11})$ satisfying the condition $\tilde{t}_1 \leq \alpha_0$, where α_0 has restriction (18). If we take into account the estimate (19), then for this solution we have the estimate

$$\|z(\tilde{t},\varepsilon)\| \leq M_2\varepsilon, \quad \tilde{t} \in (p_{10}). \tag{21}$$

Let $t \in \Pi_1$. For this case, the solution to problem (1), (5) is represented by formula (11). To (11), as in the previous case, we apply the method of successive approximations. Sequential approximations are defined as

$$z^m = \mathrm{E}(t,\tilde{t},\varepsilon)\cdot z(\tilde{t},\varepsilon)+\frac{1}{\varepsilon}\int_{\tilde{t}}^{t}E(t,\tau,\varepsilon)[(z^{m-1})^2+\varepsilon f(\tau,z^{m-1})]d\tau,$$
$$z^0(t,\varepsilon)\equiv 0,\ \ m=1,2\dots\ ,\quad \tau=\tilde{t}_1+i\tau_2. \tag{22}$$

We estimate successive approximations (22). In evaluating successive approximations, we take into account that $\|E(t,\tilde{t},\varepsilon)\|$ is bounded according to condition У.3 and estimate (21).

For the first approximation, we obtain

$$\|z^1\| \le \|E(t,\tilde{t},\varepsilon)\| \|z(\tilde{t},\varepsilon)\| + \left| \int_{\tilde{t}_2}^{t_2} \|E(t,\tau,\varepsilon\| \|f(\tau,0)\| \, d\tau_2 \right| \le$$

$$\le M_2\varepsilon + \mathrm{M} \left| \int_{\tilde{t}_2}^{t_2} \|E(t,\tau,\varepsilon\| \, d\tau_2 \right|.$$

To estimate the obtained integral, we take the integral

$$\int_{\tilde{t}_2}^{t_2} exp \frac{1}{\varepsilon} (ReA_j(\tilde{t}_1 + it_2) - ReA_j(\tilde{t}_1 + i\tau_2)) d\tau_2$$

and to the difference $(ReA_j(\tilde{t}_1 + it_2) - ReA_j(\tilde{t}_1 + i\tau_2))$ on the segment $[\tilde{t}_2, t_2]$ we apply the finite increment theorem and obtain the following representation

$ReA_j(\tilde{t}_1 + it_2) - ReA_j(\tilde{t}_1 + i\tau_2) = A'_{j1}(\tilde{t}_1, \Theta)(-t_2 - \tau_2),$

where $\tilde{t}_2 < \Theta < t_2$, $j = 1,2,3$.

According to the condition У.1 $\forall t \in \Pi_1 \left(A'_{j1}(\tilde{t}_1, \Theta) \le -m_0 = const, m_0 > 0 \right)$.

Thus

$$\int_{\tilde{t}_2}^{t_2} exp \frac{1}{\varepsilon} (ReA_j(\tilde{t}_1 + it_2) - ReA_j(\tilde{t}_1 + i\tau_2)) d\tau_2 \le$$

$$\le \int_{\tilde{t}_2}^{t_2} exp \frac{-m_0}{\varepsilon} (t_2 - \tau_2) d\tau_2 \le \varepsilon / m_0.$$

Given the calculations, we obtain

$$\|z^1\| \le M_2\varepsilon + \varepsilon M/m_0 = \varepsilon M_{20}, M_{20} = M_2 + M/m_0.$$

We estimate the second approximation

$$\|z^2\| \le \|z^1\| + \frac{1}{\varepsilon} \left| \int_{\tilde{t}_2}^{t_2} \|E(t,\tau,\varepsilon)\| [\|z^1\|^2 + \varepsilon \|f(\tau,z^1) - f(\tau,0)\| d\tau_2] \right| \le \|z^1\| +$$

$$+\varepsilon^2 (M_{20}^2 + M_{20} \cdot M)/m_0 \le \varepsilon(M_2 + \varepsilon (M_{20}^2 + M_{20} \cdot M)/m_0).$$

Let $\varepsilon \le (M_{30} - M_2)m_0/(M_{20}^2 + M_{20} \cdot M)$, где $M_{30} > M_2$.

Then $\|z^2\| \le \varepsilon M_{30}$.

For the third approximation, we have the estimate

$$\|z^3\| \le \|z^1\| + \frac{1}{\varepsilon} \left| \int_{\tilde{t}_2}^{t_2} \|E(t,\tau,\varepsilon)\| [\|z^2\|^2 + \varepsilon \|f(\tau,z^2) - f(\tau,0)\| d\tau_2] \right| \le$$

$$\le \varepsilon M_{20} + \varepsilon^2 M_{30}^2(+M_{30} \cdot M) \cdot \frac{1}{m_0}$$
$$= \varepsilon(M_{20} + \varepsilon\,(M_{30}^2 + M_{30} \cdot M)/m_0).$$

Let $\varepsilon \le (M_{30} - M_2)m_0/(M_{30}^3 + M_{30} \cdot M)$.

Then $\|z^2\| \le \varepsilon M_{30}$.

Continuing the process we get

$$\|z^m\| \le \varepsilon M_{30}, m = 1,2, \dots . \quad (23)$$

Now we prove the convergence of successive approximations (22). To do this, we estimate the difference $(z^m - z^{m-1}), m = 1,2, \dots$.

We have

$$\|z^m - z^{m-1}\| \le \varepsilon^m\,(M_{30} + MM_{30})/(2M_{30} + M) \cdot ((2M_{30} + M)/m_0)^{m-1}.$$

It follows that under the condition $\varepsilon\,(2M_{30} + M)/m_0 < 1$, the sequence (22) $\forall t \in \Pi_{10} \subset \Pi_1$, according to the constraint (18), converges uniformly to some function $z(t,\varepsilon)$, which is solving problem (1), (5). Taking into account (23) for this solution, we have the estimate

$$\|z(t,\varepsilon)\| \le \varepsilon M_{30}, \forall t \in \Pi_{10} \subset \Pi_1. \quad (24)$$

Denote $(p_{10}) \cup \Pi_{10} = \Pi_{11}$ and combining estimates (21 and (24) we obtain. $\forall \mathrm{t} \in \Pi_{11}\ (z(t,\varepsilon) \to 0\ n_0\varepsilon)$. The theorem is proved.

It follows from the theorems proved that in the considered domain the solutions $\xi_2(t), \xi_5(t), \xi_6(t), \xi_7(8)$ do not have attraction domains.

REFERENCES

1. Alybaev K.S. The method of level lines for the study of singularly perturbed equations in violation of the stability condition // Bulletin of KNU. - Series 3, Issue 6. - Bishkek, 2001 .-- S. 190-200. (in Russian)

2. K.S. Alybaev, Murzabaeva A.B. Singularly perturbed equations with analytic functions that lose their uniqueness during degeneration // Itogi Nauki In Theory and Practice 2017: Sat. scientific works of the Eurasian Scientific Association based on the materials of the XXXIV international scientific conference. No. 12 (34). Moscow, 2017 .-- S. 15-20. (in Russian)

3. Murzabaeva A.B. Systems of singularly perturbed ordinary differential equations with analytic functions that lose their uniqueness during degeneration // Theoretical and practical questions of modern science: collection of books. scientific works of the Eurasian Scientific Association based on the materials of the XLI international scientific conference. No. 7 (41). Moscow, 2018 .-- S. 12-18 (in Russian)

4. Alybaev K.S., Murzabaeva A.B. The construction of regions of attraction during the degeneration of singularly perturbed equations // International Scientific Journal. No. 9 (75). Yekaterinburg, 2018 .-- S. 7-11. (in Russian)

MSC: 34M60, 34E10, 34A12

BOUNDARY LINES FOR ATTRACTION AREAS

Tampagarov K. B., Murzabaeva A.B., Narymbetov T.K.
Medical and Social Research Institute, Jalal-Abad State University

This article explores the occurrence of boundary lines under various conditions. A new definition of the boundary line has been introduced.
Key words: singular perturbation, degenerate equation, region of attraction, boundary lines, level lines.

Бул макалада чектик сызыктардын ар түрдүү шартардан пайда болуу учурлары каралды. Чектик сызык түшүнүгүнүн жаңы аныктамасы келтирилди.
Урунттуу сөздөр: сингулярдык дүүлүгүү, кубулган теңдеме, тартылуу областы, чектик сызыктар, деңгээл сызыктар.

В данной статье исследованы возникновение пограничных линий при различных условиях. Введено новое определение пограничной линии.
Ключевые слова: сингулярное возмущение, вырожденное ураавнение, область притяжения, пограничные линии, линии уровня.

In [1], for the solutions of singularly perturbed ordinary differential equations, concepts were introduced: boundary lines, regular and singular domains and the existence of boundary lines for linear and weakly nonlinear singular equations was proved in [2].

As our studies show, boundary lines also arise in other cases.

1. Let the equation be considered

$$\varepsilon z'(t,\varepsilon) = f\big(t, z(t,\varepsilon)\big), \qquad (1)$$

with initial condition

$$z(t_0, \varepsilon) = z^0, \qquad (2)$$

where $t \in D \subset \mathrm{C}$ – set of complex numbers, and D simply connected region and t_0 her inner point; $0 < \varepsilon$ small material parameter. Assume the degenerate equation corresponding (1) at $\varepsilon = 0$ has decisions $\xi_j(t)$ $(j = 1,2.,..,n)$.

Definition 1.1. Exist $z(t,\varepsilon)$- the solution of the problem (1) – (2) defined in $D_1 \subset D$.

2. $\forall t \epsilon D_j (z(t,\varepsilon) \to \xi(t)$ by $\varepsilon)$.

By under these conditions, the domain D_j is called the attraction domain of the solution $\xi_j(t)$.

Definition 2. If:1. There is a solution to the problem $(1)-(2)$ and areas D_1, D_2having a common border (p_0).

2. $\forall t \in (p_0)(\lim_{\varepsilon\to 0} z(t,\varepsilon) -$ doesn't exist but$|\, z(t,\varepsilon)\,|$ $-$limited).

3. $\forall t \in D_1\big(z(t,\varepsilon) \to \xi_1(t)\big)$,$\forall t \in D_2\big(z(t,\varepsilon) \to \xi_2(t)\big)$.

Then (p_0) call the boundary line of the regions of attraction $D_1$and D_2.

Consider the following examples

3. Let the equation be considered

$$\varepsilon z'(t, \varepsilon) = (z - a_1)(z - a_2), \qquad (3)$$

with initial condition

$$z(t_0, \varepsilon) = z^0, \qquad (4)$$

where $a_1, a_2 \in C;\ t \in C$.

The degenerate equation corresponding to (3) has solutions

$$\xi_1 = a_1,\ \xi_2 = a_2. \qquad (5)$$

We pose the problem of studying the existence of attraction regions for solutions (5) according to the accepted definition.

To solve the problem posed, the solution of problem (3) - (4) can be represented as

$$\frac{z-a_1}{z-a_2} = \frac{z^0-a_1}{z^0-a_2} exp \frac{a_1-a_2}{z}\ (t-t_0) \qquad (6)$$

according to the theorem of the existence and uniqueness of the solution

$$z^0 \neq a_1 \text{and} z^0 \neq a_2.$$

Let be $a_1 = a_{11} + ia_{12}$, $a_2 = a_{21} + ia_{22}$, $t = t_1 + it_2$, $t_0 = t_{01} + it_{02}$, where $a_1, a_{12}, a_{21}, a_{22}, t_{01}, a_{02}$–real numbers, t_1, t_2 – valid variables.

In view of the notation introduced, we have

$$(a_1 - a_2)(t - t_0) = [(a_{11} - a_{21}) + i(a_{12} - a_{22}] \cdot [t_1 - t_{01} + i(t_2 - t_{02})] =$$

$$= [(a_{11} - a_{21})(t_2 - t_{01}) - (t_2 - t_{02})(a_{12} - a_{22})] + i[(a_{12} - a_{22})(t_1 - t_{01}) + +(a_{11} - a_{21})(t_2 - t_{02})] .$$

We introduce the notation

$$A_1(t_1, t_2) = [(a_{11} - a_{21})(t_1 - t_{01}) - (a_{12} - a_{22})(t_2 - t_{02})]$$

$$A_2(t_1, t_2) = [(a_{12} - a_{22})(t_1 - t_{01}) - (a_{11} - a_{21})(t_2 - t_{01})]$$

and consider the line

$$(P_0) = \{t \in C \mid A_1(t_1, t_2) = 0\}.$$

Line (P_0) is a straight line passing through a point t_0. (P_0) plane C divided into two half-planes.The opening of the half-plane is denoted by C_0 and C_1. In each of these half planes $A_1(t_1, t_2) < 0$ or $A_1(t_1, t_2) > 0$. For definiteness, we take

$$\forall t \in C_0(A_1(t_1, t_2) < 0).$$

Then $\forall t \in C_1(A_1(t_1, t_2) > 0)$.

Let be $t \in (P_0)$. From (6) we have

$$\frac{z - a_1}{z - a_2} = \frac{z^0 - a_1}{z^0 - a_2} e^{\frac{i}{\varepsilon}(t_1, t_2)} \tag{7}$$

It follows from (7) as $\varepsilon \to 0$ that the function $\frac{z - a_1}{z - a_2}$ has no limit, but

$\left| \frac{z - a_1}{z - a_2} \right| = \left| \frac{z^0 - a_1}{z^0 - a_2} \right|$ i.e. that is limited.

Let be $t \in C_0$. For this case for values t, when $A_1(t_1, t_2) << 0$ from (6) we get $\frac{z - a_1}{z - a_2} \to 0$. From here $z \to a_1$.

According to the adopted definition 1 exists $C_{01} \subset C_0$ and C_{01} is the domain of attraction of the solution $\xi_1 = a_1$.

The boundary of area C_{01} can be a straight line

$$(p_{0\varepsilon}^{-}) = \{t \in C_0 \mid A_1(t_1,t_2) = \varepsilon ln\varepsilon\}.$$

Let be $t \in C_1$. Were write function (6) in the form

$$\frac{z-a_2}{z-a_1} = \frac{z^0-a_2}{z^0-a_1} e^{\frac{-A_1(t_1,t_2)-iA_1(t_1,t_2)}{\varepsilon}} \qquad (8)$$

Because the $\forall t \in C_1(A_1(t_1,t_2) > 0)$, then for the half-plane $C_{11} \subset C_1$ (with border $C_{11}$is a straight line $(p_{0\varepsilon}^{+}) = \{t \in C_1 \mid A_1(t_1,t_2) = -\varepsilon ln\varepsilon\}$) from (6) we have

$$\frac{z-a_2}{z-a_1} \to 0 \text{ or } z \to a_2.$$

The half-plane C_{11} is the domain of attraction of the solution $\xi_2 = a_2$.

The line (p_0) is surrounded by lines $(p_{0\varepsilon}^{-}), (p_{0\varepsilon}^{+})$ is the boundary line according to the adopted definition 2.

In this example, a boundary line and regular regions exist, but no singular regions exist. Thus, the concept of boundary lines can be expanded.

REFERENCES

1. Pankov P.S., K.S. Alybaev K.S., Narbaev M.R., Tampagarov K.B. (2013) Bulletin of Osh State University,.- No. 1 (special issue). - P. 227-231.

2. Alybaev K.S, Tampagarov K.B. (2013) Actual problems, control theory, topology and operator equations: Materials of the II-nd international conference dedicated to 20- anniversary of KRSU and the 100th anniversary of Professor Ya. V. Bykov. – Bishkek. - P. 83-88.

3. Tampagarov K.B. (2016). Sat Art. by mater. XLVII Int. scientific-practical conf. No. 10 (45). - Novosibirsk: SibAK, - P. 67-73.

MSC 18C05

CONNECTIONS OF CATERGORY OF CORRECT EQUATIONS WITH OTHER CATEGORIES

[1]Kenenbaeva G.M., [2]Askar kyzy L., [3]Seyitbekova N.U.

[1,2]Kyrgyz State University named after J.Balasagyn,

[3]International University of Innovation Technologies

A new general notion of equation was introduced by us with assistance of the notion “predicate” on the base of the principle of preservation of solution while transformations and elements of the category of equations were constructed on the base of well-known categories. Further, we introduced the notion of the category of correct equations including the known “correctness by Hadamard” and presented examples of transformations.The aim of this paper is to connectthis notion with other categories.
Keywords: category, morphism, equation, predicate, solution, correctness.

Мурда өзгөртүүлөрдө чыгарылышты сактоо принцибинин негизинде “предикат” түшүнүгүнүн жардамы менен теңдеменин жаңы жалпы түшүнүгүн киргизгенбиз. Биз ошондой эле белгилүү болгон “Адамар боюнча корректтүүлүктү” кошуу менен корректүү теңдемелердин категорияларынын түшүнүгүн киргиздик жана корректүүлүктү сактоо менен өзгөртүүлөрдүн мисалдарын келтирдик. Бул макаланын максаты – бул түшүнүктү башка категориялар менен байланыштыруу.
Урунттуу сөздөр: категория, морфизм, теңдеме, предикат, чыгарылыш, корректтүүлүк.

Ранее нами было введено новое общее понятие уравнения с помощью понятия “предикат” на основе принципа сохранения решения при преобразованиях и построены элементы категории уравнений на основе известных категорий. Мы также ввели понятие категории корректных уравнений с включением известной «корректности по Адамару» и привели примеры преобразований с сохранением корректности. Цель настоящей статьи - связать это понятие с другими категориями.
Ключевые слова: категория, морфизм, уравнение, предикат, решение, корррректность.

1. Introduction

The approach of categories as notions being more general than sets and families of sets is used in various branches of mathematics. It was introduced in [1]. In Kyrgyzstan the first works on the category theory were [2] and [3]. We introduced the principle of preservation of solution while transformations (supra it was meant implicitly). We proposed to introduce the category *Equa* of equations and its subcategories including the category of correct equations ([5], [6], [7], [8]). (A category for one of types of equations was built in [4]). The aim of this paper is to connect this notion with other categories.

2. Review of known results and definitions in the theory of categories

In addition to mathematical objects modern mathematics investigates more and more the admissible maps defined between them. One familiar example is given by sets. Besides the sets, which form the mathematical objects in set theory, the set maps are very important. Much information about a set is available if only the maps into this set from all other sets are known.

Definition 1. A category K is defined

1) By its objects $A,B,C,\ ...$ of $Ob(K)$;

2) By its morphisms $f,g,h,\ ...$ of $Mor(K)$;

3) By the operations $dom$and cod which yield objects $dom(f)$ and $cod(f)$ (beginning and end of a morphism f).

$((f) =$ and $(f) = B) \Leftrightarrow (f : A \rightarrow B)$.

4) By operation of composition of morphism and such that $dom(f) = dom(g)$ which yields a morphism $g \circ f : A \rightarrow C$.

5) By operation I which yields the identity morphism $I_A : \rightarrow A$ by an object A. The assemblage of all morphism $\rightarrow$ in the category K is denoted as $K(A, B)$.

The following conditions are necessary:

1. Associativeness of composition

   $(\forall f, g, h, f{:}A \rightarrow B;\ g{:}B \rightarrow C;\ h{:}C \rightarrow D)(h \circ g) \circ f = h \circ (g \circ f)$.

2. Identities $\circ I_A = f,\ I_B \circ f = f$.

The main categories are the following:

The category of sets *Set*. *Ob(Set)* are non-empty sets, *Mor(Set)* are functions.

The category of functions *Func*. *Ob(Func)=Mor(Set), Mor(Func)* are transformations of functions.

These categories are used in building of the category *Equa*.

The category of topological spaces *Top*. *Ob(Top)* are topological spaces, *Mor(Top)* are continuous functions.

This category is used in building of the category *Equa-Par-Top*.

3. Building the category of equations

Supra equations were subdivided informally into algebraic ones, differential ones, integral ones, with initial or boundary conditions etc. We used the fact that equations and systems of equations of various types are equivalent. Moreover, the well known technique of reducing order of differential equations, various techniques of substitution and transforming of argument, the method of transforming of solutions developed in Kyrgyzstan, the method of additional argument and the method of reducing differential equations to systems of operator-differential equations created in Kyrgyzstan demonstrated that equations with various solutions and even in various spaces can be equivalent.

Hence, ween larged the notion of equation including «systems of equations», «with initial or boundary conditions» to formulate main notions, objects and morphisms of the category *Equa* of equations and its subcategories.

Definition 2. *Ob(Equa)*contains tuples

{Non-empty sets X, Y, predicate $P(x)$ on X, transformation $B:X\to Y$}.

If $(\exists x\in X)(P(x)\wedge(y=B(x))$ then $y\in Y$ is said to be a solution of the equation $\{X, Y, P, B\}$.

Particularly, if B is the identity operator I, then we obtain the equation *"P(x)"*only.

Mor(Equa) are such transformation so ftuples$\{X, Y, P, B\}$ that solutions (or their absence) preserve.

Example 1 of morphism (transformationof the set X). Considered the equation in *Equa*

$$\begin{cases} "P(x)", x\in X, \\ y = x\ (X\equiv Y, B\equiv I). \end{cases} \tag{1}$$

Let $\Psi:Z\to X$ be a bisection. Then the equation

$$\begin{cases} "P(\Psi(z))", z\in Z, \\ y = \Psi(z)\ (B\equiv\Psi) \end{cases} \tag{2}$$

is equivalent to (1). (Usually the second assertion is written in another place of a paper or is meant. We propose to write it evidently).

Remark**.** If Ψ is a surjection then (2) is equivalent to (1) too. If Ψ is an arbitrary function then (1) is a consequence of (2).

Some subcategories for the category *Equa*.

The category of equations forfunctions *Equa-Func*.

Definition 3 . *Ob(Equa-Func)*contains tuples

$\{X\in Ob(Func), Y\in Ob(Fuc)$, predicate $P(x)$on X, t ransformation $B:X\to Y\}$.

Mor(Equa-Func) containsтinvertible transformations of functions inherited from *Mor(Equa)*and specifictransformations.

Example 2:Transformation of argument.

Considered the equation in*Equa-Func*

$$\begin{cases} "P(x(t))", x(t): U \to V, \\ y(t) = x(t)\ (X \equiv Y, B \equiv I). \end{cases} \quad (3)$$

Let $t=\psi(s)$ be a bijection $U \to U$. Denote $z(s)=x(\psi(s)):U \to V$ and build an equivalent equation

$$\begin{cases} "P(z(s))", \\ y(t) = z(\psi^{-1}(t)). \end{cases} \quad (4)$$

Illustrative example 3 of transformation of argument.Considered the

$$\text{equation}\begin{cases} "x(t) + \int_0^4 x(v)dv = t, 0 \le t < \infty", \\ y(t) = x(t). \end{cases} \quad (5)$$

Let$t=\psi(s)=s^2.x(s^2) + \int_0^4 x(v)dv = s^2, 0 \le s^2 < \infty$. Denote $z(s)= x(s^2)$and substitute$v=w^2$: $z(s) + \int_0^2 z(w) \cdot 2wdw = s^2, 0 \le s < \infty$. $z(s)=s^2-8/5$; $y(t)=t-8/5$.

For equations with parameters

Definition 4. *Ob(Equa-Par)*are tuples

{non-emptysets*X*, *F*, *Y*, predicate *P(x,f)* on $X \times F$, transformation $B:X \to Y$}.

If $(\exists x \in X)(P(x,f) \wedge (y=B(x))$then then $y \in Y$ is said to be a solution of the equation {*X*, *F*, *Y*, *P*, *B*}.

Mor(Equa-Par) are such transformations of tuples {*X*, *Y*, *P*, *B*} (except *F*) that solutions (or their absence) preserve.

4. Category of correct equations

By our approach «correctness» can by a parameter only, hence the category of correct equations *Equa-Par-Top* is a subcategory of the category*Equa-Par*.

Definition 5.*Ob(Equa-Par-Top)* are tuples

{topological spaces *X*, *F*, *Y*, predicate *P(x,f)* on $X \times F$, continuous transformation $B:X \to Y$} such that 1) $(\forall f \in F)(\exists! y \in Y)(\exists x \in X)\ (P(x,f) \wedge (y=B(x))$;

2) the element *y* depends on the element *f* continuously.

Mor(Equa-Par-Top) are transformations preserving properties 1) and 2).

Example 4. If the predicate is written as *P(x,f)=«A(x)=f»,* where *A* is an operator an $dB = I$ then we obtain «correctness by Hadamard».

The category *Equa-Func-Par-Top* of correct equations for functions also can be defined:

Example 5. If *x(t)* is a smooth function, the predicate is written as *P(x,f) =" x'(t)=a(t)x(t); x(0)=f*$\in$**R**", *a(t)* is a given continuous function, *B =I* then we obtain «continuous dependence of the solution of initial value problem for ordinary differential equation on initial data».

The transformation $P_1(x,f)$ =" $x(t)-\int_0^t a(s)x(s)ds=f$ "reduces to Example 5 for functions. The transformation $P_2(z,f)$ =" $z(t)=a(t)\,(f+\int_0^t z(s)ds)$", $B_2(z(t))=x'(t)$ changes an initial value problem for differential equation to integral one with parameter.

Conclusion

We have proven correctness of some integral equations of the first kind [9]. We hope that the proposed presentations of equations and correct equations would make investigation of related tasks more uniform and strict.

REFERENCES

1. Eilenberg S., MacLane S. A general theory of natural equivalences // Transactions of American Math. Society, 1945, 58: pp. 231–294.
2. Медведев М.Я. Полусопряженные функторы и категории алгебр над n-тройками: Автореферат дисс. … к. ф.-м.н. (01.01.04). - Новосибирск, 1973. - 17 с.
3. Борубаев А.А. О категорных характеристиках компактных, полных равномерных пространств и полных по Райкову топологических групп // Изв. Акад. наук, вып.4, 2007. - С. 1-6.
4. Rosický J. Equational categories // Cachiers de topologie et géométrie différentielle catégoriques, vol. 22, no. 1, 1981. - Pp.85-95.

5. Кененбаева Г.М., Аскар кызы Л., Бейшебаева Ж.К., Маматжан уулу Э. Элементы категории уравнений // Вестник Института математики НАНКР, 2018, № 1. - С. 88-95.
6. Kenenbaeva G.M., Askar kyzy L. Foundations of category of equations // Тезисы докладов Международной научной конференции «III Борубаевские чтения», посвященной 35-летию со дня образования Института математики НАН КР. - Бишкек: Институт математики, 2019. - С. 46.
7. Кененбаева Г.М., Аскар кызы Л. Элементы категории корректных уравнений // Вест. Института математики НАН КР, 2019, № 1. - С. 69-74.
8. Кененбаева Г.М., Аскар кызы Л., Кененбаев Э., Абдалиев Т.С. Функторы в категории уравнений и ее подкатегориях // Вестник Института математики НАН КР, 2019, № 2. - С. 40-46.
9. Кененбаева Г.М., Аскар кызы Л. Класс интегральных уравнений первого рода, имеющих решение при любой правой части // Актуальные проблемы вычислительной и прикладной математики: труды Международной конференции, посвященной 90-летию со дня рождения академика Г. И. Марчука, Институт вычислительной математики и математической геофизики СО РАН. - Новосибирск: Абвей, 2015. - С. 321-325.

2010 MSC: 35C20, 35K05

SMAIL PARAMETER ASYMPTOTIC EXPANSIONS OF THE SOLUTIONS OF A DEGENERATE PROBLEM

Jyldyzbek Turkmanov
Bishkek state university named after K.Karasaev

In this section we consider a quasi – liner parabolic equation under the assumption that a solution of the corresponding degenerate problem has for $t > 0$ one or several lines of discontinuity of derivatives.In case of one line of discontinuity of derivatives generated by a «breaking» of the continuous initial function, we construct a complete asymptotic expansiuon of the solution of the nondegenerate problem.

Key words: Quasi–linear, parabolic equation, degenerate problem, solution, asymptotic expansion, function, continuous derivatives, point, generally speaking, standard algorithms, hogt's, estimetes.

Бул бөлүмдө биз квази–сызыктуу параболалык теңдемени t > 0 болгондогу туундунун үзгүлтүктүүлүгүнүн бир же бир канча сызыктар учурундагы кубулган маселеге тиешелуу болгон чыгарылышын карайбыз. Үзгүлтүксүз алгачкы функциянын «үзгүлтүктүүлүгү» бар, туундунун үзгүлтүктүүлүгүнүн бир сызыктын учурунда козголгон маселенин чыгарылышынын толук асимптотикалык ажыроосу тургузулат.
Урунттуу сөздөр: Квази–сызыктуу, параболалык теңдеме, кубулган теңдеме, чечим, асимптотикалык ажыроо, функция, үзгүлтүксүз түзөтүлөр жана стандарттык алгоритмдер, туунду, чекит.

В этом разделе мы рассмотрим квазилинейное параболическое уравнение в предположении, что решение соответствующей вырожденной задачи имеет при t > 0 одну или несколько линий разрыва производных. В случае одной линии разрыва производных, порожденной «разрывом» непрерывной начальной функции, строится полное асимптотическое разложение решения невырожденной задачи.
Ключевые слова: Квази–линейное, параболическое уравнение, вырожденная задача, решение, асимптотическое разложение, функция, непрерывные поправки и стандартные алгоритмы, хогтсовские, эстимейтовые.

In the strip $\Pi_T = \{(t,x)|0 < t \le T, -\infty < x < \infty\}$, let us consider the Cauchy problem

$$L_\varepsilon u \equiv \varepsilon^2 \frac{\partial^2 u}{\partial x^2} - \varphi'(u)\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0, \qquad u|_{t=0} = f(x). \tag{1}$$

The function $f(x)$ will be assumed to be continuous and bounded for $x \in (-\infty; +\infty)$, possessing for the $x \neq 0$ bounded continuous derivatives of any order and having finite limiting values as $x \to -0$, and $x \to +0$.

An asymptotic expansion of the solution of problem A_ε will be sought in the form

$$u\,(t, x) \sim \sum_{k=0}^{\infty} e^{2k} u_{2k}(t,x) + \sum_{k=1}^{\infty} e^k v_k\left(t, \frac{x}{\varepsilon}\right) \tag{2}$$

Here the functions $u_{2k}(t,x), k \geq 1$ are defined just in the preceding section as well as of their derivatives with respect to the variable x have as $t \to 0$ the order $O(t)$.

As usual, it will be assumed that the functions $v_k(t,\xi)$ as a function of the variable ξ are boundary layer character as $|\xi| \to \infty$. Taking into account the expression written out in the previous section for the derivative of the function $u_0(t,x)$, we can write a recutsion system of equations

$$L_1 v_1 \equiv \frac{\partial^2 v_1}{\partial \xi^2} - \frac{\partial v_1}{\partial t} - \frac{\partial}{\partial \xi}\left[\frac{1}{2}v_1^2 + \frac{a^{\pm}\xi}{1+a^{\pm}t}v_1\right] = 0 \tag{3}$$

$$\frac{\partial^2 v_k}{\partial \xi^2} - \frac{\partial v_k}{\partial t} - \frac{\partial}{\partial \xi}\left[\left(v_1 + \frac{a^{\pm}\xi}{1+a^{\pm}t}\right) v_k\right] = \frac{\partial}{\partial \xi}\Phi_k(t,\xi), \qquad (4)$$

$k \geq 2$. Here $a^{\pm} = \lim_{x\to\pm 0} f'(x)$ and the functions $\Phi_k(t,\xi)$ can be easily defined successively for k=2,3,… by using the standard algorithms; the functions $\Phi_k(t,\xi)$ are represented by a sum whose each summand is a product of a polynomial $P_s(t,\xi)$ of degree s with functions of the variable t as coefficients by one or several functions $v_i(t,\xi), s \leq k, i = 1,2,\dots,k-1$. [1], [2].

The equations (3), (4) are solved separately for $\xi < 0$ and $\xi > 0$. We will seek for such solutions of the equation (3), (4) which satisfy the conditions

$$[v_{2i+1}(t,\xi)] = 0, \left[(v_{2i+1}(t,\xi))'_{\xi}\right] = -[u_{2i}(t,x))'_x], \qquad (5)$$

$$[v_{2i}(t,\xi)] = -[u_{2i}(t,x)], \left[v_{2i}(t,\xi))'_{\xi}\right] = 0, \qquad (6)$$

$$v_k(0,\xi) = 0 \qquad (7)$$

$$[z(t,y)] \equiv z(t,+0) - z(t,-0), i = 1,2,\dots.$$

The fulfillment of the conditions (5), (6) implies the continuity (along with the first order derivatives) of the formal asymptotic expansion (2) of the solution of the problem (1).

Considerthe problem (3), (5), (7) for $i = 0$. The change of the unknown function $\omega_1(t,\xi) = v_1(t,\xi) + \frac{a^{\pm}\xi}{1+a^{\pm} * t}$ leads us to the equation

$$\omega''_{1\xi} - \omega_1 * \omega'_{1\xi} - \omega'_{1t} = 0 \qquad (8)$$

Whose solution must satisfy the additional conditions $\omega_1(0,\xi) = a^{\pm}\xi$, $\omega_1(t,+0) = \omega_1(t,-0), \left[\omega'_{1\xi}(t,\xi)\right] = 0$.

Thus the solution of the equation (8) must be continuous in the domain t > 0 and possess in that domain the continuous derivative with respect to the variable ξ. Note that the equation (8)

$$\omega_1(t,\xi) = \left\{ \frac{(a^+ - a^-)\sqrt{t}}{\sqrt{(1+a^-t)(1+a^+t)}} \exp\left(-\frac{\xi^2}{4t}\right) + \xi \frac{a^-\sqrt{1+a^+t}}{1+a^-t} exp\left[-\frac{a^-\xi^2}{4(1+a^-t}\right] \int_{\delta(t,\xi)}^{\infty} e^{-\omega^2} d\omega + \xi \frac{a^+ * \sqrt{1+a^-t}}{1+a^+t} exp\left[-\frac{a^+\xi^2}{4(1+a^+t}\right] \int_{-\delta^+(t,\xi)}^{\infty} e^{-\omega^2} d\omega \right\} * \left\{ \sqrt{1+a^+t}\, exp\left[-\frac{a^-\xi^2}{4(1+a^-t)}\right] \int_{\delta(t,\xi)}^{\infty} e^{-\omega^2} d\omega + \sqrt{1+a^-t}\, exp\left[-\frac{a^+\xi^2}{4(1+a^+t}\right] \int_{-\delta^+(t,\xi)}^{\infty} e^{-\omega^2} d\omega \right\}^{-1}, \qquad (9)$$

Where $\delta^+(t,\xi) = \frac{\xi}{2\sqrt{1+a^+t}}$, $\delta(t,\xi) = \frac{\xi}{2\sqrt{1+a^-t}}$. The expression (9) implies that the equality $v_1(t,0) = 0\ (\sqrt{t})$ is fulfilled.

To investigate the behavior of the function $\omega_1(t,\xi)\ \ as\ \ |\xi| \to \infty$, we will use the well-known asymptotic formulas for the integrals appearing (9)

Applying these formulas, for $|\, \xi \,| \gg 1$ we can get

$$\omega_1(t,\xi) = \frac{a^\pm\xi}{1+a^\pm t} + \frac{2t^{\frac{3}{2}}(a^+ - a^-)}{\sqrt{\pi\xi^2\sqrt{(1+a^-t)(1+a^+t)}}} exp\left\{-\frac{\xi^2}{[4t(1+a^\pm t)]}\right\}(1 + o(1)),$$

Where the symbol «$\pm$" takes the values «-» for $\xi \ll -1$ and «+» for $\xi \gg 1$. On the basis of the above- obtained asymptotic representations can formulate the following.

Lemma 1. A solution of the problem (3), (5), (7) exist and exponentially tends to zero as $|\, \xi \,| \to \infty$; moreover, for that solution there holds the estimate

$$|v_1(t,\xi) \le M\sqrt{t}\left\{exp\left[-\frac{\xi^2}{4t(1+a^\pm t}\right] + exp\left[\frac{\xi^2}{4t}\right]\right\}.$$

For our further investigation we have to study the behavior of the derivatives of the function $v_1(t,\xi)$ as $|\, \xi \,| \to \infty$ and $t \to 0$

Lemma 2. For the derivatives of the solution of the problem (3), (5), (7) with respect to the variable ξ the estimates

$$\left|\frac{\partial v_1(t,\xi)}{\partial \xi}\right| \le M\left\{exp\left[-\frac{\xi^2}{4t(1+a^{\pm}t}\right] + exp\left[-\frac{\xi^2}{4t}\right]\right\},$$

$$\left|\frac{\partial^2 v_1(t,c)}{\partial \xi^2}\right| \le Mt^{-\frac{1}{2}}\left(1+\frac{\xi^2}{t}\right)\left\{exp\left[-\frac{\xi^2}{4t(1+a^{\pm}t}\right] + exp\left[-\frac{\xi^2}{4t}\right]\right\}.$$

are valid.

We can prove that lemma by means of an explicit expression for the function $v_1(t,\xi)$. From Lemmas 1 and 2 and the equation (3) in follows the estimate for the function $\frac{\partial v_1}{\partial t}$.

Let us pass to the consideration of the functions $v_k(t,\xi), k \ge 2$. Suppose that the estimates

$$\left|\frac{\partial \Phi_k}{\partial \xi}\right| \le M(1+|\xi|^{n_k}\left\{exp\left[-\frac{\xi^2}{4t(1+a^{\pm}t}\right] + exp\left[-\frac{\xi^2}{4t}\right]\right\},$$

$$\left|\frac{\partial^2 \Phi_k}{\partial \xi^2}\right| \le M\sqrt{t}(1+|\xi|^{n_k+1}\left\{exp\left[-\frac{\xi^2}{4t(1+a^{\pm}t}\right] + exp\left[-\frac{\xi^2}{4t}\right]\right\}$$

Hold for the right- hand side of the equation (4). Note that by the change of variables

$$y = \frac{\xi}{1+a^{\pm}t}, \tau = \frac{t}{1+a^{\pm}t}, \tilde{\upsilon}_k = (1+a^{\pm}t)v_k \qquad (10)$$

The equations (3), (4) are reduced to those with bounded coefficients which, generally speaking, are discontinuous for $y = 0$:

$$\tilde{\upsilon}''_{1yy} - \tilde{\upsilon}_1\tilde{\upsilon}'_{1y} - \tilde{\upsilon}'_{1\tau} = 0.$$

$$\tilde{\upsilon}''_{kyy} - (\tilde{\upsilon}_1\tilde{\upsilon}_k)'_y - \tilde{\upsilon}^{+1}_{k\tau} = \{\Phi_k(\tau,y)/[1-a^{\pm}\tau]\}'_y.$$

The existence of bounded solutions of either equation can be substantiated, for example, by the methods developed in [3]

Let us show that functions $v_k(t,\xi)$ and their derivatives are of boundary layer character as $|\xi| \to \infty$.

Theorem1. For $y \le 0$ and Φ'(b) r $\le y$, for the difference

z_0(r, y) = u (r, y , ε) $- u_0$ (r, y) the inequality

$|z_0(r,y)| \leq M\{\exp[-y^2/(4r)] + \exp[-(y-\Phi'(b)r)^2/(4r)]\}$ is fulfilled

Proof. Let us consider an auxiliary function $\psi(r,y) = \exp[-y^2/(4r)]$ for $r > 0$, $\psi(r,y) = 0$ for r = 0. Obviously.

$L_1\psi \equiv \frac{\partial^2\psi}{\partial y^2} - \Phi'(u)\frac{\partial\psi}{\partial\psi} - \frac{\partial\psi}{\partial r} = \frac{\psi(r,y)}{2r}[-1 + y\Phi'(u)]$ and hence for y<0 the relation $L_1(M\psi \pm z_0) = M\psi[-(2r)^{-1} + \Phi'(u)y/(2r)] < 0$ holds, since the function $\Phi'(u)$ for the initial function is nonnegative. Moreover, the function $M\psi \pm z_0$ is nonnegative for r = 0, and for large values of y, if the constant M is sufficiently large. According to the maximum principle, the function $M\psi \pm z_0$ is positive for $y \leq 0$,which implies the validity of the assertion of the lemma for $y \leq 0$.

Using the function $\psi(r,y) = \exp[-(y-\Phi'(b)r)^2/(4r)]$, we can analogously prove the assertion of the lemma for the case $y \geq \Phi'(b)r$.

Theorem2. For the solution of problem A_ε under the above- mentioned conditions the asymptotic expansion (2) is valid. Moreover, the estimate

$$\|u(t,x,\varepsilon) - u^{(N)}(t,x,\varepsilon)\|_{C^1} \equiv \|u(t,x,\varepsilon) - \sum_{k=0}^{N}\varepsilon^{2k}u_{2k}(t,x) - \sum_{k=1}^{2N+1}\varepsilon^k v_k(t,x,\varepsilon\|_{C^1} \leq M\varepsilon^{2N}$$

holds

Proof. Consider the difference $z_N(t,x,\varepsilon) = u(t,x,\varepsilon) - u^N(t,x,\varepsilon)$. The function $z_N(t,x,\varepsilon)$ satisfies the zero initial condition for $t = 0$ and is twice continuosly differentiable for $t > 0, x \neq 0$.

Everywhere in the strip Π_T, with the exception of the points of the axis $x = 0$, the function $z_N(t,x,\varepsilon)$ satisfies the equation

$$\varepsilon^2\frac{\partial^2 z_N}{\partial x^2} - \Phi'(u)\frac{\partial z_N}{\partial x} - \Phi''_u\frac{\partial u^{(N)}}{\partial x}z_N - \frac{\partial z_N}{\partial t} = -\psi_N(t,x,\varepsilon) \qquad (11)$$

Where

$$\Phi''_u = \int_0^1 \Phi''\left(u^{(N)}(1-\theta) + u\theta\right)d\theta,$$

$$\left|\psi_N(t,x,\varepsilon\right| \leq M\varepsilon^{2N+2}$$

According to the maximum principle, everywhere in the strip Π_T the estimate $z_N(t,x,\varepsilon)$ for the function $|z_N(t,x,\varepsilon)| \le M\varepsilon^{2N+2}$ is valid.

$$\left|\widetilde{\psi}_N(t,x,\varepsilon) - \widetilde{\psi}_N(t,y,\varepsilon)\right| \le M\varepsilon^{2N+1} t^{-\frac{1}{2}}|x-y|.$$

Using this estimate, we can, as when proving Theorems

$|[z_N(t,x,\varepsilon)]'_{xx}| \le M\varepsilon^{2N}\left(\sqrt{t} + \varepsilon^{-2}\right)$.

Which provides us with the estimate for the function $\frac{\partial z_N(t,x,\varepsilon)}{\partial t}$.

Getting back to the variables t and x, we can rephrase the obtained results in terms of the following [4].

Theorem 3. Everywhere outside some neighborhood of the origin the estimate

$$\left|u(t,x) - u_1(t/\varepsilon^2, y/\varepsilon^2)\right| \le$$

$$\le \begin{cases} M\varepsilon^{2\gamma} t^{-\gamma} \exp(m_3 x^2/\varepsilon^2 t & if \quad x \le 0, \\ M\varepsilon^{2\gamma} t^{-\gamma} & if \quad 0 \le x \le \phi'(b)t, \\ M\varepsilon^{2\gamma} t^{-\gamma} \exp\{-m_3[x - \phi'(b)t]^2/\varepsilon^2 t & \text{if } \phi'(b)t \le x \end{cases}$$

Is valid (m_3 *is* a positive constant), note that the radius of that neighborhood is of order O(ε^2).

Obviously, Theorem 3 may be considered to be valid for all t>0 if the constant M is sufficiently large; note once more that one can obtain the corresponding estimates for finite values of t by using the same techniques we have used in the first section.

Theorem 3 gives us an idea of the character of variation of the solution of the problem under consideration as $t \to \infty$.

REFERENCES

1.Sushko V.G., On the asymptotics with respect to a small parameter a one parabolic equation. Dokl.Akad.Nauk SSSR 205(1972), №4, 794-797.

2. Oleinik O.A. and Kruzhkov S.N., Quasilinear parabolic equations of second order with several independent variables. (Russian) Uspekhi Mat/Nauk 16 (1961), №5, 115-155.

3.Ladyzhenskaya O.A., Boundary value problems of mathematical physics. Nauka, Moskow, 1973.

4.Oleinik O.A. and Ventzel T.D., The first boundary value problem and the Cauchy problem for quasilinear equation of parabolic type. (Russian) Mat. Sb. 4(1957), №1, 105-128.

MSC 34A26, 35A16

FUNCTIONAL RELATIONS FOR ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Kenenbaev E.
Institute of Mathematics of NAS of KR

There are considered such differential equations of various types in the paper that some functional relations connect values of a solution in different points. For examples, even, odd and periodical solutions, Vallée-Poussin's assertion, Lagrange interpolation polynomial, Hermite interpolation polynomial, spline-functions for ordinary differential equations, Asgeirsson's identity and its generalizations for partial differential equations of hyperbolic type. An application of such relations to solve some problems is demonstrated.

Keywords: functional relation, ordinary differential equation, partial differential equation, solution.

Макалада, чыгарылыштары үчүн, алардын маанилери ар кайсы чекиттерде бири бири менен байланышта болгондой, функционалдык өз ара байланышы бар ар кандай типтеги дифференциалдык теңдемелер каралат. Мисал катары так, жуп жана мезгилдүү чыгарылыштар, Валле-Пуссендин өз ара байланышы, Лагранж интерполирлөө көп мүчөсү, Эрмит интерполирлөө көп мүчөсү, кадимки дифференциалдык теңдемелер үчүн сплайн-функциялар, Асгейрссон теңдештиги жана анын гиперболалык типтеги дифференциалдык теңдемелер үчүн жалпыланышы келтирилген. Кээ бир маселелерди чыгаруу үчүн ушундай өз ара байланыштар көрсөтүлгөн.

Урунттуу сөздөр**:** функционалдык өз ара байланыш, чыгарылыш, кадимки дифференциалдык теңдемелер, жекече туундулуу дифференциалдык теңдемелер.

В статье рассматриваются дифференциальные уравнения различных типов, для решений которых имеют место функциональные соотношения, связывающие между собой значения решения уравнения в различных точках. В качестве примеров приведены четные, нечетные и периодические решения, соотношение Валле-Пуссена, интерполяционный многочлен Лагранжа и интерполяционный многочлен Эрмита, сплайн-функции для обыкновенных дифференциальных уравнений, тождество Асгейрссона и его обобщения для дифференциальных уравнений гиперболического типа. Показано применение таких соотношений для решения некоторых задач.

Ключевые слова: функциональное соотношение, решение, обыкновенное дифференциальное уравнение, дифференциальное уравнение в частных производных.

Introduction

To investigate differential equations of various types we propose to use the following fact. Solutions of some types of differential equations have functional relations connecting their values in different points. By given values of solutions in several points one can find their values in other points.

For examples in the first section, even, odd and periodical solutions, Vallée-Poussin's assertion, Lagrange interpolation polynomial, Hermite interpolation polynomial, spline-functions are considered for ordinary differential equations are considered.Their approximations are in the second section. Asgeirsson's identity and its generalizations for partial differential equations of hyperbolic type are described in the third section.

In this paper we will use functional denotations of type $x[n]$instead of x_n.

1. Functional relations for ordinary differential equations

Denote the functional relation number F for every equation as the minimal number of connected points (if it exists):

1.1. The simplest differential equation of the first order with the zero initial value$y'(x)=a, y(0)=0, a\neq 0$:$F=2$:

$$y(x[1])x[2]-y(x[2])x[1]=0. \quad (1)$$

1.2. The same equation with arbitrary initial value: $y(0)=y_0$: $F=3$:

$$(y(x[1])-y(x[3]))(x[1]-x[2])-(y(x[1])-y(x[2]))(x[1]-x[3])=0. \quad (2)$$

1.3. The linear differential equation of the k-thorder$y^{(k)}(x)=0$, or a polynomial of $(k-1)$-th order: $F=k+1$. Let numbers $x[1], x[2],\ldots, x[k+1], y[1], y[2],\ldots, y[k+1]$ be given. Construct the Lagrange interpolation polynomial of the $(k-1)$-th order by the values $x[1], x[2],\ldots,x[k]$ и$y[1], y[2],\ldots,y[k]$ then

$$L(x[k+1])-y[k+1]=0. \quad (3)$$

If$x[1], x[2],\ldots,x[k]$forman arithmetic progression then (3) can be rewritten as

$$\sum_{j=1}^{k+1} C_{k+1}^{j-1}(-1)^j y(x[j])=0. \quad (4)$$

1.4. Introduce*a generalized functional relation*: between values of solution and its derivatives. The first example is the Hermite interpolation polynomial: for given numbers $x[j], y[j,k], j=1..m, k=0..p[j]-1,$ there is the unique solution (polynomial) of the equation $y^{(n)}(x)=0$ where $n=\sum_{j=1}^{m} p[j]+1$ meeting the conditions

$$P^{(k)}[n](x[j]) = y[j,k], j = 1..m, k = 0..n[j]-1. \qquad (5)$$

1.5. The first result on functional relations (inour terms) for a linear ordinary differential equation was obtained by C. J. de la Vallée Poussin (for instance see [1]): the multipoint value problem

$y^{(n)}(x)+p_1(x)\, y^{(n-1)}(x)+\ldots+ p_n(x)\, y(x) = 0,\ a\le x\le b,$

$p_k(x)\in C[a,b],\ y(x[i]) = c[i],\ i=1,\ldots, n$

has a unique solution when

$\| p_1\|_{[a,b]}(b- a)+ \| p_2\|_{[a,b]} (b- a)^2/2! +\ldots + \| p_n\|_{[a,b]} (b-a)^n/n! < 1.$

2. Approximate functional relations for ordinary differential equations

Below ε is a small positive parameter and the "small" function $|\zeta(x)|<\varepsilon$.

2.1. See 1.1. If "*a*" is known approximately then we obtain the following differential inequality: $|y'(x)-a|<\varepsilon,\ y(0)=0$, or $y'(x)=a+\zeta(x),\ y(0)=0.$

Hence $y(x) = ax + \int_0^x \zeta(s)ds, |y(x[1])x[2]-y(x[2])x[1]| =$

$$= \left|\left(ax[1] + \int_0^{x[1]} \zeta(s)ds\right)x[2] - \left(ax[2] + \int_0^{x[2]} \zeta(s)ds\right)x[1]\right| \le$$

$$\le \varepsilon|x[1]|\cdot|x[2]-x[1]| + \varepsilon|x[1]|\cdot|x[2]-x[1]| \le 2\varepsilon|x[1]|\cdot|x[2]-x[1]|.$$

2.2. See 1.2. We have

$$|(y(x[1])-y(x[3]))(x[1]-x[2])-(y(x[2])-y(x[3]))(x[2]-x[3])| =$$

$$\left|\left(a(x[3]-x[1]) + \int_{x[1]}^{x[3]} \zeta(s)ds\right)(x[2]-x[1]) - \left(a(x[2]-x[1]) + \right.\right.$$

$$\left.\left. + \int_{x[1]}^{x[2]} \zeta(s)ds\right)(x[3]-x[1])\right|$$

$$= \left|\int_{x[1]}^{x[3]} \zeta(s)ds\,(x[2]-x[1]) - \int_{x[1]}^{x[2]} \zeta(s)ds\,(x[3]-x[1])\right| \le$$

$$\le 2\varepsilon|x[3]-x[2]|\cdot|x[2]-x[1]|.$$

3. Functional relations for partial differential equations

Denote: $x := (x_1, x_2, \ldots, x_m) \in R^m, \Delta_x := \sum_{i=1}^{m} \frac{\partial^2}{\partial x_i^2}$.

3.1. Consider the Laplace equation $\Delta_x u(x)=0$. Let $m=2$, points $x[1], x[2],\ldots,x[k]$ and numbers $u[1], u[2],\ldots,u[k]$ be given. Construct the Lagrange interpolation polynomial *L(x)*as a function of complex variable: $L(x[j])=u[j], j=1,\dots,k$, and define the harmonic function $U(x)=Re\ L(x)$. Then $U(x[j])=Re\ L(x[j])= Re u[j]= u[j],\ j=1,\dots,k$. Hence the Laplace equation does not have finite functional relations.

3.2. A solution of the hyperbolic equation $\frac{\partial^2}{\partial x_1\,\partial x_2} u(x_1,x_2)=0$ meets the Asgeirsson's identity *(F=4)*:

$$u(w_1, v_1)+ u(w_2, v_2)-u(w_1, v_2)-u(w_2, v_1)\equiv 0. \qquad (6)$$

3.3. A solution of the wave equation $\frac{\partial^2}{\partial x_1^2} u(x_1,x_2)=\frac{\partial^2}{\partial x_2^2} u(x_1,x_2)$ meet sthesimilar Asgeirsson's identity *(F=4)*: for four vertices of a rectangle obtained by means of rotation of the rectangle (6) on 45^0.

3.4. A solution of the multi dimensional hyperbolic equation $\frac{\partial^m}{\partial x_1\,\partial x_2 \dots \partial x_m} u(x_1,x_2,\dots,x_m)=0$ has the form

$$u(x)= g_1(x_2,\dots, x_m)+ \dots + g_q(x_1,\dots, x_{q-1}, x_{q+1},\dots, x_m) + \dots + g_m(x_1,\dots,x_{m-1})$$

and meets the meets the generalized Asgeirsson's identity $(F=2^m)$[15].

3.5. A solution of the two-dimensional wave equation $\frac{\partial^2}{\partial x_1^2} u(x_1,x_2,x_3)=\frac{\partial^2}{\partial x_2^2} u(x_1,x_2,x_3)+\frac{\partial^2}{\partial x_3^2} u(x_1,x_2,x_3)$ in contrast to 3.3 does not have finite functional relations.

4. Conclusion

Are view of publications [2], [3], [5], [6], [12], [18], [20], [21] and other ones demonstrates that there does not existaunified classification of multi dimensional partial differential equations and some existing classifications were base don formal writings of them. Authors of[23], [24] proposed to classify equations by properties of their solutions. Examples in this papers ubstitute this point of view.

REFERENCES

1. Vallée-Poussin Ch.J. Sur l'equation différentielle linéaire du second ordre. Détermination d'une intégrale par deux valeursassignées. Extension aux equations d'ordre // Journal Math. Pures Appl., **8,** 1929, pp. 125–144.

2. Бицадзе А.В. Уравнения смешанного типа. - Москва: Изд-во Академии наук СССР, 1959. - 164 с.

3. Векуа И.Н. Дифференциальное уравнение с частными производными; методы комплексного переменного. – В кн.: Математическая энциклопедия, том 2. – Москва: Советская энциклопедия, 1979. – С. 311-318.

4. Гельфонд А.О. Исчисление конечных разностей. – Москва: Гос. изд-во физико-математической литературы, 1959.

5. Джураев Т.Д. Краевые задачи для уравнений смешанного и смешанного-составного типов. – Ташкент: Фан, 1979. – 240 с.

6. Джураев Т.Д., Попелек Я. О классификации и приведении к каноническому виду уравнений с частными производными третьего порядка // Дифференциальные уравнения. - 1991. - Т. 27. - № 10. - С. 1734-1745.

7. Джураев Т.Д., Сопуев А. К теории дифференциальных уравнений в частных производных четвертого порядка. – Ташкент: Фан, 2000. – 144 с.

8. Джураев Т.Д., Сопуев А., Мамажанов М. Краевые задачи для уравнений параболо-гиперболического типа. – Ташкент: Фан, 1986. – 220 с.

9. Кененбаева Г. Эффект аналитичности для дифференциальных и интегральных уравнений. – Saarbrücken, Deutschland: LAP Lambert Academic Publishing, 2015. – 64 с.

10. Комленко Ю.В. Характеристика. – В кн.: Математическая энциклопедия, Т.5. – М.: Советская энциклопедия, 1985. – С. 753-755.

11. Курант Р., Гильберт Р. Методы математической физики, 2-е издание. – Москва-Ленинград: Гостехтеориздат, 1951. – 544 с.

12. Михлин С.Г. Линейные уравнения в частных производных. – Москва: Высшая школа, 1977. – 431 с.

13. Панков П.С. Доказательные вычисления на электронных вычислительных машинах. - Фрунзе: Илим, 1978. - 179 с.

14. Панков П.С., Матиева Г.М., Сабирова Х.С. Аксиоматическая теория характеристик и ее применение к аналитическим функциям // Исследования по интегро-дифференциальным уравнениям, вып. 33. – Бишкек: Илим, 2004.

– С. 37-42.
15. Панков П.С., Сабирова Х.С. Составление функционально-характеристических уравнений с аналитическими функциями // Вестник Казахского национального технич. Ун-та им. Сатпаева, 2006. - № 5. – С. 135-141.
16. Сабирова Х.С. Влияние младших членов дифференциальных уравнений с частными производными на их характеристичность // Исслед. по интегро-дифференц. уравнениям, вып. 38. – Бишкек: Илим, 2008. – С. 107-111.
17. Сабирова Х.С. Различие в характеристических свойствах волновых уравнений с различным количеством переменных // Вестник Междунар. ун-та Кыргызстана, № 1(20), 2011. – С. 58-61.
18. Рождественский Б.Л. Гиперболического типа уравнение. – В кн.: Матем. энциклопедия, том 1. – Москва: Советская энциклопедия, 1977. – С. 992-993.
19. Смирнов М.М. Дифференциальные уравнения в частных производных второго порядка. – Москва: Наука, 1964. – 208 с.
20. Солдатов А.П. Параболического типа уравнение. – В кн.: Математическая энциклопедия, том 4. – Москва: Советская энциклопедия, 1984. – С. 195.
21. Тихонов А.Н., Самарский А.А. Уравнения математической физики, 4-е издание. – Москва: Наука, 1972. – 288 с.
22. Благовещенский А. С. О характеристической задаче для ультра гиперболического уравнения // Матем. сборник, 63(105):1 (1964). - С. 137–168.
23. Кененбаева Г.М., Аскар кызы Л., Бейшебаева Ж.К., Маматжан уулу Э. Элементы категории уравнений // Вестник Института математики НАН КР, 2018, № 1. - С. 88-95.
24. Кененбаева Г.М., Аскар кызы Л. Элементы категории корректных уравнений // Вестник Ин-та матем. НАН КР, 2019, № 1. - С. 69-74.

MSC 45D05

SPECTRAL PROPERTIES OF NON-LINEAR VOLTERRA INTEGRAL EQUATIONS WITH ANALYTIC FUNCTIONS

Muratalieva V.T.
International University of Kyrgyzstan

Supra the author constructed and implemented the following algorithms on a computer. Given an equation with power coefficients by integral summands, the algorithm presents data to detect existence of solutions and occurrence of arbitrary constants in it; also, given an equation with a coefficient, specific values of the coefficient are found. In this paper these items are considered for non-linear equations.
Keywords: integral equation, non-linear equation, Volterra equation, algorithm, analytical function.

Автор төмөнкү алгоритмдерди түзүп жана компьютерде жүзөгө ашырган. Даражалуу көбөйтүндүлүү интегралдык кошулуучулары бар теңдеме берилген. Теңдемеде үчүн алгоритм чыгарылышынын жашоосун аныктоо жолуна мүмкүндүгүн жана анда каалагандай турактуу сан бар экендигин аныктоо үчүн малыматты берет. Ошондой эле коэффициент менен теңдеме сунушталган, бул теңдемеде коэффициенттин өзгөчө маанисин табуу, анализдөө маселеси каралат. Бул макалада бул сыяктуу маселелер сызыктуу эмес теңдемелер үчүн каралат.
Урунттуу сөздөр: интегралдык теңдеме, сызыктуу эмес теңдеме, Вольтерра тибиндеги теңдеме, алгоритм, аналитикалык функция.

Ранее автор построила и реализовала на компьютере следующие алгоритмы. Дано уравнение со степенными сомножителями при интегральных слагаемых, алгоритм представляет данные для определения существования решения и наличия в нем произвольных постоянных; также дано уравнение с коэффициентом, находятся особые значения коэффициента. В данной статье такие вопросы рассматриваются для нелинейных уравнений.
Ключевые слова: интегральное уравнение, линейное уравнение, уравнение типа Вольтерра, алгоритм, аналитическая функция.

Introduction

Supra the author constructed and implemented the following algorithms on a computer [1-4]. Given a linear equation with power coefficients by integral summands, the algorithm presents data to detect existence of solutions and occurrence of arbitrary constants in it; also, given an equation with a coefficient, specific values of the coefficient are found.

In this paper these items are considered for non-linear equations.

1. Survey of preceding results

As is customary, if the equation can be rewritten as $u^{(m)}(t) = Y(u(s), 0 \le s \le t)$ where the right hand part depends (continuously) on past only and contains lower derivatives and integrals then it is said to be a Volterra equation of the second kind; one of type $b(t)u^{(m)}(t) = Y(u(s), 0 \le s \le t)$ (the function $p(t)$ sometimes vanishes but is not zero) is said to be a Volterra equation of the third kind.

We will use denotations

$R := (-\infty;\infty);\ R_+ := [0;\infty);\ R_{++} := (0;\infty);\ Z := \{..., -2, -1, 0, 1, 2, 3, ...\};$

$N_0 := \{0, 1, 2, 3, ...\};\ N := \{1, 2, 3, ...\}.$

We will use the term ”Algorithm” as it is usually understood in Analysis: arithmetical operations and comparison over numbers in *R* (for rational numbers this definition coincides with the strict one).

We will write discrete arguments in brackets to bring denotations nearer to algorithmic ones and to bypass the common ambiguity of expressions such as a_{2j}.

We will consider given and unknown real-valued analytical functions in the form

$$f(t) = f_0 + f_1 t + f_2 t^2 + \ldots, \tag{1}$$

$$u(t) = u_0 + u_1 t + u_2 t^2 + \ldots \tag{2}$$

Denote $h[j]=0$ $(j<0)$; $h[j]=1$ $(j\geq 0)$. We introduced the function

$A[m,n] = \frac{n!}{(n-m)!} h[n]h[n-m]$.

<u>Example 1</u> of applying the algorithm for linear integral equations. Consider the equation

$$tu(t) + 3\int_0^t u(s)ds - \int_0^t \int_0^s u(v)dvds = f(t). \tag{3}$$

Input of initial data in the program:

Venera Ordinary IDE 2019

Input number of summands 2<= K <=5: 3

Input coef. b[1], t^p[1], int/dif m[1]: 1 1 0

Input coef. b[2], t^p[2], int/dif m[2]: 3 0 -1

Input coef. b[3], t^p[3], int/dif m[3]: -1 0 -2

Result:

Equation

+t u(t) +3 int_0^t u(s)ds - (int_0^t)^2 u(s)ds = f(t)*

System of equations for coefficients

*+A(0,n-1) u[n-1] +3*A(-1,n-1) u[n-1] -A(-2,n-2) u[n-2] = f[n]*

First equations for coefficients

0 = f[0]

*+0!/0! * u[0] +3*0!/1! * u[0] = f[1]*

*+1!/1! * u[1] +3*1!/2! * u[1] -0!/2! * u[0] = f[2]*

*+2!/2! * u[2] +3*2!/3! * u[2] -1!/3! * u[1] = f[3]*

... (similar summands)

*+8!/8! * u[8] +3*8!/9! * u[8] -7!/9! * u[7] = f[9]*

*+9!/9! * u[9] +3*9!/10! * u[9] -8!/10! * u[8] = f[10]*

...

It is seen that firstly *u[0]* is defined, further *u[1]* is done by means of *u[0]*, *u[2]* is done by means of *u[1]* etc. Coefficients by *u[n]* are greater than 1 wherein coefficients by *u[n–1]* are less than 1. Hence,

Theorem 1. If *f(0)=0* then the equation (3) has an analytical real-valued solution *u(t)*. Its radius of convergence is the same as one of the function *f(t)*.

The following examples demonstrate that sometimes linear integral equations do not have analytical solutions.

Example 2. The following equation *tu(t) = 1* does not have an analytical solution.

Example 3. Consider the following equation

$$-t^2u(t) + \int_0^t u(s)ds = t, t \in R_+. \quad (4)$$

The formal series $u(t) = 1 + 2t + 2*3t^2 + ... + (n + 1)!t^n + ...$ exists but does not converge.

2. Some auxiliary results

Lemma 1. The sum γ_n of numerical coefficients by t^n in expansion of a function $u^2(t)$ equals *(n+1)*.

Proof. If *n=2k* is even then we have the coefficient

$$C_n(u) = u_k^2 + 2u_0u_{2k} + \ ... + 2u_{k-1}u_{k+1};$$

the sum γ_n of numerical coefficients is *1+2(k+1)=n+1*.

If *n=2k+1* is odd then we have the coefficient

$$C_n(u) = 2u_0u_{2k+1} + \ ... + 2u_ku_{k+1};$$

the sum γ_n of numerical coefficients is *2(k+1)=n+1*.

Lemma 2. Formally $\int_0^t (u_0 + u_1s + u_2s^2 + \cdots)^2 ds =$

$$= u_0^2t + u_0u_1t^2 + (u_1^2 + 2u_0u_2)t^3/3 + \cdots = \sum_{n=1}^{\infty} C_{n-1}(u)t^n /n.$$

3. Algorithm for quadratic integral equations

Consider the following type of equations

$$P(t)u(t) + \lambda \int_0^t u^2(s)ds = f(t), t \in R_+ \qquad (5)$$

where $P(t)$ is a polynomial (if $P(0)=0$ then $f_0=0$).

The algorithm is the following.

3.1. Substitute (1) and (2) in (5).

3.2. Substitute $u^2(s)$ by means of $C_n(u)$.

3.3. Calculate the integral and equalize coefficients by powers of t.

3.4. Consequently, for t^0, t^1, t^2,... obtain equations for λ.

3.5. By condition of non-solvability with respect to λ define the set of values of λ.

3.6. By using Lemma 1, estimate domain of convergence of the series (2) with respect to one of the series (1).

3.7. Intersection of the set of 3.5 and the domain of 3.6 yield the spectrum for λ.

4. Examples of applying the algorithm

Example 4. Consider the equation which can be solved in evident form:

$u(t) + \lambda \int_0^t u^2(s)ds = a;\ u'(t) + \lambda u^2(t) = 0;\ u(t) = a/(1 + \lambda at)$.

A spectrum is absent; the series converges for $|t|<1/|\lambda a|$.

Example 5. Consider the equation

$$tu(t) + \lambda \int_0^t u^2(s)ds = f(t),\ f(0) = 0. \qquad (6)$$

$u_0t + u_1t^2 + u_2t^3 + \cdots + \lambda \int_0^t (u_0 + u_1s + u_2s^2 + \cdots)^2 ds =$

$= f_1t + f_2t^2 + f_3t^3 + \ldots$

$u_0t + u_1t^2 + u_2t^3 + \cdots + \lambda \int_0^t (u_0^2 + u_1^2s^2 + u_2^2s^4 + \cdots + 2u_0u_1s + \cdots)ds =$

$= f_1t + f_2t^2 + f_3t^3 + \ldots$

$u_0 + \lambda u_0^2 = f_1;\ u_1 + \lambda u_0u_1 = f_2;\ u_2 + 1/3 \cdot \lambda u_1^2 + 2/3 \cdot \lambda u_0u_2 = f_3;$

$u_3 + 2/4 \cdot \lambda u_0u_3 + 2/4 \cdot \lambda u_1u_2 = f_4; \ldots$

The first equation:

$$\text{if } \lambda f_1 \geq -1/4 \quad \text{then } u_{0,12} = (-1 \pm \sqrt{1 + 4\lambda f_1})/(2\lambda). \qquad (7_1)$$

The second equation:

$$\text{if } q_{2,12} := 1 + \lambda u_{0,12} \neq 0 \text{ then } u_{1,12} = f_2/q_{2,12}. \qquad (7_2)$$

The third equation:

if $q_{3,12} := 1 + 2/3 \cdot \lambda u_{0,12} \neq 0$ then $u_{2,12} = (f_3 - 1/3 \cdot \lambda u_1^2)/q_{3,12}$. (7$_3$)

… The n-th equation: denote $q_{n,12} := 1 + 2/n \cdot \lambda u_{0,12}$;

if $q_{n,12} \neq 0$ then $u_{n-1,12} = \left(f_n - \lambda\left(C_{n-1}(u) - 2u_{0,12}u_{n-1,12}\right)/n\right)/q_{n,12}$. (7$_n$)

Remark. "$u_{n-1,12}$" is absent in the right side.

Theorem 2. If (7$_1$), all $q_{n,12} \neq 0$ and $|f(t)|$ is sufficiently small in a neighborhood of zero then (5) has a solution in a neighborhood of zero.

Proof. Let $|f_n| < pw^n$ $(n > 0)$.

Choose $n_0 > 4|\lambda u_0|$ then $q_n > 1/2 (n \geq n_0)$ and denote

$v_0 = min\{1/2, min\{|q_n|: 2 \leq n < n_0\}\}$

then $|q_n| \geq v_0$;

$$|u_{n-1}| \leq (|f_n| + |\lambda| \cdot |C_{n-1}(u) - 2u_0 u_{n-1}|/n)/v_0 (n \geq 2). \quad (8)$$

Choose $u_0 = (-1 + \sqrt{1 + 4\lambda f_1})/(2\lambda)$. We have $|u_0| \leq |f_1| < pw$. Further, $|u_1| = |f_2/q_2| \leq pw^2/v_0$.

Denote $v_1 := max\{1; 1/v_0\}$. Then $|u_0| < pv_1 w$; $|u_1| < pv_1 w^2$. Let us prove by induction that $|u_n| < 2pv_1 w^{n+1}$.

Estimate by Lemma 1:

$|C_{n-1}(u) - 2u_0 u_{n-1}| \leq \gamma_{n-1} n \, max\{|u_k u_{n-1-k}|: 1 \leq k \leq n-2\} <$

$< \gamma_{n-1} \, max\{4p^2 v_1^2 w^{k+1} w^{n-1-k+1}: 1 \leq k \leq n-2\} = \gamma_{n-1} 4p^2 v_1^2 w^{n+1}$;

$|u_{n-1}| \leq (pw^n + 4|\lambda| \cdot p^2 v_1^2 w^{n+1})/v_0 \leq pv_1 w^n (1 + 4|\lambda| \cdot pwv_1^2)$ $(n \geq 2)$.

If p is small: $4|\lambda| \cdot pwv_1^2 < 1$ then $|u_{n-1}| < 2pv_1 w^n$.

The series (2) converges for $|t| < 1/w$.

Theorem is proven.

5. Conclusion

Theorem 2 demonstrates that there exist non-linear Volterra integral equations of the third type which have infinite spectrum.

REFERENCES

1. Мураталиева В.Т. Спектральные свойства линейных вольтерровских интегро-дифференциальных уравнений третьего рода второго порядка //

Наука вчера, сегодня, завтра: сб. статей по матер. XXXIV междунар. научно-практ. конф. № 5(27). Часть I. – Новосибирск: СибАК, 2016. – С. 57-61.

2. Панков П.С., Мураталиева В.Т. Спектральные свойства линейных задач с аналитическими функциями // Доклады Национальной академии наук Кыргызской Республики, 2016, № 1. – С.11-14.

3. Мураталиева В.Т. Алгоритм для исследования спектральных свойств линейных задач с аналитическими функциями // Вестник Жалал-Абадского государственного университета, 2016, № 1 (32). – С. 55-59.

4. Muratalieva V.T. Method and algorithm to investigate integro-differential equations with analytical functions // Тезисы докладов Международной научной конференции «III Борубаевские чтения», посвященной 35-летию со дня образования Института математики НАН КР. - Бишкек: Институт математики, 2019. - С. 48.

MSC 49J15

MATHEMATICAL MODELS OF INCREMENT OF ENTROPY IN AFFECTABLE SYSTEMS

Pankov P.S., Akerova Dzh.A.
Institute of Mathematics of NAS of KR

As a specification of the second law of thermodynamic for isolated systems, supra the authors introduced new definitions, proposed general hypotheses and derived estimations from below on increasing of entropy while motion of a material point both without friction and with friction over definite distance on depending on time in permanently unstable (affect table) systems. Such estimations are generalized in this paper.

Keywords: entropy, control, differential equation, affectable system, friction, motion.

Четтетилген системада термодинамиканын экинчи законун тактыктоо катары, буга чейин авторлор жаңы аныктамаларды киргизишкен, жалпылаган гипотезаларды сунушташ-кан жана таасир этилүүчү системада материалдык чекитти сүрүлүүсүз жана сүрүлүүнүн негизинде кандайдыр бир аралыкка жылдырууда убакыттан көз каранды болгон энтропия-нын өсүүсүнүн төмөнкү баасын алышкан. Бул макалада жогоруда көрсөтүлгөн баалоолор жалпылатылган.

Урунттуу сөздөр: энтропия, башкаруу, дифференциалдык теңдеме, таасир этилүүчү система, сүрүлүү, кыймылдоо.

Как уточнение второго закона термодинамики для изолированных систем, ранее ав-торы ввели новые определения, предложили общие гипотезы и получили оценки снизу для возрастания энтропии при передвижении материальной точки без трения и с трением на определенное расстояние в зависимости от времени в перманентно неустойчивых системах. В данной статье такие оценки обобщены.

Ключевые слова: энтропия, управление, дифференциальное уравнение, перманентно неустойчивая система, трение, движение.

Introduction

The second law of thermodynamicfor isolated systems states increment of entropy but does not give quantitative estimations.

One estimation was proven in [2]: the minimal energy (increment of entropy) to treat one bit of information is the Shannon-von Neumann-Landauer boundary:

$E_{bit} = k_B\ T\ ln\ 2$ where k_B is the Boltzmann constant and Θ is an absolute temperature.

A hint to such estimation in general case was in [3]: “In any mechanical system the energy that must be expended to work against friction is equal to the product of the frictional force and the distance through which the system travels. Hence the faster a swimmer travels between two points, the more energy he or she will expend, although the distance traveled is the same whether the swimmer is fast or slow.”

Basing on the notion of economical (cruising) speed, taking into account ide-as of Ecological Rallies and Shell Eco-marathon for cars we specified the second law of thermodynamic for “almost isolated”, “permanently unstable”, “affect table” sys-tems. We proved some estimations in mathematical models describing such concrete systems. In this paper we generalize some these results.

1. Definition and formula for entropy

By one of definitions, entropy is the measure of a system’s thermal energy per unit temperature that is unavailable for doing useful work. Because work is obtained from ordered molecular motion, the amount of entropy is also a measure of the molecular disorder, or randomness, of a system.

In processes considered below increment of entropy is de fined by the formula $\Delta H = \sum \Delta Q / \Theta$ where ΔQ is a quantity of transferred heat energy or energy that was converted to heat irreversibly and Θ is an absolute temperature [1].

2. Definition of almost isolated systems

Definition 1. If low energetic outer influences can cause sufficiently various reactions and changings of the inner state of the system then it is said to be an “almost isolated”, “permanently unstable”, “affecttable” system.

Such outer influences are said to be commands (these reactions and changings are implemented by means of inner energy of the object or of outer energy entering into object besides of commands).

3. Main hypotheses

Let there is an almost isolated physical system. Let it is in any stationary state *A* now and there it can pass to any other stationary state *B*.

Hypothesis 1. There exists such time T_0 (the adiabatic time of the system), depending only on the initial state of the system, that the increasing ΔH of the entropy of the system is not less than any positive value for any transition from the state *A* to the state *B* during $T<T_0$. Moreover, there also exists such positive constant C_0 that $\Delta H \geq C_0 / T^2$ (the dimension of C_0 is mass × length ×length / temperature).

Remark. If we accept the principle of determinism then there is only scenario of the future for any isolated system, that is, there cannot be different possibilities of transitions. Hence, the system is to be almost isolated: different possible actions within the system, transforming it from the state *A* to the state *B* are controlled by any outer impetus (control) of sufficiently small energy.

Give a more concrete hypothesis. Let any point of mass *m* does not move in any inertial coordinate system at the moment t_1 and it is at the distance *d* from its initial state and does not move at the moment $t_2 = t_1 + T$.

Hypothesis 2. There exists such time T_0 (the adiabatic time of the system), depending only on the initial state of the system, that the increasing of the entropy fulfills the inequality $\Delta H \geq G_0\, m\, d^2 / T^2$ for any transition from the state *A* to the state *B* during $T<T_0$ (the dimension of G_0 is 1 / temperature).

Denote ΔH for adiabatic time as ΔH_0.

Substantiation. To move the point has to acquire velocity ~ d/T, i.e. kinematical energy E_{kin}~md^2/T^2. As the point is motionless in the final state, this energy has to pass into other kinds of energy. But possibilities of kinematical energy to pass into potential, chemical etc. ones during bounded time are bounded. Hence a greater part of kinematical energy has to pass into heat, i.e. the increment of entropy must be $\Delta H \sim md^2/T^2/\Theta$.

Another substantiation. If any estimation on ΔH exists then it must be of dimension of entropy: mass × length ×length /(time ×time×temperature).

Remark. The notions of adiabatic time and corresponding optimal speeds of the system generalize the well-known notion of cruising speed (which is applicable to homogeneous motion on sufficiently large distances only).

4. Examples

Example 4.1. Let B be over A at the height h in gravitation field. There is the load of mass m being based on many compressed short almost ideal springs at the point A, the massive brake with the absolute temperature Θ along the segment AB and the catcher at the point B. It is necessary to deliver the load from the point A to the point B during the time T.

The adiabatic time $T_0 := \sqrt{(2h/g)}$ coincide switch the free fall time from B to A. Correspondingly, $\Delta H_0 \approx 0$ in this case. We release some springsto launch the load with initial velocity $v_0 := \sqrt{(2gh)}$ necessary to deliver the load to the point B with zero velocity. Falling back the load will com press same spring sand the system will return in almost initial state.

If $T<T_0$ then the initial velocity $v_1>v_0$. Braking near the point B we obtain

$$v_1 T - gT^2/2 \approx h, \tag{1}$$

and the velocity before braking approximately is

$$v_B := v_1 - gT. \tag{2}$$

We have from (1) and (2):

$$v_1 = h/T + gT/2, \;\; v_B = v_1 - gT = h/T - gT/2. \tag{3}$$

Hence, before braking the kinetic energy of the load is $E_{kin}:=mv_B^2/2$. All it must turn to heat,

$\Delta H \sim \Delta Q/\Theta = E/\Theta = mv_B^2/2/\Theta = m\,(h/T - gT/2)^2/2/\Theta;$

$\Delta H \sim\ mh^2/T^2\,(1 - gT^2/(2h))^2/2/\Theta = mh^2/T^2\,(1 - T^2/T_0{}^2)^2/2/\Theta.$

Hence $\Delta H \sim\ mh^2/T^2/2/\Theta$ as $T \to 0; G_0 = 1/(2\Theta)$.

Example 4.2. Let A be over B at the height h in gravitation field and B be on an absolutely elastic plane. There is the load of mass m at the point A, the massive brake with the absolute temperature Θ along the segment AB and the catcher at the point A. We can apply unbounded force to the load. It is necessary to return the load to the point A by repelling at the point B during the time T.

We have the following problem of optimal control:

$$y''(t) = \frac{u(t)}{m} - g(0 \le t \le T), y(0) = h; y'(0) = 0;\ \ y(T) = h, y'(T) = 0 \quad (4)$$

with the additional condition: there exists such moment $w \in (0,T)$ that $y(w)=0; y'(w-0)>0; y'(w+0) = -y'(w-0)$.

Here the controlling (piecewise continuous) function $u(t)$ has the dimension of force$[u(t)] = [m][L]^2[\mathrm{T}]^{-2}$. The goal is to minimize ΔH.

We obtain the adiabatic time (with $u(t) \equiv 0$)$T_0 = 2\sqrt{2gh}$.

Obviously, the optimal control is: $u(t)$ is very large at the initial moment and further $u(t) \equiv 0$. Let $y'(+0) = -v$, then $y(t) = h - vt - \frac{1}{2}gt^2\ (y(t) > 0)$.

Hence $h - vw - \frac{1}{2}gw^2 = 0$, by symmetry $w = \frac{1}{2}T$ and

$$v = (h - \frac{1}{2}gw^2)/w\ = 2(h - \frac{1}{8}gT^2)/T.$$

Kinetic energy of the load in the end almost equals kinetic energy of the load in the beginning and all it transforms to heat. Hence

$$\Delta H \sim \frac{1}{2}m\left(2(h - \frac{1}{8}gT^2)/T\right)^2/\Theta = 2m\left((h - \frac{1}{8}gT^2)/T\right)^2/\Theta, T < 2\sqrt{2gh}.$$

As $T \to 0$ we obtain $\Delta H \sim 2mh^2/T^2/\Theta$. This substitutes Hypothesis 2.

Example 4.3. We consider the example of swimmer [2]. Let the segment *[A, B]=[0, h]* be horizontal. Suppose that friction *F* in the water is proportional to any power of velocity. Obviously, for optimal motion $x'(t)\geq 0$.

$$x''(t) = -p\left(x'(t)\right)^k + \frac{u(t)}{m}(0 \leq t \leq T), p > 0, k > 0,$$

$$x(0) = 0; x'(0) = 0;\ x(T) = h, x'(T) = 0, \quad (5)$$

$$\Delta H \sim J(x,u) := \int_0^T p(x'(t))^k\, x'(t)dt \to min.$$

Pass to dimensionless variables: $t=T\tau$ $(0\leq\tau\leq 1)$, $x=h\xi$; $x'(t)=h\xi'(\tau)/T$; $dt=Td\tau$.

$$J = \int_0^1 p(h\xi(\tau)/T)^{k+1}\, Td\tau = ph^{k+1}/T^k \int_0^1 \left(\xi(\tau)\right)^{k+1} d\tau.$$

Hence, $\Delta H \sim ph^{k+1}/T^k/\Theta\cdot\gamma$ where γ is an absolute dimensionless constant. On one hand, it is well-known that *k~2* for high velocities; on other hand, the value *k=2* corresponds to Hypothesis 2.

Conclusion

We hope that these hypotheses would be substituted by investigation of other processes. We see: the less is the time of transition the more is spending of energy for braking and the more is the (unavoidable) increment of entropy which is proportional to pollution of environment. Probably, it explains some processes of modern civilization.

REFERENCES

1. Мартин Н. Ингленд Дж. Математическая теория энтропии. – Москва: Мир, 1988. – 350 с.

2. Landauer R. Irreversibility and heat generation in the computing process // IBM J. of Research and Development, vol. 5, 1961, pp. 183-191.

3. Landauer R., Bennett C. H. The Fundamental Physical Limits of Computation // Scientific American, July, 1985, pp. 48-56.

4. Панков П.С. Адиабатические показатели замкнутых систем // Вестник КНУ им. Ж.Баласагына. Серия 3. Естественно-технические науки. Физика и физическое образование, 2003. – С. 146-147.

5. Панков П., Баячорова Б., Жураев М. Кыргыз тилин компьютерде чагылдыруу. – Бишкек: Турар, 2010. – 172 б.

6. Панков П.С. Энергетическое определение понятия «язык», его приме-нение и возможности технической реализации // Известия НАН КР. Серия физико-технических, математических, горно-геологических наук, № 2, 2011. – С. 66-71.

7. Pankov P., Akerova Dzh. Minimization in mathematical model of increment entropy in almost closed systems // Abstracts of V Congress of the Turkic World Mathematicians (Kyrgyzstan, Bulan-Sogottu, 5-7 June, 2014) / Ed. Acad. A. Borubaev. – Bishkek: Kyrgyz Mathematical Society, 2014. – P. 252.

8. Кененбаева Г. М., Акерова Дж.А., Кененбаев Э. Математические модели для оценки приращения энтропии // Тезисы докладов II Борубаевских чтений (г. Бишкек, 1 марта 2018 года). - Бишкек: Кыргызское математическое общество, 2018.

9. Панков П.С., Акерова Дж.А. Дифференциальные уравнения с управлением в модели возрастания энтропии в почти замкнутых системах с трением // Вестник Института математики НАН КР, 2018, № 1. - С. 23-30.

MSC 91F20

MATHEMATICAL MODELS OF HUMAN CONTROL, CLASSIFICATION AND APPLICATION

[1]Pankov P.S., [2]Bayachorova B.J., [3]Karabaeva S.Zh.

[1,3]Institute of Mathematics of NAS of KR, [2]KNU named after J.Balasagyn

In this paper we consider a general mathematical model for interactive software. We carry out a survey of existing and conceivable software, including one proposed and developed with author’s participation. The paper highlights the objectives and specifics of various software including “independent presentation” of an object.

Keywords: human control, mathematical model, computer model, classification, application, training, learning.

Интерактивдик программалык жабдуу үчүн жалпы математикалык моделдер каралды. Программалык жабдуунун учурдагы жана мүмкүн болгон түрлөрүнө, анын ичинде авторлор сунуштаган жана иштеп чыккан түрлөрүнө серер жүргүзүлгөн. Макалада ар кандай түрдөгү программалык жабдуунун, анын ичинде объектти “көз карандысыз чагылдыруунун” максаттары жана өзгөчөлүктөрү баса белгиленет.
Урунттуу сөздөр: адамдын башкаруусу, математикалык модель, компьютердик модель, классификациялоо, колдонуу, машыгуу, үйрөнүү.

Рассмотрена общая математическая модель для интерактивного программного обеспечения. Проведен обзор существующих и возможных видов программного обеспечения, в том числе предложенного и разработанного авторами. В статье подчеркиваются цели и особенности различных видов программного обеспечения, в том числе “независимое представление" объекта.
Ключевые слова: человеческое управление, математическая модель, компьютерная модель, классификация, приложение, тренировка, обучение.

1. Introduction

Training devices for hunting, horsemanship and war are known since ancient times. Mechanical flight simulators appeared together with the development of aviation. Computers gave the opportunity to create simulations with real-time feedback and elements of virtual reality. These ideas were also implemented in computer games. Educational computer software and educational games developed together with the development of personal computers.

The paper contains a general mathematical model of such software, remarks on its implementation as a computer model, a list of known and possible kinds of software (some of which has been implemented with authors’ participation).

2. Mathematical model for human control

$N_0:=\{0,1,2,\ldots\}$ contains values of discrete time t; $R_+:=[0,\infty)$;

Denote X as the space of states x (including virtual media and objects in it); $X_0 \subset X$ as the set of targets; $Q: X \to R_+$ as the target function to be minimized;

V as the set of observable (affectable by human interaction) elements of X; $W: X \to V$ is a given function;

P as the set of random elements p;

U as the set of possible actions u by the user (control).

We will consider discrete models. Continuous models are obtained from discrete ones by setting time divisions/steps to zero.

We propose the system

$x[0]$ $(x[0]=Z(p[0]))$ *is given (either* $x[0] \notin X_0$ *or* $Q(x[0]) > 0)$; (1)

$$v[t]=V(x[t]);\ x[t+1]=F(x[t], u[t], p[t]),\ \text{or}\ x[t+1]=x[t]+G(x[t], u[t], p[t]),\ t \in N_0 \quad (2)$$

where $u[t]$ is the action of the user influenced by information $v[t]$; $Z(p):P \rightarrow X$ is a function of random generation of initial data.

The goal is either to reach $x[t+1] \in X_0$ in minimal time or to minimize $Q(x[t+1])$ in a given time.

Two options of input *$x[0]$ by the user* and *random* give us two modes: learning mode and exam mode.

In advanced software *TaskLang* [6] the user can choose functions *F (*or *G)* too.

It may be $x=\{x_1,\dots,x_n\}$, $x_1,\dots,x_n$ are input independently; it is a necessary-collective task for *n* users.

The principle of duality [1]: (narrow *V* and wide *U*) and (wide *V* and narrow *U*) yield similar efficiency.

This principle extends for different kinds of human activity: Duality of available information and available goal achievement capacity.

3. Computer model specifics

3.1. Input of information $v[t]$

- common (by means of eyesight, hearing, vibration - vestibular apparatus);
- by means of special devices (earphones, binocular displays);
- to brain immediately.

3.2. Output of control $u[t]$:

- common (by means of hands, foots and voice; by top of head in diving suits);
- by reading nerve impulses in hands [2];
- from brain immediately.

General conclusion from [2]: using appropriate equipment for feedback, each physiological display (breath, pulse etc.) by human or animal with cognitive ability (ape, dolphin, dog) can be used for control.

3.3. There is Galileo-Einstein's principle of relativity: if we observe uniform movement of an object towards us then we cannot detect whether the object is moving, or we are.

For virtual motion the condition of uniformity (i.e. no acceleration) is not necessary. Hence, we receive a principle of relativity in virtual motion: if we observe movement in a kinematical space [3] then we can interpret it either movement of space toward us or our movement toward space.

The first interpretation prevails in software for scientific purposes (Mathematica, MathLab, MathCad) and the second one does in computer games.

4. Some cases of computer-human control

4.1. Simulation cases of real control where real training is too difficult, expensive or dangerous include: spaceship, aircraft, boat, U-boat, artillery, launching big rockets, manufacturing processes, medical operations. They consist of random generation of media and random generation of emergences. Simulation made for a crew (for instance, pilot, co-pilot and navigator at aircraft) is an example of necessary-collective activity.

Remark. Some simulators are mixed computer-mechanical solutions that involve vibration and physical inclinations.

4.2. Computer games. Notes:

- some computer games arose from items listed in 4.1;
- computer games involving simulations of real objects (geographical map, concrete vehicles) may be considered educational;
- there are some hints in computer games useful to forthcoming proposals.

Remark. We do not consider games "person versus person" and "team versus team" by means of computers.

4.3. Imitation of physical-chemical experiments - "virtual laboratory".

4.4. Enhancing virtual reality. We [3] proposed to present abstract spaces in form close to presentation of the metric space.

Definition. A pair: a set X of points and a set K of **routes** is said to be a **kinematic space (**each route M, in turn, consists of the positive real number T_M

(**time** of route) and the function $m_M : [0, T_M] \to X$ (**trajectory** of route)) if the following conditions are fulfilled: (K1) For $x_0 \neq x_1 \in X$ there exists such $M \in K$ that $m_M(0) = x_0$ and $m_M(T_M) = x_1$, and the set of values of such T_M is bounded with a positive number below (infinitely fast motion is impossible); (K2) If $M= \{T_M , m_M(t)\} \in K$ then the pair $\{T_M , m_M(T_M - t)\}$ is also a route of K (the reverse motion is possible); (K3) If $M= \{T_M, m_M(t)\} \in K$ and $T^* \in (0, T_M)$ then the pair: T^* and function $m^*(t)=m_M(t)$ $(0 \le t \le T^*)$ is also a route of K (one can stop at any desired moment); (K4) concatenation.

We implemented controlled motion in Riemann surfaces, Moebius band, projective plane and topological torus.

4.5. Experimental mathematics [4]. On one hand, it is using well-known software (Mathematica, MathLab, MathCad), on the other hand a search for new mathematical facts (hypothesis) - a separate direction of investigations.

4.6. Training in deciphering the simplest ciphers alongside with evaluating the knowledge of a language [5].

4.7. Complex examination (for example, [7]) including multimedia tasks, interactive tasks of optimization and solving equations, tasks with objects with-out. Primary versions of such software for Kyrgyz language, mathematics and informatics were implemented and are in use.

4.8. Measuring imagery [8]. **Definition**. The problem is said to be intellectual eye measurer (or measuring imagery) - its conditions are strict but the approximate answer is permissible; using any tool (computer, pen-and-paper, reference book) is forbidden; in sciences the time given to answer is about 20 - 30 seconds to avoid immediate mental counting. If the answer is a real number then $Q(x) = |x - x_0|$ or $Q(x)=|log(x/x_0)|$ (for $x_0 > 0$) where x_0 is the exact answer.

We have introduced competitions on students' capacities in this subject matter.

4.9. Necessarily-collective tasks [9]. For example such task includes: transformation of sign systems: the first teammate is given a drawing (a set of similar drawings); s/he describes it in a prescribed language (during 15-20 minutes) and this text is sent to the second teammate by an intermediary; s/he restores the drawing (the consequence of drawings) (during 10-15 minutes).

4.10. Software to correct pronunciation.

4.11. Independent interactive presentation of objects. If a computer presentation does not depend on the user's knowledge and skills on similar objects then it is said to be independent.

4.11a. Interactive presentation of some mathematical objects [10].

4.11b. Interactive presentation of basic of language. Earlier, learning a living language was implemented with the assistance (including bilingual dictionaries and text-books) of persons who had a complete command of it; investigating of a dead language was done by means of remained bilingual texts and texts with additional implicit suggestions and conclusions. Invention of recording sounds gave possibility to fix examples of an oral language objectively. Invention of talking pictures fixed examples of phrases with connection to situations and actions. Computer games gave the user the opportunity to choose actions with corresponding phrases.

Before our publications, existed software to learn languages were based on languages native to the user.

We proposed [11-15] **Definition**. Let any "notion" (word of a language) be given. If an algorithm acting at a computer: - performs (generating randomly) sufficiently large amount of situations covering all essential aspects of the "notion" to the user; - gives a command involving this "notion" in each situation; - perceives the user's actions and performs their results clearly on a display; - detects whether a result fits the command, then such algorithm is said to be a computer interactive presentation of the "notion".

Simple mathematical models consist of fixed (F_i) and movable (M_j) sets and temporal sequence of conditions of types $(M_j \subset F_i)$, $(M_j \cap F_i = \varnothing)$, $(M_j \cap F_i \neq \varnothing)$.

Remark. 4.10) can also be involved.

Sketches of such software were implemented for Kyrgyz, English and Turkish languages. A proposal for Chinese language was in [16].

5. Conclusion

We hope that developing this method would yield new types of educational software both interesting and useful for students. For instance, combination of 4.3) and 4.11a) can give independent presentation of some physical notions; adding of mathematical tasks with physical content can give a complex examination in physics.

REFERENCES

1. Панков П.С. Двойственность параметров управления и наблюдения при получении гарантированных оценок // Проблемы теоретической кибернетики: Тез. докладов IX всесоюзной конференции (сентябрь 1990 года). - Волгоград, 1990. - Часть 1(2), с. 57.

2. Панков П.С. Проект автоматизации ввода дискретной информации с помощью биотоков, считываемых с мышц человеческих рук // Проблемы автоматики и управления: Научно-технический журнал / НАН КР. – Бишкек: Илим, 2005. – С. 125-127.

3. Борубаев А.А., Панков П.С. Компьютерное представление кинематических топологических пространств. - Бишкек: КГНУ, 1999. – 131 с.

4. Grenander U. Mathematical experiments on the computer. - New York: Academic Press, 1982. - 525 p.

5. Борубаев А.А., Панков П.С. Дискретная математика (допущено МОН КР в качестве учебного пособия для преподавателей вузов). - Бишкек: изд. КРСУ, 2010. – 123 с.

6. Долматова П.С. Автоматизированная система интерактивного операционного представления понятий естественных языков/ Автореферат дисс. … к.т.н., специальность 05.13.06 – автоматизация и управление технологическими процессами (образование). - Бишкек, 2013. - 21 с.

7. Pankov P., Dolmatova P. Software for Complex Examination on Natural Languages // Human Language Technologies as a Challenge for Computer Science

and Linguistics: Proceedings of 4th Language and Technology Conference, 2009, Poznan, Poland. – P. 502-506.
8. Pankov P.S. Independent learning for Open society // Collection of papers as results of seminars conducted within the frames of the program «High Education Support». Bishkek: Foundation «Soros-Kyrgyzstan», 1996. - Issue 3, pp. 27-38.
9. Pankov P.S. Necessarily-Collective Computer Competitions for Schoolchildren // Information Technologies at Schools: Proceedings of the Second International Conference on Informatics in Secondary Schools "Evolution and Perspectives", 2006, Vilnius, Lithuania. – Pp. 585-588.
10. Панков П.С., Барыктабасов К.К. Инварианты при бесконечномерных геометрических преобразованиях и их компьютерная реализация // Вестник КГПУ им. И.Арабаева, 2004, серия 3, выпуск 2. – С. 56-60.
11. Pankov P.S., Aidaraliyeva J. Sh., Lopatkin V. S. Active English on computer // Conference «Improving Content and Approach in the Teaching of English Language in the Context of Educational Reform», Bishkek, 1996. - pp. 25-27.
12. Pankov P.S., Alimbay E. Virtual Environment for Interactive Learning Languages // Human Language Technologies as a Challenge for Computer Science and Linguistics: Proceedings of 2nd Language and Technology Conference, Poznan, Poland, 2005. – Pp. 357-360.
13. Панков П., Баячорова Б., Жураев М. Кыргыз тилин компьютерде чагылдыруу. – Бишкек: Турар, 2010. – 172 б.
14. Pankov P.S., Bayachorova B.J., Juraev M. Mathematical Models for Independent Computer Presentation of Turkic Languages // TWMS Journal of Pure and Applied Mathematics, Volume 3, No.1, 2012. – Pp. 92-102.
15. Карабаева С. Единый алгоритм словоизменения и представление пространства в кыргызском языке. – Saarbrűcken, Deutschland: Lap Lambert Academic Publishing, 2016. – 62 с.
16. Pankova M. Mathematical models of notions of Chinese language // Abstracts. Issyk-Kul International Mathematical Forum, Bozteri, Kyrgyzstan, 2015. - P. 80.

MSC 70F99

EXISTENCE AND STABILIZATION OF SOLUTION OF SYSTEM OF DIFFERENTIAL EQUATIONS DESCRIBING ARRANGEMENT OF REPELLING POINTS ON A SEGMENT

Tagaeva S.B.
Institute of Mathematics of NAS of KR

Considered the system of ordinary differential equations with discontinuous right hand parts presenting mutual repelling of points charges in very viscous media. Existence and stabilization of a smooth solution on a half-axis is proven in come cases.

Keywords**:** mathematical model, particle, repelling, ordinary differential equation, system of equations, smooth solution, stabilization.

Абдан жабышкак чөйрөдө чекиттердин түртүшүүнү чагылдыруучу, оң жактары үзгүл болгон кадимки дифференциалдык теңдемелер системасы каралат. Кээ бир учурда жылма чыгарылыштын жарым окто бар болуусу жана туруктуу абалга келтирүүсү далилденген.

Урунттуу сөздөр: математикалык модел, бөлүкчө, түртүшүү, кадимки дифференциалдык теңдеме, теңдемелер системасы, жылма чыгарылыш, туруктуу абалга келтирүү.

Рассматривается система дифференциальных уравнений с разрывными правыми частями, представляющая взаимное отталкивание точек в очень вязкой среде. В некоторых случаях доказаны существование и стабилизация гладкого решения на полуоси.

Ключевые слова: математическая модель, частица, отталкивание, обыкновенное дифференциальное уравнение, система уравнений, гладкое решение, стабилизация.

Introduction

Distributions of continuous electrical (similar, repelling) charges on conductors were considered in many papers. Also, theorems on uniqueness of such distributions were proven. For instance, continuous charges on surfaces were considered in [1], ones on lines were done in [2]. We considered distribution of discrete electrical charges of same sign. We found that such distributions have other properties, including absence of uniqueness. For example, two charges have three stable arrangements within a regular triangle.

In [3] we put a general problem to investigate arrangements of discrete electrical charges of same sign, considered motion of a charge under sum of all repelling forces from other charges within the interior of a domain and along its boundary. In [4] we proved existence of solution of task for two moving charges on an axis; in [5] we also did for one immovable charge and one moving charge on a half-axis. We also proved existence of solution of evident task for two immovable charges and one moving charge on a segment. In [6] we considered an arbitrary number of charges on a segment. Here we consider points repelling by a more general law.

1. Stating of the problem

Denote $\boldsymbol{R}:=(-\infty,\infty)$, $\boldsymbol{R}_+:=[0,\infty)$, $\boldsymbol{S}:=[0,1]$. Let the time $t\in\boldsymbol{R}_+$.

We will consider $(n+1)$, $n\geq 2$ points moving on the segment $\boldsymbol{S}$. Denote their coordinates as $x_i(t)$, $i=0..n$.

They are mutually repelling by the law: the repelling force between two points is $F = ad^{-\beta}$ where d is the distance between the charges, $a = const >0$, $\beta = const >0$. Because of repelling, the marginal points are immobile:

$$0 \equiv x_0(t) < x_1(t) < x_2(t) < \dots < x_{n-1}(t) < x_n(t) \equiv 1$$

(while these functions exist).

Suppose that the media of $\boldsymbol{S}$ is very viscous. Then forces of inertia can be neglected and a force pushes a body in the media as well as it is immobile each moment. Also, suppose that the velocity of a body in the media is proportional to the force. Thus we obtain the following system of ordinary differential equations of the first order

$$x_k'(t) = \sum_{i=0}^{k-1} \frac{ab}{(x_k(t)-x_i(t))^\beta} - \sum_{i=k+1}^{n} \frac{ab}{(x_k(t)-x_i(t))^\beta}, \quad k = 1..n-1, b = const \tag{1}$$

with initial conditions

$$0<x_1(0)=z_1<x_2(0)=z_2<\dots<x_{n-1}(t)=z_{n-1}<1. \tag{2}$$

Without loss of generality suppose that $ab=1$. Denote the first sum in (1) as $F[left,k]$ and do the second sum as $F[right,k]$. Then

$$x_k'(t) = F[left,k] - F[right,k].$$

The task is to prove existence of a solution of the initial value problem (1)-(2).

2. Auxiliary estimations

L e m m a 1. There exist such positive constant γ_{n1} that if $z_1< \gamma_{n1}$ then

$$x_1(t)< \gamma_{n1} \tag{3}$$

(while a solution of (1)-(2) exists), one can take

$$\gamma_{n1} = \frac{1}{n-2}\left((n-1)^{1/\beta} - 1\right). \tag{4}$$

P r o o f (by contradiction). (For brevity $\gamma_{n1}=\gamma$). Suppose that (3) does not take place. Then there exists such $t_1 \in \boldsymbol{R}_+$ that (for the first time) $x_1(t_1)=\gamma$. Then $F[left,1] = \frac{1}{\gamma^\beta}$. "Putting" all $(n-1)$ right charges as far as possible (to the point $x=1$) we have $F[right,1] > \frac{n-1}{(1-\gamma)^\beta}$. Thus, if $\frac{1}{\gamma^\beta} = \frac{n-1}{(1-\gamma)^\beta}$ then

$x_1'(t_1) = F[left,1] - F[right,1] < 0$

and we have a contradiction. The number (4) is the solution of this equation. Lemma is proven.

Lemma 2. There exist such positive constant $\chi_{31}<\gamma_{31}$ that for $n=3$

$$\text{if} \quad z_1> \;\; >\chi_{31} \quad \text{then } x_1(t)> \chi_{31} \tag{5}$$

(while a solution of (1)-(2) exists), one can take (for brevity $\chi_{31}=\chi$, $\gamma_{31}=\gamma$) the constant χ as the (unique) positive solution of the equation $\frac{1}{\chi^\beta} = \frac{1}{(1-\gamma-\chi)^\beta} + \frac{1}{(1-\chi)^\beta}$.

Proof (by contradiction). Suppose that (5) does not take place. Then there exists such $t_0 \in \boldsymbol{R}_+$ that (for the first time) $x_1(t_0)= \chi$. Then $F[left,1] = \frac{1}{\chi^\beta}$.

"Putting" the right moving (third) point as close as possible (to the point $x=1-\gamma$) we have $F[right,1] < \frac{1}{(1-\gamma-\chi)^\beta} + \frac{1}{(1-\chi)^\beta}$. Thus, if χ fulfills the equation then

$x_1'(t_0) = F[left,1] - F[right,1] < 0$

and we have a contradiction. Lemma is proven.

Lemma 3. There exist such positive constant $\chi_{n2} >\gamma_{n1}$ that for $n>3$ *if* $z_2< \chi_{n2}$ then $x_2(t)< \chi_{n2}$ (6)

(while a solution of (1)-(2) exists), one can take (for brevity $\chi_{n2}=\chi$, $\gamma_{n1}=\gamma$) the constant χ as the (unique) positive solution of the equation $\frac{1}{\chi^\beta} + \frac{1}{(\chi-\gamma)^\beta} = \frac{n-2}{(1-\chi)^\beta}$.

Proof (by contradiction). Suppose that (6) does not take place. Then there exists such $t_2 \in \boldsymbol{R}_+$ that (for the first time) $x_2(t_2)= \chi$. "Putting" the left moving (first) point as close as possible by Lemma 1 we obtain $F[left,2] < \frac{1}{\chi^\beta} + \frac{1}{(\chi-\gamma)^\beta}$.

"Putting" the *(n–2)* right moving charges as far as possible (to the point *x=1*) we have $F[right,2] > \frac{n-2}{(1-\chi)^\beta}$. Thus, if χ fulfills the equation then

$$x_2{}'(t_2) = F[left,2] - F[right,2] < 0$$

and we have a contradiction. Lemma is proven.

3. System of algebraic equations for stationary states

Stationary states of the system (1) can be found from the system

$$\sum_{i=0}^{k-1} \frac{1}{(x_k - x_i)^\beta} = \sum_{i=k+1}^{n} \frac{1}{(x_k - x_i)^\beta}, \quad k = 1..n-1. \quad (7)$$

They can be either stable or non-stable.

Theorem 1. If $n=3$ then the system (1) has one stable solution.

P r o o f. We have the system of two equations:

$$H_1(x_1,x_2) := 1/\,x_1^\beta - 1/(x_2 - x_1)^\beta - 1/(1 - x_1)^\beta = 0;$$

$$H_2(x_1,x_2) := 1/\,x_2^\beta + 1/(x_2 - x_1)^\beta - 1/(1 - x_2)^\beta = 0. \quad (8)$$

Obviously, $x_1<1/2<x_2$. Denote $x=x_1$, $y=1-x_2$. Adding we obtain

$$1/\,x_1^\beta - 1/(1 - x_1)^\beta + 1/\,x_2^\beta - 1/(1 - x_2)^\beta = 0;$$

$$1/\,x^\beta - 1/(1 - x)^\beta = 1/\,y^\beta - 1/(1 - y)^\beta. \quad (9)$$

Denote the function $G(x) := 1/\,x^\beta - 1/(1 - x)^\beta$. We have

$$G'(x) = -\beta/\,x^{\beta+1} - \beta/(1 - x)^{\beta+1} < 0.$$

Hence, the equality (9) implies $x=y$.

Then the system (8) reduces to one equation due to symmetry:

$$\frac{1}{x^\beta} = \frac{1}{(1-x-x)^\beta} + \frac{1}{(1-x)^\beta}. \quad (10)$$

Denote $H(x) = \frac{1}{x^\beta} - \frac{1}{(1-2x)^\beta} - \frac{1}{(1-x)^\beta} \left(0 < x < \frac{1}{2}\right)$.

We have: for $x=+0$ $H(x)>0$; for $x=2-0$ $H(x)<0$. The right hand side of (10) increases; the left hand side of (10) decreases. Hence, there exists one root x_{01} of the equation (10). Denote $x_{02}=1-x_{01}$. The root $\{x_{01}, x_{02}\}$ is stable.

The theorem is proven.

4. Existence of solutions of system of differential equations

It is obvious that the initial value problem (1)-(2) has a local solution (for small t) and the task is to prove that it can be continued for any initial values (fulfilling Lemmas 1, 2, 3).

The case $n=2$ is obvious: one equation $x_1'(t) = \frac{1}{x_1^{\beta}(t)} - \frac{1}{(1-x_1(t))^{\beta}}$.

Substituting $y(t) = x_1(t) - \frac{1}{2}$ we obtain: $y'(t) = \frac{1}{\left(\frac{1}{2}+y(t)\right)^{\beta}} - \frac{1}{\left(\frac{1}{2}-y(t)\right)^{\beta}}$;

$y'(t) = -\frac{\left(\frac{1}{2}+y(t)\right)^{\beta} - \left(\frac{1}{2}-y(t)\right)^{\beta}}{\left(\frac{1}{2}+y(t)\right)^{\beta}\left(\frac{1}{2}-y(t)\right)^{\beta}} y(t)$. The "coefficient" by $y(t)$ is strictly negative.

Hence, $y(t)$ tends to 0, $x_1(t)$ tends to $\frac{1}{2}$ as t tends to ∞.

Theorem 1. If $n=3$ then the initial value problem (1)-(2) has a solution for all $t \in \mathbf{R}_+$; it tends to (x_{01}, x_{02}) as t tends to ∞.

P r o o f. (Non-formally): Consider the vector field defined by (1) in the triangle $(0< x_1<1,\ 0< x_2<1,\ \ x_1 < x_2)$ with one stationary point (x_{01}, x_{02}) being a stable node. All angles between vectors and directions to the stationary point are less than 90^0.

Conclusion

By the obtained results we suggest a hypothesis that the initial boundary problem (1) -(2) has a solution on a half-axis for all $n \geq 2$ and it stabilizes as t tends to ∞.

REFERENCES

1. Kolahal Bhattacharya. On the Dependence of Charge Density on Surface Curvature of an Isolated Conductor // Physica Scripta, 2016, Volume 91, Number 3, pp. 1-6.

2. http://teacher.pas.rochester.edu/PHY217/LectureNotes/Chapter4/Lecture Notes Chapter4.html\sharp318

3. Pankov P., Tagaeva S. Mathematical modeling of distribution of discrete electrical charges // Abstracts of the V International Scientific Conference "Asymptotical, Topological and Computer Methods in Mathematics" devoted to

the 85 anniversary of Academician M. Imanaliev / Ed. by Academician A.Borubaev. - Bishkek, 2016. – P. 58.

4. Тагаева С.Б. Исследование систем обыкновенных дифференциальных уравнений с разрывными правыми частями, описывающих отталкивание частиц // Вестник Жалал-Абадского государственного университета, 2016, № 1(32). – С.78-82.

5. Тагаева С.Б. Исследование систем обыкновенных дифференциальных урав-нений с разрывными правыми частями, описывающих отталкивание частиц с различными зарядами // Естественные и математические науки в современном мире / Сб. статей по материалам XLIX междунар. научно-практ. конф. № 12 (47). Новосибирск: Изд. АНС «СибАК», 2016. 136 с. – С. 85-91.

6. Tagaeva S.B. Existence and stabilization of solution of system of differential equations describing arrangement of many discrete electrical charges on a segment //Интернет-журнал ВАК КР, 2017, № 1. - 6 с.

MSC 34E05

CONTINUOUS DEPENDENCE OF SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH NON-MONOTONIC DELAY ON INITIAL DATA IN INTEGRAL NORM

Zheentaeva Zh. K.
Kyrgyz-Uzbek University

Supra, the author proved that solutions of initial value problems for linear differential equations with monotonic delay depend on initial data continuously in integral norm. Further, the author proved an analogous result for non-linear delay-differential equations and for equations with some delays. In this paper an analogous phenomenon is demonstrated for equations with non-monotonic delay.

Keywords: delay-differential equation, initial value problem, monotonicity, integral norm.

Мурда автор монотондук кечигүүлүү сызыктуу дифференциалдык теңдемелер үчүн баштапкы маселелердин чыгарылыштарынын баштапкы берилгендерден интегралдык норма боюнча үзгүлтүксүз көз карандылыгын далилдеди. Андан кийин автор кечигүүлүү сызыктуу эмес дифференциалдык теңдемелер үчүн окшош натыйжаны далилдеди. Бул макалада монотондук эмес кечигүүлүү болгон теңдемелер үчүн окшош кубулушу көрсөтүлдү.

Урунттуу сөздөр: кечигүүчү аргументтүү дифференциалдык теңдеме, баштапкы маселе, монотондук, интегралдык норма.

Ранее автор доказала, что решения начальных задач для линейных дифференциальных уравнений с монотонным запаздывающим аргументом непрерывно зависят от начальных данных по интегральной норме. Далее, автор доказала аналогичный результат для нелинейных уравнений. В этой статье продемонстрировано аналогичное явление для уравнений с немонотонным запаздыванием.
Ключевые слова: дифференциальное уравнение с запаздывающим аргументом, начальная задача, монотонность, интегральная норма.

Introduction

Preceding results on asymptotic of delay-differential equations are in [1], [2]. In [3] the author proved that solutions of initial value problems for linear differential equations with monotonic delay depend on initial data continuously in integral norm. Further, the author proved [4] an analogous result for non-linear delay-differential equations and for equations with some delays. In this paper an analogous phenomenon is demonstrated for equations with non-monotonic delay.

1. Review of preceding results

Since publication [1] linear differential equations both with

concentrated and distributed delays are considered in general form (we will be restricted with bounded delay):

$$x(t) = \int_{t-h}^{t} x(s) d_s M(t,s) ds + f(t),\ t \in R_+, \tag{1}$$

with the initial condition

$$x(t) = \varphi(t) \in C[-h, 0]\,. \tag{2}$$

In [3] we demonstrated new peculiarities of differential equations with concen-trated delays in linear case.

Considered the equation

$$x'(t) = P(t)x\big(\sigma(t)\big), t \in R_+; P(t), \sigma(t) \in C(R_+), t-h \le \sigma(t) \le t. \tag{3}$$

Introduce the following norms in $C[-h, 0]$:

$$||\varphi||_* := \int_{-h}^{0} |\varphi(s)| ds;\ ||\varphi||_{**} := ||\varphi||_* + |\varphi(0)|.$$

Theorem 1. If $\sigma(t) \in C^1(R_+)$ and $\sigma'(t) > 0, t \in R_+$, then the solution $X\big(t, \varphi(\cdot)\big)$ of (3)-(2) depends on $\varphi(t)$ continuously in the integral norm:

If $\lim \left\{ \left\| \varphi_0(t) - \varphi(t,\varepsilon) \right\|_{**} : \varepsilon \to 0 \right\} = 0$ then

$\lim\{ |X\left(t, \varphi_0(\cdot)\right) - X\left(t, \varphi(\cdot,\varepsilon)\right)| : \varepsilon \to 0 \} = 0.$

In [4] we extended this result for non-linear equations.

An ordinary delay-differential equation with bounded delay can be written in the form

$$x'(t)=F(t, x(t-\tau), 0 \le \tau \le h\}, t \in R_+ \tag{4}$$

where $h>0$, $F:R_+ \times C[-h,0] \to R$ is a continuous operator.

Denote the norm $||\cdot||_{[a,b]}$ in the space $C[a,b]$.

The following result is well-known:

Theorem 2. If the operator $F(t,v(\cdot))$ fulfills the Lipschitz condition: there exists such $L(t) \in C(R_+)$ that

$|F(t,v_1(\cdot))-F(t,v_2(\cdot))| \le L(t)\, ||\, v_1(\cdot)-v_2(\cdot)||_{[-h,0]}$

then 1) the initial value problem (4)-(2) has a unique solution;

2) $X(t; \varphi(\cdot))$ is continuous with respect to $\varphi(\cdot)$ by the norm $||\cdot||_{[-h,0]}$.

In general, dependence $X(t; \varphi(\cdot))$ on the function $\varphi(t)$ by the norm $||\cdot||_{**}$ can be discontinuous.

E x a m p l e 1. Consider the initial value problem

$$x'(t)= |x(-1)|, t \in [1,2], \tag{5}$$

$$x(t)= \varphi(t, \varepsilon)= \exp(-(t-1)^2/\varepsilon), t \in [-1,0]. \tag{6}$$

The solution $X(t; \varphi(\cdot, \varepsilon))=x(0)+t|x(-1)|= \exp(-1/\varepsilon)+t.$

We have $\lim \{|| \varphi(t, \varepsilon) ||_{**}: \varepsilon \to 0 \} = 0$ but $\lim \{X(t; \varphi(\cdot, \varepsilon))\varepsilon \to 0 \} = t$ does not tend to zero.

The problem is to find conditions ensuring continuous dependence $X(t; \varphi(\cdot))$ on the function $\varphi(t)$ by the norm $||\cdot||_*$.

Consider the equation with concentrated delay

$$x'(t)=G(t, x(\sigma(t))), t \in R_+ \tag{7}$$

where $G:R_+ \times R \to R$ is a continuous function.

Theorem 3. If $\sigma(t) \in C^1(R_+)$, $\sigma'(t) > 0$, $t \in R_+$ and the function $G(t,x)$ fulfills the Lipschitz condition: there exists such $L(t) \in C(R_+)$ that

$|G(t,x_1) - G(t,x_2)| \leq L(t)\,|x_1 - x_2|$

then the solution of the initial value problem (7)-(2) depends on the function $\varphi(t)$ continuously by the integral norm as well as in Theorem 1.

Theorem 3 was generalized to equations with some concentrated delays, as follows

$$x'(t)=G(t, x(\sigma_1(t)), x(\sigma_2(t)),\dots, x(\sigma_n(t))),\ t \in R_+ \tag{8}$$

where $G: R_+ \times R^n \to R$, $\sigma_k(t): R_+ \to R$ are continuous functions,

$(t-h \leq \sigma_k(t) \leq t)$, $k=1..n$.

Theorem 4. If $\sigma_k(t) \in C^1(R_+)$, $\sigma_k'(t)>0$, $t \in R_+$ and the function $G(t,x_1, x_2,\dots,x_n)$ fulfills the Lipschitz condition: there exists such $L_k(t) \in C(R_+)$ that

$|G(t,x_{11},x_{21},\dots,x_{n1}) - G(t, x_{12},x_{22},\dots,x_{n2}))| \leq L_1(t)|x_{11}-x_{12}| + L_2(t)|x_{21}-x_{22}| +$

$+\dots+ L_n(t)|x_{n1}-x_{n2}|$

then the solution of the initial value problem (8)-(2) depends on the function $\varphi(t)$ continuously by the integral norm: if $\lim_{\varepsilon\to 0} \|\varphi(t,\varepsilon)-\varphi_0(t)\|_{**}= 0$ then $\lim_{\varepsilon\to 0} |X(t;\varphi(\cdot,\varepsilon)) - X(t;\varphi_0(\cdot))|_{**}= 0$ for all $t \in R_+$.

2. Equations with non-monotonic delay

Consider the equation (3) with $h=3$, $P(t) = 1$, $\gamma(t)$ is a 3-periodical function, $\sigma(t) = t - \gamma(t)$.

Define $\gamma(t)$ and calculate $\sigma(t)$ as follows:

t	$[0;1)$	$[1;2)$	$[2;3)$
$\gamma(t)$	$2t$	2	$6-2t$
$\sigma(t)$	$-t$	$t-2$	$3t-6$

We have: $\gamma(t) \geq 0$; $\gamma(0)=0=\gamma(3-0)$; $\gamma(t)$ is continuous.

Hence, $\sigma(t) \leq t$; $\sigma(t)$ is continuous; $\sigma(t)$ decreases for $0<t<1$; increases for $1<t<3$.

By (3) and (2) $x(t) = \varphi(0) + \int_0^t x(\sigma(s))ds$ $(0 \leq t \leq 3)$.

For $0 \leq t \leq 2$

$$|x(t)| \leq |\varphi(0)| + \left|\int_0^1 \varphi(-s)ds\right| + \left|\int_1^2 \varphi(s-2)ds\right| =$$

$$= |\varphi(0)| + \left|\int_{-1}^0 \varphi(s)ds\right| + \left|\int_{-1}^0 \varphi(s)ds\right| \leq 2\|\varphi(0)\|_{**}. \quad (9)$$

For $2 < t \leq 3$

$$|x(t)| \leq |\varphi(0)| + \left|\int_0^1 \varphi(-s)ds\right| + \left|\int_1^2 \varphi(s-2)ds\right| + \left|\int_2^3 x(3s-6)ds\right| =$$

$$= 2\|\varphi(0)\|_{**} + \left|\int_2^t x(3s-6)ds\right|; \quad (10)$$

$$0 \leq 3t-6 \leq t. \quad (11)$$

Due to (11), (10) implies the following estimation in virtue of Gronwall-Bellman inequality:

$|x(t)| \leq 2\|\varphi(0)\|_{**} \exp(t-2),\ 2 < t \leq 3.$

Thus, by (9) and this estimation, $x(t)$ depends on $\varphi(t)$ continuously in the integral norm.

Conclusion

The results obtained demonstrate that there can be various norms (topologies) in spaces of solutions and spaces of initial data for various classes of initial value problems for delay-differential equations.

REFERENCES

1. Myshkis A.D. (1972) Linear Differential Equations with Delayed Argument (in Russian). - Moscow: Nauka, 352 p.

2. Kharatishvili G.L., Tadunadze T.A. Formulas of variation of solution and tasks of optimal control for differential equations with retarded argument (in Russian) // Modern Math. and its Applications, Tbilisi , 2005, vol. 25, pp. 3-166.

3. Pankov P., Zheentaeva Zh. Conditions of continuous dependence of solutions of delay-differential equations on initial data in integral norm // Abstracts of V Congress of the Turkic World Mathematicians / Ed. A.Borubaev. Bishkek: Kyrgyz Mathematical Society, 2014, p. 129.

4. Zheentaeva Zh. Conditions of continuous dependence of solutions of nonlinear delay-differential equations on initial data in integral norm //

Proceedings of V Congress of the Turkic World Mathematicians / Ed. A.Borubaev. – Bishkek: Kyrgyz Mathematical Society, 2014. – pp. 116-120.

MSC 54F45

DEFINITIONS OF DIMENSIONS IN KINEMATICAL SPACES

Zhoraev A.H.
Kyrgyz-Uzbek University

This paper deals with controlled motion in non-Euclidean topological spaces which can be implemented by means of computer. It contains a survey of definitions to provide motion of a point, definitions of motion of a lengthy object and three definitions of dimension based on motion in kinematical spaces.
Key words: topological space, metrical space, kinematical space, computer, motion, rotation, dimension.

Компьютер аркылуу жүзөгө ашырылуучу, эвклиддик эмес мейкиндиктерде башка-рылуучу кыймылдоо бул макалада каралат. Макалада чекиттин жана узун-туурасы бар объекттин кыймылдоону жабдуучу аныктамалар жана кинематикалык мейкиндиктерде кый- мылдоонун негизинде өлчөмдү үч аныктама бар.
Урунттуу сөздөр: топологиялык мейкиндик, кинематикалык мейкиндик, компьютер, кыймылдоо, айландыруу, өлчөм.

В статье рассматривается управляемое движение в неэвклидовых топологических про-странствах, которое может быть реализовано на компьютере. Статья содержит определения, обеспечивающие движение точки и протяженных объектов и три определения размерности, основанные на движении в кинематических пространствах.
Ключевые слова: топологическое пространство, кинематическое пространство, компьютер, движение, вращение, размерность

Introduction

Since it is known, S.Ulam [6] was the first to propose an active work on computer to present a virtual (four-dimensional Euclidean) space, but he did not propose any concrete methods of implementation.

An another way to perform non-Euclidean spaces visually by means of com-puter was proposed [7]. His idea can be demonstrated by the following example. If we put the figure $\subseteq$ onto a common ring band and we can look "along" the band suffi- ciently far then we will see the sequence of diminishing figures

$\subseteq \quad \subseteq \quad \subseteq \subseteq \ldots$

If we do same for a Möbius band then we will see the sequence of diminishing figures $\subseteq \quad \supseteq \quad \subseteq \supseteq \ldots$

We [4] proposed to use controlled (interactive) motion in non-Euclidean topological spaces by means of computer. We implemented the Möbius band as follows. We are standing on a band and see the figure $\subseteq$ (the horizon is less than half of the length of the band). We go and soon we see the figure $\supseteq$.

We [1] introduced general conception of a kinematical space and implemented some kinematical spaces (Riemann surfaces, Möbius band, projective plane, topological torus) with search in them. Methods of constructing such spaces and marking to facilitate motion in them were proposed in [2] and applied in [5].

A similar definition, independently of us, was proposed in [3]. We do not know whether it was implemented by computer.

Kinematical investigation of unknown spaces defined by differential and algebraic equations was proposed in [8].

New types of dimensions based on motion were announced in [9] and [10].

In this paper we expound this approach and give definitions of three new types of dimensions: successful observation and "almost observation" from observable domains; possibility of rotation of lengthy sets.

1. Review of preceding definitions on motion and dimensions

Let $Q^k := [0; 1]^k, k = 1, 2, 3, \ldots$ is a k-dimensional cube (segment, square, cube, ...); ε is a small positive parameter. Also, we will extend functions to sets with same denotations.

Natural motion of points (also implemented on computer) is presented by the following system of axioms [2] based on the notion of *time.*

Definition 1. *A pair: a set K of points and a set Φ of routes is said to be a kinematical space (each route ϕ, in its turn, consists of "time" $T_\phi > 0$ and "trajectory" $M_\phi : [0,T] \to K$ if*

(K1) $(\forall z_0 \neq z_1 \in K)(\exists \phi \in \Phi)((M_\phi(0) = z_0) \wedge (M_\phi(T_\phi) = z_1))$ *and the set of times $\{T_\phi\}$ for all such routes has a positive minimum (the kinematical distance $\rho_K(z_0, z_1)$);*

(K1) (if $\rho_K(z_0 , z_1)$ is always attainable then the space is said to be "flat"; the corresponding route is said to be "straight");*

(K2) if $\phi = \{T_\phi ,M_\phi) \in \Phi$ then $\phi' := \{T_\phi ,M'(t) = M_\phi(T_\phi - t)\} \in \Phi$ (the reverse motion with same time is possible);

(K3) if $\phi = \{T_\phi ,M_\phi) \in \Phi$ and $T" \in (0,T_\phi)$ then $\phi" := \{T";M"(t) = M_\phi(t) : [0,T"] \to K\} \in \Phi$ (during a route one can stop at any desired moment);

(K4) if $M_\phi(T_\phi) = M_\psi(0)$ then ϕ and ψ can be concatenated with time $T = T_\phi + T_\psi$.

Remark 1. After our publication [2] another version of presenting "motion" based on the notion of "path" was proposed.

Denote the set of connected subsets of R as *In.* A *path* is a continuous map $\gamma : In \to X$ (a topological space).

Definition 2. *The following definition is composed of some definitions in [3] (briefly) reduced to a "a priori" bounded, path-connected space; denotations are slightly unified.*

A length structure in X consists of a class A of admissible paths together with a function (length) $L : A \to R_+$.

The class A has to satisfy the following assumptions:

(A1) The class A is closed under restrictions: if $\gamma \in A$, $\gamma : [a, b] \to X$ and $[u, v] \subset [a, b]$ then the restriction $\gamma|_{[u, v]} \in A$ and L is continuous with respect to u,v;

(A2) A is closed under concatenations of paths and the function L is additive correspondingly.

(A3) A is closed under (at least) linear reparameterizations and L is invariant correspondingly: for a path $\gamma \in A$, $\gamma : [a, b] \to X$ and a homeomorphism $\phi : [c, d] \to [a, b]$ of the form $\phi(t) = \alpha t + \beta$, the composition $\gamma(\phi(t))$ is also a path.

(A4) (similar to (Kl)).

$\rho_L(z_0 , z_1) := \inf\{L(\gamma) : \gamma : [a, b] \to X; \gamma \in A; \gamma(a) = z_0; \gamma(b) = z_1 \}$.

(A4) is similar to (K1*); the authors proposed the term "complete".*

Remark 2. In Definition 2 any topological structure is presumed while in Definition 1 such (metrical) one arises from itself. By this reason functions M_ϕ are "per se" continuous $(\rho_K (M_\phi(x), M_\phi(y)) < \backslash x - y\backslash)$ while continuity of the function L is to be required.

We mention some known definitions briefly (we restrict with metric sets):

Definition 3. *Dim-dimension (or "cover"- or Lebesque one): it is defined to be the minimum value of n, such that every open cover (set of open sets) C of X has an open refinement with number of overlappings being (n + 1) or below.*

Ind-dimension: by induction $Ind(\varnothing) = -1$; *Ind(X) is the smallest n such that, for every closed subset F of every open subset U of X, there is an open V in "between F and U " such that* $Ind(Boundary(U)) < (n - 1)$.

Minkovski (Min)-dimension. $Min(X) := \lim\{ (-\log N_\varepsilon / \log \varepsilon) : \varepsilon \to 0\}$ *where* N_ε *is the minimal cardinality of* ε*-sets in X. If lim does not exist then lim inf* (Min_-) *and lim sup* (Min_+) *to be considered.*

Remark 3. For metrical spaces *Dim*-dimension and *Ind*-dimension coincide. Obviously, $Min(Q^k) = k$.

2. Motion of lengthy objects in kinematical spaces

Definition 1 is not sufficient for motion of point sets. One of possible extensions of Definition 1 is the demand of isometric of all shifts of a set during motion but it is too binding. We proposed [11]

Definition 4. *Given a set* $S \in K$. *A set of routes with functions* $\{M(p) : p \in S\}$ *with a same time T is said to be a motion of S with bounded deformation if there are such constants* $0 < a_- < 1 < a_+$ *that*

(M1) $(\forall p \in S)(M(p)(0)=p)$;

(M2) $(\forall p_1 \neq p_2 \in S)(\forall t \in [0,T])(\rho_K(M(p_1)(t),M(p_2)(t)) \in [a_-,a_+]\rho_K(p_1,p_2))$.

Definition 5. *If additionally*

(R1) there exists such non-empty set ("axis") $C \in S$ *that* $M|_c$ *is the identity operator;*

(R2) $(\forall p \in S)\{M(S)(0) = M(S)(T))$ *(initial and final sets coincide);*

(R3) $(\forall t_1 \neq t_2 \in (0,T))(M(S)(t_1) \cap M(S)(t_2) = C)$ *(the set S is "thin" and does not pass by itself excluding the axis);*

then such motion is said to be a "proper rotation" (with "bounded deformation" correspondingly) around C.

Remark 4. To define "rotation" of a general (spacious) objects in a space without geometry is very complicated. For our purposes such "proper rotation" is sufficient.

3. Dimensions defined by motion in kinematical spaces

Definition 6. *A set B of a kinematical space K is said to be "fully observable" if there exists a route including all this set.*

Definition 7. *A kinematical space K is said to be "locally observable" if each its point has a "fully observable " neighborhood.*

Definition 8. *A locally observable kinematical space K is said to be "observable" if each its bounded set is "fully observable ".*

As usually, we will call a bijective continuous image of a segment $[0,T]$ a "segment in kinematical space". Also, we will call the trace of bijective motion of a segment with one of endpoints fixed "triangle" etc.

Definition 9. *"Orientation dimension" Ori is 1 for observable spaces. If there exists such "segment" with endpoints z_1 and z_2 and an inner point z_0 and such rotation with bounded deformation around z_0 that z_1 passes to z_2 and vice versa then $Ori(K) > 2$; if there exists a "triangle" with vertices z_1,z_2 and z_3 and a point z_0 within "segment" z_1-z_2 which can be rotated around segment z_0-z_3 with bounded deformation such that Z\ passes to z_2 and vice versa then $Ori(K) > 3$ etc.*

Obviously, $Ori(Q^k) = Dim(Q^k), k = 1, 2, 3, \ldots$.

Remark 5. "Motion" of such lengthy sets into themselves is not sufficient for such definition because a triangle z_1-z_2-z_3 can be transformed continuously into triangle z_2-z_1-z_3 by motion along the Möbius band but its dimension is 2.

The next definition also begins with observable spaces.

Definition 10. *Kinematical (Kin-) dimension is 1 for observable spaces. By in- duction: If $Kin(K) \neq n$ and there exists such $K_1 \in K$, $Kin(K_1) = n$ and function $D : K_1 \to \Phi$ that 1)$(\forall x \in K_1)(M_{D(x)}(Q) = x)$; 2) $\cup\{ M_{D(x)}[0,T_{D(x)}] : x \in K_1\} = K$. 3) $(\forall x_1 \neq x_2 \in K_1)(\forall t_1 \in [0,T_{D(x1)}], t_2 \in [0,T_{D(x2)}])(M_{D(x1)}(t_1) \neq (M_{D(x2)}(t_2))$.*

It is obvious that $Kin\{Q^1) = 1$.

Theorem 1. $Kin(Q^2) = 2(= Dim(Q^2))$.

Proof. By contradiction. $Kin(Q^2) = 1$ then there exists a trajectory S covering all Q^2. Choose a natural n and divide Q^2 into n x n little squares. S passes though all centers of squares and has the length within each square not less than $1/n$. Hence, its total length is not less than $n \cdot n \cdot 1/n = n$ and tends to infinity as $n \to \infty$.

Conclusion

The paper demonstrates that various new definitions of "dimension" conforming with known ones can be introduced on the base of "motion" and "rotation" in kinematical spaces.

REFERENCES

1. Borubaev A.A., Pankov P.S., Computer presentation of kinematic topological spaces. - Bishkek: Kyrgyz State National University, 1999 (in Russian).
2. Borubaev A.A., Pankov P.S., Chekeev A.A. Spaces Uniformed by Coverings.- Budapest: Hungarian-Kyrgyz Friendship Society, 2003.
3. Burago D., Burago Yu., Ivanov S. *A* Course in Metric Geometry // Graduate Studies in Mathematics, Volume 33, American Mathematical Society Providence, Rhode Island, 2001.
4. Pankov P.S., Bayachorova B.J. Using computers to perform non-Euclidean topological spaces // The 6-th conference and exhibition on computer graphics and visualization "Graphicon-96", Saint-Petersburg, 1996, vol. 2, p. 232.
5. Pankov P.S., Joraev A.H. Recognizability and local computer presentation of topological spaces // Problems of modern topology and applications: abstracts of the international conference. Tashkent: Nizami Tashkent State Pedagogical University, 2013, pp. 70-72.
6. Ulam S.M. A Collection of Mathematical Problems. - New York: Interscience Publishers, 1960.
7. Weeks J.R. The Shape of Space. – N. Y.: Marcel Dekker, Inc., 1985.
8. Zhoraev A. Investigation of topological spaces by kinematic method (in Russian).- Saarbrücken: Lap Lambert Academic Publishing, 2017.

9. Zhoraev A.H. Motion of sets and orientation dimension of kinematical spaces. Abst. of the VI Congress of the Turkic World Math. Society. - Astana: L.N. Gumilyov Eurasian National University, 2017. - P. 124.
10. Zhoraev A. Orientation dimension and orientation constants of kinematical spaces // Abstracts of the Third International Scientific Conference "Actual problems of the theory of control, topology and operator equations". - Bishkek: Kyrgyz Math. Society, 2017. - P. 36.
11. Zhoraev A.H. Motion of lengthy objects in kinematical spaces // Science. Education. Technology. - Osh: Kyrgyz-Uzbek University, 2018, No. l. - Pp. 47-50.

MSC PDE

APPLICATION OF THE METHOD OF ADDITIONAL ARGUMENT TO QUASILINEAR DIFFERENTIAL EQUATIONS OF THE FIRST ORDER WITH THE INITIAL CONDITION

Shaymbek Egemberdiev
Institute of Mathematics of National Academy of Sciences of Kyrgyz republic

A study using the method of additional argument of first-order quasilinear differential equations with an initial condition. Using the methodof additional argument to reduce first-order quasilinear differential equations with the initial condition to systems of integral equations.
Keywords: quasilinear differential equation, additional argument, initial condition, integral equation.

Кошумча аргумент ыкмасын колдонуу менен квазисызыктуу дифференциалдык теңдемелер үчүн коюлган баштапкы шарттык маселелерди изилдөө. Кошумча аргумент ыкмасын колдонуу менен квазисызыктуу дифференциалдык теңдемелер үчүн коюлган баштапкы шарттык маселелерди изилдөөнүн негизинде каралган маселени нтегралдык теңдемелер системасына келтирү ыкмасын киргизүү.
Урунттуу сөздөр: квазисызыктуу дифференциалдык теңдеме, кошумча аргумент, баштапкы шарт, интегралдык теңдеме.

Исследование с применением метода дополнительного аргумента квазилинейных дифференциальных уравнений первого порядка с начальным условием. С помощью метода дополнительного аргумента свести квазилинейных дифференциальных уравнений первого порядка с начальным условием к системам интегральных уравнений.
Ключевые слова: квазилинейные дифференциальные уравнение, дополнительный аргумент, начальное условие, интегральные уравнение.

In [1,2] it was shown that a system of differential equations with different initial-boundary conditionscan be exploredby using the method of an additional argument.

The present paper is devoted to an investigation of the following quasilinear partial differential equations:

$$\frac{\partial u}{\partial t}+\sum_{i=1}^{n} a_i(t,x_1,x_2,...,x_n,u)\frac{\partial u}{\partial x_i} = f_1(t,x_1,x_2,...,x_n,u) \tag{1}$$

with initial conditions: $u(0,x_1,x_2,\dots,x_n)=\varphi(x_1,x_2,\dots,x_n)$ (2)

$t\in[o,T]$, $T=const$, $(x_1,x_2,\dots,x_n)\in R^n$.

It is assumed, that the functions $f(t,x_1,x_2,\dots,x_n,u(t,x_1,x_2,\dots,x_n))$, $\varphi(x_1,x_2,\dots,x_n))$ are continuously differentiable with respect to all their arguments.

We study the solvability of (1) - (2) by the method of additional argument [1,2].

An extended characteristic system is introduced:

$$\begin{cases} \dfrac{dp_1(s,t,x_1,x_2,...,x_n)}{ds} = a_1(s,p_1(s,t,x_1,x_2,...,x_n),...,p_n(s,t,x_1,x_2,...,x_n),w(s,t,x_1,x_2,...,x_n)) \\ \dfrac{dp_2(s,t,x_1,x_2,...,x_n)}{ds} = a_2(s,p_1(s,t,x_1,x_2,...,x_n),...,p_n(s,t,x_1,x_2,...,x_n),w(s,t,x_1,x_2,...,x_n)) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{dp_n(s,t,x_1,x_2,...,x_n)}{ds} = a_n(s,p_1(s,t,x_1,x_2,...,x_n),...,p_n(s,t,x_1,x_2,...,x_n),w(s,t,x_1,x_2,...,x_n)) \\ \dfrac{dw(s,t,x_1,x_2,...,x_n)}{ds} = f(s,p_1(s,t,x_1,x_2,...,x_n),...,p_n(s,t,x_1,x_2,...,x_n),w(s,t,x_1,x_2,...,x_n)) \end{cases} \tag{3}$$

with additional terms:

$$\begin{cases} p_1(t,t,x_1,x_2,...,x_n) = x_1; \\ p_2(t,t,x_1,x_2,...,x_n) = x_2; \\ \dots\dots\dots\dots\dots\dots \\ p_n(t,t,x_1,x_2,...,x_n) = x_n \end{cases} \tag{4}$$

$w(0,t,x_1,\dots,x_n)=\varphi(p_1(0,t,x_1,\dots,x_n), p_2(0,t,x_1,\dots,x_n),\dots, p_n(0,t,x_1,\dots,x_n))$. (5)

Integrating the first n equations of system (3) over s from s to t, we obtain n integral equations taking into account conditions (4)

$p_1(s,t,x_1,\dots,x_n)=x_1- \int_s^t a_1(\nu,p_1(\nu,t,x_1,\dots,x_n),\dots,p_n(\dots),w(\nu,t,x_1,\dots,x_n))d\nu,$

$p_2(s,t,x_1,\dots,x_n)=x_2- \int_s^t a_2(\nu,p_1(\nu,t,x_1,\dots,x_n),\dots,p_n(\dots),w(\nu,t,x_1,\dots,x_n))d\nu,$

………………………………………………………………………………………

$p_n(s,t,x_1,\dots,x_n)=x_n-\int_s^t a_n(\nu,p_1(\nu,t,x_1,\dots,x_n),\dots,p_n(\dots),w(\nu,t,x_1,\dots,x_n))d\nu,$

(6)

We integrate the last equation of system (3) over s from 0 to s, and taking (5), (6) into account, we obtain the integral equation

$$w(s,t,t,x_1,\dots,x_n)=\varphi\begin{pmatrix} x_1-\int_0^t a_1(\nu,p_1(\nu,t,x_1,\dots,x_n),\dots,p_n(\dots),w(\dots))d\nu, \\ x_2-\int_0^t a_2(\nu,p_1(\nu,t,x_1,\dots,x_n),\dots,p_n(\dots),w(\dots))d\nu, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ (x_n-\int_0^t a_n(\nu,p_1(\nu,t,x_1,\dots,x_n),\dots,p_n(\dots),w(\dots))d\nu, \end{pmatrix}+$$

$$+\int_0^s f(\nu,p_1(\nu,t,x_1,\dots,x_n),\dots,p_n(\nu,t,x_1,\dots,x_n),w(\nu,t,x_1,\dots,x_n))d\nu, \qquad (7)$$

So, we have obtained that the system of integral equations (6), (7) follows from problem (3), (4), (5).

By direct differentiation with respect to s of equations (6), (7), we obtain system (3). Setting s = t in (6), we obtain conditions (4), and setting s = 0 we obtain conditions (5).

Thus, problem (3), (4), (5) and the system of integral equations (6), (7) are equivalent.

Now let $u\ (t,\ x_1,\ \dots,x_n)$ be a solution to problem (1), (2).Defining $p_1(s,t,x_1,\dots,x_n)$, $p_2(s,t,x_1,\dots,x_n)$, … , $p_n(s,t,x_1,\dots,x_n)$from the system of differential equations (3), with conditions (4), we obtain that the function

$w(s,t,t,x_1,\dots,x_n)=u(s,p_1(s,t,x_1,\dots,x_n),p_2(s,t,x_1,\dots,x_n),\dots,p_n(s,t,x_1,\dots,x_n))$

will satisfy the last equation of system (3) and condition (5).

By this we proved the following lemma.

Lemma. If each solution $u(t,x_1,\dots,x_n)$ of problem (1), (2) gives a solution to problem (3), (4), (5), and therefore a solution to the system of integral equations (6), (7).

On the contrary, each continuously differentiable and bounded together with its first derivatives solution of the system of integral equations (6), (7) gives a solution to the problem (1), (2) at $0<t<T_0\leq T$, where T_0 is some constant determined by the given data.

REFERENCES

1. Egemberdiev Sh. A. (2002) Solving nonlinear partial differential equations by the additional argument method. *Analytical and approximate methods.* International Conference at the Kyrgyz-Russian-Slavic University. Bishkek, Kyrgyzstan.-P.55-61.
2. Egemberdiev Sh. A. (2014) Reductions of the system of partial differential equations with initial-boundary values description to systems of integral equations by means of an additional argument. *Abstracts of V Congress of the Turkic World Mathematicians*. Kyrgyz Mathematical Society, Bishkek, p. 175.

MSC 49M37

MATHEMATICAL MODEL AND METHOD FOR CALCULATING THE OPTIMIZATION PROBLEM OF LIVESTOCK PRODUCTION

Zhusupbaev A., Asankulova M., Zhusupbaeva G.A., Iskandarova G.S.
Institute of Mathematics NAS KR

In the work, a mathematical model of the problem of choosing the optimal breed of cattle (cattle) of a farm is developed with an indicative plan for the production of livestock products and an algorithm for solving it. The performance of the model is shown in a numerical example.
Key words: mathematical model, sown area, production, productivity, consumption, income, economy.

Бул жумушта мал чарба продукцисынын индикативдик планын аткарууда чарбадагы ийри мүйүздү малдын оптималдуу породасын аныктоо маселесине математикалык модель жана анын чугаруу алгоритмасы иштелип чыккан.
Урунтуу сөздөр: математикалык модель, айдоо аянты, өндүрүш, түшүмдүлүк, чыгым, киреше, чарба.

В работе разработана математическая модель задачи выбора оптимальной породы крупнорогатого скота (КРС) хозяйства при индикативном плане производства продукции животноводства и алгоритм ее решения. Работоспособность модели показана на числовом примере.
Ключевые слова: математическая модель, посевная площадь, производство, урожайность, расход, доход, хозяйство.

Formulation of the problem. Suppose that a farm that has sown areas of various categories (irrigated, rainfed, etc.) in the amount of s_k, $k \in K$ and, with sufficient financial ability, planned to produce livestock products in an amount not less than b^h, $h \in H$ by optimizing the breeds of animals for keeping, where h is the type produced on the farm animal products.

It is assumed that for each type of animal breed, productivity and the corresponding daily diet are known.

Also known is the yield of crops on each category of cultivated area used by the farm in the animal feed ration.

It is required to determine the optimal composition of animals on the farm, allowing to ensure the production of livestock products in the planned volume, with minimal total costs.

The mathematical model of the problem can be represented as.

Find a minimum

$$L(x,y) = \sum_{k \in K} \sum_{j \in J_0} c_{kj} x_{kj} + \sum_{h \in H} \sum_{l \in L} c_l^h y_l^h \tag{1}$$

under conditions

$$\sum_{j \in J_0} x_{kj} \le s_k, \quad k \in K, \tag{2}$$

$$\sum_{k \in K} a_{kj} x_{kj} = \sum_{h \in H} \sum_{l \in L} \alpha_{lj}^h y_l^h, \quad j \in J_0, \tag{3}$$

$$\sum_{l \in L} \theta_l^h y_l^h \ge b^h, \quad h \in H, \tag{4}$$

$$x_{kj} \ge 0, \quad k \in K, \quad j \in J_0, \tag{5}$$

$$y_l^h \ge 0, \quad l \in L, \ integer \quad h \in H, \tag{6}$$

where x={ $x_{kj} \ge 0$, $k \in K$, $j \in J_0$}, y={ $y_l^h \ge 0$ - integer, $h \in H$, $l \in L$},

j - index of the type of agricultural products, crop production used in the daily ration of animal feeding, $j \in J_0=\{1,2,\ldots,n\}$;

J_0 - many types of crop production aimed at animal feed, $j \in J_0$;

k - index of the type of crop area category in the farm, $k \in K$;

K- many types of crop areas, $K=\{1,2,\ldots,p\}$;

h - index of the type of livestock products produced on the farm, $h \in H$;

H - many types of livestock products, $H=\{1,2,\ldots, \bar{H}\}$;

l - index of the species of animal breed on the farm, $l \in L$;

L- many species of animal breeds, $L=\{1,2,\ldots,\ \bar{L}\}$;

Known parameters:

s_k - sown area of k-category on the farm, $k\in K$;

a_{kj} – productivity of j-th type of crop on the k category of sown area of the economy $k\in K,\ j\in J_0$;

α_{lj}^h - annual demand for the j-th type of crop production per animal of the l-th breed in the production of the h-th type of product, where $\alpha_{lj}^h=\beta_{jl}^h\gamma_{jl}^h,\quad j\in J_0,\ l\in L,\ \ h\in H$;

β_{jl}^h - the share of the j-crop production in the daily ration per animal of the l-th breed in the farm for the production of the h-type of products, $j\in J_0,\ l\in L,\ h\in H$;

γ_{jl}^h - the number of days in the diet of the crop production of the j-th species for the l-th breed of animal in the production of the h-th type of product, $j\in J_0,\ l\in L,\ h\in H$;

θ_l^h - the volume of production of the h th type received by the farm from one animal of the l-th breed, $l\in L,\ h\in H$;

b^h - the planned output of the h-type livestock produced by the farm, $h\in H$;

c_{kj} - costs per unit size of the k-th category of sown area under the j-th type of culture, $j\in J_0,\ k\in K$;

c_l^h - annual consumption per animal of the l-th breed in the production of the h-th type of livestock products, $h\in H,\ l\in L$;

Searched variables:

x_{kj} – size from the k-th category of sown area allocated for the j-th type of culture, $j\in J_0,\ k\in K$;

y_l^h - the number of animals of the l-th breed on the farm for the production of the h-th type of product, $h\in H,\ l\in L$.

The objective function (1) determines the minimum total expenditure of the economy for the cultivation of feed crops and for the maintenance of animals for the production of products in the planned volume;

Limitations (2) determines that the total size of the sown area of a farm allocated for fodder crops in each category should not exceed the size of the sown area of this category;

The restriction (3) shows that the volume of agricultural production
the production of each type of feed should be equal to the volume of farm needs for domestic needs (feed);

Restriction (4) requires that the volume of livestock products for each species should be no less than the planned volume of production of these products;

Constraint (5) requires non-negative variables;

Constraint (6) requires that the value of the variables must be an integer. The mathematical model (1) - (6) can be presented in the form of table 1.

Table 1

Presentation of the conditions of problem (1) - (6) in the form of a table

x_{11}	x_{12}	…	x_{1n}	x_{21}	x_{22}	…	x_{2n}	…	x_{p1}	x_{p2}	…	x_{pn}
1	1	…	1									
				1	1	…	1					
								…				
									1	1	…	1
a_{11}				a_{21}					a_{p1}			
	a_{12}				a_{22}					a_{p2}		
		…				…		…			…	
			a_{1n}				a_{2n}					a_{pn}
c_{11}	c_{12}	…	c_{1n}	c_{21}	c_{22}	…	c_{2n}	…	c_{p1}	c_{p2}	…	c_{pn}

Continuation of table 1

y_1^1	y_2^1	$\cdots$	y_l^1	y_1^2	y_2^2	$\cdots$	y_l^2	$\cdots$	y_1^h	y_2^h	$\cdots$	y_l^h		
													$\le$	S_1
													$\le$	S_2
													$\cdots$	$\ldots$
													$\le$	S_p
$-\alpha_{11}^1$	$-\alpha_{12}^1$	$\cdots$	$-\alpha_{1l}^1$	$-\alpha_{11}^2$	$-\alpha_{12}^2$	$\cdots$	$-\alpha_{1l}^2$	$\cdots$	$-\alpha_{11}^h$	$-\alpha_{12}^h$	$\cdots$	$-\alpha_{1l}^h$	=	0
$-\alpha_{21}^1$	$-\alpha_{22}^1$	$\cdots$	$-\alpha_{2l}^1$	$-\alpha_{21}^2$	$-\alpha_{22}^2$	$\cdots$	$-\alpha_{2l}^2$	$\cdots$	$-\alpha_{21}^h$	$-\alpha_{22}^h$	$\cdots$	$-\alpha_{2l}^h$	=	0
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\ldots$
$-\alpha_{n1}^1$	$-\alpha_{n2}^1$	$\cdots$	$-\alpha_{nl}^1$	$-\alpha_{n1}^2$	$-\alpha_{n2}^2$	$\cdots$	$-\alpha_{nl}^2$	$\cdots$	$-\alpha_{n1}^h$	$-\alpha_{n2}^h$	$\cdots$	$-\alpha_{nl}^h$	=	0
θ_1^1	θ_2^1	$\cdots$	θ_l^1										$\ge$	b_1
				θ_1^2	θ_2^2	$\cdots$	θ_l^2						$\ge$	b_2
								$\cdots$					$\cdots$	$\ldots$
									θ_1^h	θ_2^h	$\cdots$	θ_l^h	$\ge$	b_p
c_1^1	c_2^1	$\cdots$	c_l^1	c_1^2	c_2^2	$\cdots$	c_l^2	$\cdots$	c_1^h	c_2^h	$\cdots$	c_l^h	$\rightarrow$	min

The algorithm for solving the problem. Calculations begin by determining the value of the parameters α_{lj}^h, $j\in J_0$, $l\in L$, $h\in H$ by equality (7). Using known parameters a_{kj}, c_{kj}, s_k, $k\in K$, $j\in J_0$ и ϑ_l^h, c_l^h, b^h, $h\in H$, $l\in L$, the numerical model of the problem is formulated according to (1)-(6).

From the solution of the problem, the quantitative composition of animals is determined $y=\{y_l^h,\ h\in H,\ l\in L\}$ farms for the production of livestock products of each type and the size of the cultivated area for each type of crop $x=\{x_{kj},\ k\in K,\ j\in J_0\}$ at the lowest total cost. The decision algorithm ends.

Let us verify the operability of the mathematical model and the algorithm for solving the problem using a numerical example.

Example. Let the farm have acreage in the amount of S = 366 ha, of which irrigated S1 = 280 ha, and rainfed S2 = 86 ha.

The main activity of the farm is the production of livestock products: milk and beef. The farm has the opportunity to choose two breeds of animals (cows) of the dairy direction and two breeds of cows of the meat direction.

Known: - daily ration of feeding dairy cows of the first breed with milk yield of 3600 kg (Table 2), i.e. ($\theta_{l=1}^{h=1}$ = 3600 kg);

- daily ration of feeding dairy cows of the second breed with a milk yield of 4,500 kg (Table 3), i.e. ($\theta_{l=2}^{h=1}$ = 4500 kg);
- daily ration of feeding cows with a live weight of 300 kg - the first type of breed for meat (Table 4), i.e. ($\theta_{l=1}^{h=2}$ = 300 kg);
- daily ration of feeding cows with a live weight of 450 kg - the second type of breed for meat (table 5), i.e. ($\theta_{l=2}^{h=2}$ = 450 kg):

Table 2

Daily ration of feeding milk cows of the first breed with milk yield 3600 kg of milk

	Name stern		Daily ration kg (1 head)		Fodder units	Total units	Total For 1 head qty	Days Qty
1	Lucerne (hay)		4		0,5	2,0	720,0	180
2	straw	wheat	3	1	0,2	0,6	180.0	180
		barley		2			360.0	
3	haylage		6		0,3	1,8	1080,0	180
4	The conc. feed	wheat	2,4	0,3	1	2,4	109,5	365
		barley		1,5			547,5	
		corn		0,6			219,0	
5	Silo (corn)		10		0,3	3,0	1800,0	180
6	Mineral feed		0,010		-	-	-	365
7	salt		0,030		-	-	-	365
8	Grazing feed		40		-	-	7200,0	180
	Green feed							
	Total		-		-	9,8	-	-

Table 3

Daily ration of feeding milk cows of the second breed with milk yield 4500 kg of milk

	Name of feed	**Daily ration (1 animal)**	**Days Qty**	**Per year (1 animal)**
1.	Lucerne (hay)	10 kg	180	1800 kg

2.	Straw	Wheat	3 kg	1 kg	180	180 kg
		Barley		2 kg		360 kg
3.	Hay		8 kg		180	1440 kg
4.	Conc. feed	Barley	3 kg	2 kg	365	730 kg
		Wheat		0,5 kg		182,5 kg
		Grain (corn)		0,5 kg		182,5 kg
5.	Silo (corn)		12 kg		180	2160 kg
6.	Mineral feed		-		365	3,6 kg
7.	Соль		-		365	10,8 kg
8.	Grain (corn)		50 kg		180	9000 kg
	Green feed					

Table 4

Daily ration for feeding cattle (bulls, heifers) with live weight 300 kg

	Name of feed		**Daily ration (1 animal), kg**		**Feed unit**	**General unit**	**Ttl per 1 animal**	**Days qty**
1.	Lucerne (hay)		3		0,5	1,5	540,0	180
2.	Straw	Wheat	2	0,5	0,2	0,4	90,0	180
		Barley		1,5			270,0	
3.	Hay		6		0,3	1,8	1080,0	180
4.	Conc. feed	Barkey	1,5	0,5	1	1,5	182,5	365
		Wheat		0,5			182,5	
		Grain		0,5			182,5	
5.	Silo (corn)		5,0		0,3	1,5	900,0	180
6.	Mineral feed		0,010		-	-	-	365
7.	Salt		0,030		-	-	-	365
8.	Grazing feed		30		-	-	5400,0	180
	Green feed							
	Total		-		-	6,7	-	-

Table 5

Daily ration for feeding cattle (bulls, heifers) for meat with a live weight of 450 kg

<table>
<tr><th></th><th colspan="2">Name of feed</th><th colspan="2">Daily ration (1 head)</th><th>Days qty</th><th>Per year (1 head)</th></tr>
<tr><td>1.</td><td colspan="2">Lucerne (hay)</td><td colspan="2">5 kg</td><td>365</td><td>1825 kg</td></tr>
<tr><td>2.</td><td colspan="2">Straw (Barley)</td><td colspan="2">1 kg</td><td>365</td><td>365 kg</td></tr>
<tr><td>3.</td><td colspan="2">Hay</td><td colspan="2">10 kg</td><td>180</td><td>1800 kg</td></tr>
<tr><td rowspan="3">4.</td><td rowspan="3">Conc. feed</td><td>Barley</td><td rowspan="3">3 kg</td><td>1 kg</td><td rowspan="3">365</td><td>365 kg</td></tr>
<tr><td>Grain (corn)</td><td>1 kg</td><td>365 kg</td></tr>
<tr><td>Wheat</td><td>1 kg</td><td>365 kg</td></tr>
<tr><td>5.</td><td colspan="2">Silo (corn)</td><td colspan="2">10 kg</td><td>180</td><td>1800 kg</td></tr>
<tr><td>6.</td><td colspan="2">Mineral feed</td><td colspan="2">-</td><td>365</td><td>-</td></tr>
<tr><td>7.</td><td colspan="2">Salt</td><td colspan="2">-</td><td>365</td><td>-</td></tr>
<tr><td rowspan="2">8.</td><td colspan="2">Grazing feed</td><td colspan="2" rowspan="2">30 kg</td><td rowspan="2">180</td><td rowspan="2">5400 kg</td></tr>
<tr><td colspan="2">Green feed</td></tr>
</table>

- crop yields on irrigated fields (I) and rainfed (II), included in the diet, a_{kj}, k=1,2, j=1,2,…,7, table 6

Table 6

	wheat	barley	alfalfa hay	haylage	Green feed	Silo (corn)	Grain (corn)
	1	2	3	4	5	6	7
I	2070.0	1962.2	2380.0	6281.0	5730.0	12340.0	20280.0
II	1500.0	0	1700.0	0	0	0	0

- the costs of growing crops per unit size (I) and (II) fields, $|c_{kj}|_{2,7}$, table.7.

Table 7

	1	2	3	4	5	6	7
I	2279.0	1096.0	2618.0	5071.0	9225.0	13574.0	7743.0
II	2000.0	1000.0	2000.0	5000.0	9000.0	13000.0	7000.0

The farm plans to produce 125 tons of milk and 25 tons of beef meat.

In addition, the known consumption for the content of one cattle of each breed in the production of milk and meat, respectively $c_{l=1}^{h=1}$ =50040.0 som, $c_{l=2}^{h=1}$ =55080.0 som, $c_{l=1}^{h=2}$ =18030.0 som, $c_{l=2}^{h=2}$ =25020.0 som.

It is required to determine the optimal composition of animals of the dairy and meat direction in the economy, which allows to ensure the planned volume of milk and meat production at the minimum total cost.

For the mathematical formalization of the problem, we determine the annual feed requirement for one animal of each breed in production α_{lj}^{h}, $j \in J_0$, $l \in L$, $h \in H$.

Using the daily diet, we determine the annual demand of each type of agricultural product included in the feed for one dairy cow with a milk yield of 3600 kg of milk and one dairy cow with a milk yield of 4500 kg. We also determine the annual demand of each type of agricultural product for feed for one cow of the first and second type of breed for meat (see table 8).

Table 8

Annual feed requirement per animal, depending from breed and productivity

Name stern	Feed requirement per dairy cow		Feed requirement per cow for meat	
	1 breed with 3600 kg milk yield	2 breed with 3600 kg milk yield	1 breed with a live weight of 300 kg	2 species of live weight breed 450 kg
1.wheat	289,5	362,5	272,5	365,0
2.barley	907,5	1090,0	452,5	730,0
3. perennial grass 3.1. hay (alfalfa)	720,0	1800,0	540,0	1825,0
3.2. haylage	1080,0	1440,0	1080,0	1800,0
3.3. Green feed	7200,0	9000,0	5400,0	5400,0
4. Corn 4.1. silage	1800,0	2160,0	900,0	1800,0
4.2.grain	219,0	182,5	182,5	365,0

We formulate a numerical model of the problem.

Find a minimum

$L(x,y)$=2279.0x_{11}+1096.0x_{12}+2618.0x_{13}+5071.0x_{14}+9225.0x_{15}+13574.0x_{16}+

$+7743.0x_{17}+2000.0x_{21}+1000.0x_{22}+2000.0x_{23}+5000.0x_{24}+9000.0x_{25}+$

$+13000.0x_{26}+7000.0x_{27}++50040.0\,y_1^1+55080.0\,y_2^1+18030.0\,y_1^2+$

$$+25020.0\,y_2^2 \qquad (7)$$

with conditions of

$$\sum_{j=1}^{7} x_{1j} = x_1 \le 280, \qquad \sum_{j=1}^{7} x_{2j} = x_2 \le 86, \qquad (8)$$

$2070,0x_{11}+1500,0x_{21}=289,5\,y_1^1+362,5\,y_2^1+272,5\,y_1^2+365,0\,y_2^2,$

$1962,0x_{12}+0x_{22}=907,5\,y_1^1+1090,0\,y_2^1+452,5\,y_1^2+730,0\,y_2^2,$

$2380,0x_{13}+1700,0x_{23}=720,0\,y_1^1+1800,0\,y_2^1+540,0\,y_1^2+1825,0\,y_2^2,$

$6281,0x_{14}+0x_{24}=1080,0\,y_1^1+1440,0\,y_2^1+1080,0\,y_1^2+1800,0\,y_2^2,$

$5730,0x_{15}+0x_{25}=7200,0\,y_1^1+9000,0\,y_2^1+5400,0\,y_1^2+5400,0\,y_2^2,$

$12340,0x_{16}+0x_{26}=1800,0\,y_1^1+2160,0\,y_2^1+900,0\,y_1^2+1800,0\,y_2^2,$

$$20280,0x_{17}+0x_{27}=219,0\,y_1^1+182,5\,y_2^1+182,5\,y_1^2+265,0\,y_2^2, \qquad (9)$$

$3600\,y_1^1+4500\,y_2^1 \ge 125000,$

$$300\,y_1^2+450\,y_2^2 \ge 25000, \qquad (10)$$

$$x_{kj} \ge 0, \quad k=1,2, \quad j=1,2,\dots,7, \qquad (11)$$

$$y_l^h - inteeger, \quad l=1,2, \quad h=1,2. \qquad (12)$$

We write the numerical model of problem (7) - (12) in the form of a table 9.

Table 9

Presentation of the conditions of problem (7) - (12) in the form of a table

x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{21}	x_{22}	x_{23}	x_{24}
1	1	1	1	1	1	1				
							1	1	1	1
2070.0							1500.0			
	1962.0							0		
		2380.0							1700.0	
			6281.0							0
				5730.0						
					12340.0					
						20280.0				
2279.0	1096.0	2618.0	5071.0	9225.0	13574.0	7743.0	2000.0	1000.0	2000.0	5000.0

Continuation of table 9

x_{25}	x_{26}	x_{27}	y_1^1	y_2^1	y_1^2	y_2^2	0	
							≤	280
1	1	1					≤	86
			-289,5	-362,5	-272,5	-365,0	=	0
			-907,5	-1090,0	-452,5	-730,0	=	0
			-720,0	-1800,0	-540,0	-1825,0	=	0
			-1080,0	-1440,0	-1080,0	-1800,0	=	0
0			-7200,0	-9000,0	-5400,0	-5400,0	=	0
	0		-1800,0	-2160,0	-900,0	-1800,0	=	0
		0	-219,0	-182,5	-182,5	-265,0	=	0
			3600,0	4500,0			≥	125000
					300,0	450,0	≥	25000
9000,0	13000,0	7000,0	50040,0	55080,0	18030,0	25020,0	→	min

Having solved the problem (7) - (12), using the MS EXCEL spreadsheet [1], we determine the optimal plan for the distribution of sown areas for fodder crops (see Appendix) x={ $x_{11}=14,7;$ $x_{12}=36,2;$ $x_{13}= 63,1;$ $x_{14}=22,3;$ $x_{15}= 96,4;$ $x_{16}=13,0$; $x_{17}=1,0$}

and the composition of cattle in the dairy and meat sector

y={ $y_1^1 = 1;$ $y_2^1 = 27;$ $y_1^2 = 1;$ $y_2^2 = 55$ },

L(x,y) =4356053.0 som.

Conclusion. From the optimal solution, it follows that for the production of the planned production volume of 125 tons of milk and beef meat in the amount of 25 tons, the farm should have 27 cows of the 2nd breed and one cow of the 1st breed for milk production. For meat production, the farm must have cows of the 2nd breed type in the amount of 55 and one cow of the 1st breed. At the same time, out of the available 280 ha of irrigated and 86 ha of rainfed sowing areas, the economy uses only 246.7 ha of irrigated sown areas for crops for forage, i.e. 14.7 ha for wheat; 36.2 hectares under barley; under perennial grass, only 181.8 ha., of which 63.1 ha under hay (alfalfa); 22.3 ha for hay; 96.4 ha is used as green feed; 14.0

hectares are used for corn, of which 13.0 hectares are used for silage and 1 hectare for corn grain. The total cost of growing crops for feed and animal care to obtain the planned volume of production, i.e. 125 tons of milk and 25 tons of meat amounted to 4356053.0 soms.

REFERENCES

1. Kalugina O.B., Lutsarev V.S. Work with spreadsheets. Microsft Office Excel 2003. - INTUIT. - 2006. - 222s.

MSC 49M37

DATA STREAM OPTIMIZATION IN HIGHLY LOADED ENVIRONMENT USING PYTHON

Mambetova A., Sidorenko S.

Kyrgyz National State University

The paper describes the method that helps to build a client-server communication using modern services by means of the most popular programming language Python. Messages are stored in MySQL and NoSQL database REDIS works as a caching server. The main goal is to keep software setup as simple as possible without running expensive solutions.

Key words: MySQL, Python, Redis, Performance optimization, Trading solutions architecture.

Бул жумушта белгилүүлөрдү сактоонун классикалык DBMS системасынын көз карашы менен, мисалга MySQL, ошондой эле азыркы учурга дал келген NoSQL, мисалга REDIS сервери менен сактоо ыкмалары көрсөтүлгөн. Сактоо боюнча БД менен өз ара байланышы өзгөчө мисалда көрсөтүлгөн. Эки окшош мисалдарды аткаруусуна анализ жүргүзүлгөн.

Урунтуу сөздөр: MySQL, Python, Redis, өндурумдулукту оптимизациялоо, соода чечимин архитектурасы.

Ключевые слова: MySQL, Python, Redis, оптимизация производительности, архитектура торговых решений.

В данной работе демонстрируется подход к хранению данных как с точки зрения классических DBMS систем, на примере MySQL, так и более современных NoSQL на примере сервера REDIS. Продемонстрирован типичный пример взаимодействия с БД по хранению данных. Проведен анализ проиводительности при выполнении двух идентичных задач.

Ключевые слова: MySQL, Python, redis, оптимизация производительности, архитектура торговых решений.

Formulation of the problem. Consider a simple example when there is a need to make a single purchase of some goods. For example the command could be “buy goods with id 0012345, the amount is 2 pieces and the price is USD $10”. Such a command is transmitted to the server using the “dictionary” data type in Python:

{‘ItemId’: 1234, ‘quantity’: 2, ‘price’: 10, ‘currency’: ’USD’, 'userId': ‘John Smith’}

Example HTTP GET request in python with parameters sent in the header:

1. # import module

2. import requests

3. # api-endpoint

4. URL = “https://myapi.myserver.com/v3/”.

5. # form a request

6. my_order = {‘itemId’: 1234, ‘quantity’: 2, ‘price’: 10, ‘currency’: ’USD’, 'userId': ’John Smith’}

7. # add the order field to the request parameters equal to the previously described

8. # order according to API documentation

9. PARAMS = {‘order’: my_order}

10. # send a request and get an object with a response to the variable r

11. r = requests.get (url = URL, params = PARAMS)

12. # convert the response to json format

13. data = r.json ()

Description of the architecture.

On the client side, the following software will be used:

1) MySQL server as a data warehouse.

2) Python application. Main purpose – order control system. It is responsible for creating, sending, storing, performing state requests and processing them. It also provides connectivity with the database as a storage system not directly but through REDIS.

3) REDIS server will be a high-speed buffer between the python application and the database.

A step-by-step algorithm when it necessary to perform an operation using external server will look like this:

a) The order comes from upper algorithm or user and goes to the Python control module.

b) The control module stores the corresponding data in the database, but not directly. Order comes to REDIS server and then another part of the application picks it and stores it in the database.
c) Having completed all the necessary save and checks, the control module makes a request to the server, as already described above.
d) The remote server processes the call, gives a response. The response comes with server ID of the order. This ID is later used to track the state of the order.
e) The control module saves the result of the server response in REDIS and continues to work, waiting for further commands.
f) Module that synchronizes the data between MySQL and REDIS saves the data in DB taken from buffer.
Let us count how many times the control program must access the database in order to process a single purchase request?
1) Check for active order routines.
2) Create a journal entry with the status “created” (which means a new order exists)
3) Update status from “created” to “sent” (when the order was sent)
4) Update status from “sent” to “processing” (when remote server accepted the order, e.g. no error returned)
5) Update the status from “processing” to “completed” (when the remote server confirmed that the execution succeeded)
6) Save the fields from the server response to the database.

The algorithm has to make queries to database six times. If database has to do a lot of work with hard drives and the server are congested overall speed of the algorithm will be low.

REDIS and MySQL performance tests.

import timeit

SETUP_CODE = '''import redis

conn = redis.Redis ('localhost') '''

```
TEST_CODE = """user = {"status": "New", "quantity": 22, "item_name": "shoes",
"Price": 0.3}
rset = conn.hmset ("pythonDict", user)
rget = conn.hmget ("pythonDict", user)
rdel = conn.hdel ("pythonDict", 1) """
times = timeit.repeat (setup = SETUP_CODE, stmt = TEST_CODE, repeat = 3,
number = 100)
print (times)
```

The code above produces:

[0.05709504522383213, 0.026847911067306995, 0.021604058798402548]

At doing the same thing with MySQL database:

```
SETUP_CODE = """import mysql.connector
cnx = mysql.connector.connect (user = 'user', password = 'pass', host =
'127.0.0.1', database = 'db')
cursor = cnx.cursor (dictionary = True, buffered = True)
c2 = cnx.cursor ()"""
TEST_CODE = """stmt =" insert into user.orders (itemID, quantity, price, status,
order_hash) values (1,2,33, 'new', 'abcdefg') "
a = cursor.execute (stmt)
cnx.commit ()
query = "SELECT * from greed.orders;"
cursor.execute (query)
if cursor.rowcount> 0:
   for rec in cursor:
      stmt = "delete from user.orders where id = {};". format (rec ['id'])
      c2.execute (stmt)
      cnx.commit () """
times = timeit.repeat (setup = SETUP_CODE, stmt = TEST_CODE, repeat = 3,
number = 100)
print (times)
```

And the result is:

[0.7270211200229824, 0.7100321920588613, 0.7069365601055324]

The traditional DB loses 28 times to NoSQL in terms of performance.

Server side example of message publication:

```
import redis
server_r = redis.Redis (host = 'localhost', port = 6379, db = 0)
server_r.publish ('messages', 'here is the message')
```

Client side example:

```
import redis
client = redis.Redis (host = 'localhost', port = 6379, db = 0)
client_m = client.pubsub ()
# subscribe to messages channel
client_m.subscribe ('messages')
msg = client_m.get_message ()
```

As soon as the messages appear, insert them into a remote database, for example, in MySQL:

```
while true:
    msg = client_m.get_message ()
    if msg:
        order_dict = msg [‘data’]
        stmt = """insert into user.orders
            (id, orderId) """. format (order_dict ['id'], order_dict [‘orderId’])
        c2.execute (stmt)
        cnx.commit ()
```

Interaction steps to complete the service call:

Pros and Cons

Pros

A) The transparency of the state of the remote call. It is very easy to build a web interface that helps to track the exchange process and identify if there are problems in the messages exchange.
B) The speed. Introductions of REDIS as an intermediate link between a relational DBMS and the application greatly increases the speed, as the database is not a bottleneck anymore.
C) Robustness. The detailed information about the state of a particular order allows the program to handle the control and keep on running depending on the required behavior even if some orders have failed.
D) Detailed logging. Simplifies the search for errors and improves development speed.
D) Ease of development. Python is one of the most popular with huge community and simple languages with mature structure and environment, which greatly simplifies the development.
E) The cost of software components. MySQL, REDIS and Python are free tools and they do not require commercial licensing.
Cons
A) The introduction of the REDIS system, in addition to the ordinary DBMS, increases the number of system failure points.
B) Additional software requires higher staff qualification, despite the simpler syntax of all NoSQL databases.
B) Security requirements. If REDIS and Python are running on independent servers, secure network infrastructure is required. For example, a physically isolated network segment would be a good idea if it is necessary to prevent traffic interception.
Conclusion
The architecture described above is a balanced solution, a compromise between the off-the-shelf products and development efforts, which allows building highly loaded systems with a considerable degree of reliability and fault tolerance on

relatively inexpensive (should be interpreted as simple) servers without extra processing power.

REFERENCES

1. Redis Cookbook, Practical Techniques for Fast Data Manipulation, By Tiago Macedo, Fred Oliveira, Publisher: O'Reilly Media, (Release Date: July 2011)
2. MySQL Cookbook: Solutions for Database Developers and Administrators 3rd Edition by Paul DuBois (Author). Publisher: O'Reilly Media; 3 edition (August 18, 2014)
3. Learning Python, 5th Edition Powerful Object-Oriented Programming, By Mark Lutz, Publisher: O'Reilly Media (Release Date: June 2013)

MSC 49M37

MODELING THE FUNCTIONING OF ECONOMIC SYSTEMS USING PRODUCTION VES FUNCTIONS

Choroev K., Suynalieva N.K., Zhusupbaeva N.A.
Institute of Mathematics of NAS of KR

This article examines the problem of constructing a production function of variable elasticity of substitution (VES-functions). The results of modeling the functioning of economic systems using production functions with variable elasticity of labor replacement by capital (VES-functions) are presented. A comparative analysis of the simulation results obtained using the already known analytical dependencies for VES functions and the proposed algorithm for constructing production functions of this type showed the feasibility of this algorithm for solving similar problems. To test this algorithm for constructing production functions of the VES type, statistical data published in the open press were used.

Keywords: production function; VES-function; elasticity of replacement of labor with capital; variable elasticity.

Бул статьяда өзгөрүлмө ийкемдүү алмаштыруу негизиндеги өндүрүш функциясын (VES-функция**)** куруу проблемасы изилденент. Эмгек менен капиталды алмаштыруунун өзгөрмөлүү икемдилүүк көрсөткүчтүү өндүрүш функциясын колдонуу менен экономикалык системанын иштешинин моделдештирүүнүн жыйынтыгы берилген. Бул функциянын жардамы менен моделштирүүнүн жыйынтыгы жана белгилүү VES-функция үчүн аналитикалык көз карандылкты салыштырмалуу анализ, биздин сунуштаган өндүрүш функциясын куруу алгоритм коюлган максатка ылайык экендигин көргөздү. VES-функция түрүндөгү өндүрүш функциясын куруудагы берлиген алгоритм ачык басылып чыккан статистикалык берилиштердин негизинде апробация кылынды.

Урунттуу сөздөр: өндүрүш функциясы, VES-функция, эмгекти капитал менен алмаштыруу ийкемдүүлүгү, өзгөрүлмө ийкемдүүлүк.

В данной статье исследуется проблема построения производственной функции переменной эластичности замещения (VES-функций). Представлены результаты моделирования функционирования экономических систем с использованием производственных функций с

переменной эластичностью замещения труда капиталом (VES-функции). Сравнительный анализ результатов моделирования, полученных с применением уже известных аналитических зависимостей для VES-функций, и предложенного нами алгоритма построения производственных функций этого вида показал целесообразность данного алгоритма для решения аналогичных задач. Для апробации данного алгоритма построения производственных функций вида VES-функций использовались статистические данные, опубликованные в открытой печати.
Ключевые слова: производственная функция, VES-функция, эластичность замещения труда капиталом, переменная эластичность.

Determining the main indicators of the functioning of economic systems is often carried out using production functions (PF). The latter are one of the tools of economic and mathematical modeling of the production process, if it is considered as an open system, the inputs of which are the costs of resources (material and human), and the outputs are the products produced. Production functions are also used to analyze the impact of a number of key factors (inputs) on the results of the production process (outputs). This is due to the fact that PF generally reflect fairly stable quantitative ratios between the inputs and outputs of economic and production systems.

The complexity of economic systems that use neoclassical production functions of the form CES-functions to describe their functioning does not always allow us to say that the values of elasticity of labor replacement by capital σ in the systems under consideration are constant, since this situation is not so common in the real conditions of economic systems functioning.

VES - production functions (*variable elasticity substitution production function*) are a type of neoclassical production functions that takes into account changes in the values of elasticity of labor replacement by capital σ in economic systems. Currently, there are various variants of analytical representation of a production function of the form VES-function.

A number of authors have accepted that the marginal rate of replacement of labor with capital γ is characterized by the following dependence on the Fund capacity *k* of the considered economic system:

$\gamma = \boldsymbol{\alpha + \beta k},$ where $\beta > 0,$ and $\frac{\alpha}{\beta} < k$

Then σ – is defined by the dependency

$$\sigma(k) = 1 + \left(\frac{\alpha}{\beta}\right)k^{-1}$$

where $\sigma(k) < 1$ and $\frac{d\sigma(k)}{dk} < 1$, if $\alpha < 0$;

$\sigma(k) > 1$ and $\frac{d\sigma(k)}{dk} > 1$, if $\alpha > 0$.

and the VES-function is defined as:

$$Y = ae^{\lambda t}\left[(1+\beta)KL^{\beta} + \alpha L^{1+\beta}\right]^{1/1+\beta} \quad (1)$$

the value of γ is estimated by the expression

$$\gamma = k\left(\frac{1}{\alpha + \beta k}\right) - 1$$ where $0 < \alpha < 1, 0 < \alpha + \beta k < 1$.

Based on this, the dependencies for *σ* and VES- functions were obtained:

$$\sigma(k) = 1 - \frac{\beta k}{(\alpha + \beta k)^2 - \alpha}$$

if $\sigma(k) < 1$ and $\frac{d\sigma(k)}{dk} < 1$, if $\beta < 0$;

$\sigma(k) > 1$ and $\frac{d\sigma(k)}{dk} > 1$, if $\beta > 0$.

$$Y = ae^{\lambda t}K^{\alpha}L^{1-\alpha}e^{\beta k} \quad (2)$$

If we assume that the elasticity of replacement of labor with capital *σ* is represented as $\sigma(k) = \alpha + \beta k$, we get a General view of the VES-function for this case of the relationship between *α* and *k*:

$$Y = aK^{\frac{\alpha}{1+c}}\left[L + \left(\frac{b}{1+c}\right)K\right]^{\frac{\alpha c}{1+c}}$$

All parameters of the dependencies that define the VES-function presented above were evaluated based on statistical analysis of the initial retrospective data that characterize the functioning of the economic system.

The assumptions made by the authors concerning the nature of the relationships between *γ*, *σ* and *k* ensure changes in the values of o depending on the value of *k*, as well as the fulfillment of the requirements for neoclassical production functions [4. P.91]. The possibility of using the above-mentioned variants of VES-functions also implies the need for additional justification for the possibility of describing changes in the values of y and o by the accepted dependencies.

The structure of the production function is identified by solving the following system of differential equations [4]:

$$\begin{cases} \frac{g'(k)}{g(k)} = \frac{\delta}{\gamma(k)+k} \\ \frac{\gamma'(k)}{\gamma(k)} = \frac{1}{k\sigma(k)} \end{cases} \quad (3)$$

Here δ is the index of homogeneity of the production function; k is the stock ratio: $k = K/L$; $g(k) -$ modified production function:

$$Y = f(K,L) = Lf(1,K) \rightarrow \frac{Y}{L} = y = f(1,k) = g(k) \quad (4)$$

γ(k) – the marginal rate of replacement of labor with capital:

$$\gamma(k) = \frac{\delta g(k) - kg'(k)}{g'(k)} \quad (5)$$

σ(k) – elasticity of replacement of labor with capital for *σ*-homogeneous production function:

$$\sigma(k) = \frac{1}{k}\left(\frac{d\gamma(k)}{dk}\right)^{-1} \quad (6)$$

The value σ (k) is given by some function, and $\gamma(k)$ and $g(k)$ are determined from the solution of the system (3). Directly *f(K,L)* is determined by the function $g(k)$ according to (4). For example, you can choose a continuous function, including a piecewise linear function, as *σ(k)*.

The initial data for constructing a neoclassical *δ*-homogeneous production function of type VES-function are the sets of output volume values $Y = f(K,L) - Y = \{Y_i\}, (i = \overline{1,n})$ and the corresponding values $K = \{K_i\},\ L = \{L_i\}$ in value or index terms, in kind. They characterize the functioning of the economic system under consideration at each moment of time T_i for a certain time interval $[T_i, T_n]$. The values of the uniformity index are also set

$\delta_j: \delta_j \in [0,1]$. These data are used to determine the values of the capital strength of the considered economic system $k_i = {K_i}/{L_i}$ and the values of the function $g(k_i)$.

The algorithm described above was tested when constructing σ-homogeneous production functions of the VES-function type based on data describing the functioning of the KR economy in the period 2001-2018. This data is a time series of the following values for the specified time period: Y – gross domestic product in million som. (in comparable prices for 2001); L – average annual number of workers, thousand people.; K – gross accumulation of fixed assets (buildings and equipment) less intangible assets in million som. (in comparable prices for 2001). The algorithm was implemented using MS EXCEL data Analysis applications. The table shows the main characteristics of production functions.

Table 1 shows:

- production functions of the form CES-function (PF1 - hereafter the authors ' designations) and VES-function (PF2 (1), PF3 (2)), identified by regression analysis of data table 1;
- o values for the CES function and regression dependences for estimating the values of this value for the VES functions PF2 (1) and PF3 (2).

Dependencies for CES and VES Table 1.

PF	Interval	Type of production function	Values σ
PF CES	2001-2018	$Y = 3{,}32e^{0{,}02}(0{,}92K^{-1{,}12} + 0{,}24L^{-1{,}18})^{-0{,}27}$	0,474
PF 1 VES	2001-2018	$Y = 3{,}12e^{0{,}01}(2{,}91K^{3{,}17} - 0{,}32L^{5{,}21})^{0{,}56}$	$1 - 0{,}23k^{-1}$
PF 2 VES	2001-2018	$Y = 7{,}25e^{0{,}19}K^{0{,}47}L^{0{,}65}e^{-2{,}51k}$	$1 + \frac{ak}{[b-ak]^2 - b}$ $a = 2{,}53$ $b = 0{,}56$

Table 2.

	PF CES	PF 1 VES	PF 2 VES
The average value of $\bar{\boldsymbol{\varepsilon}}$	0,0332	0,0248	0,0283
Standard deviation $\boldsymbol{S_{\varepsilon}}$	00184	0,0172	0,0163

Comparison of mean and standard deviation values (see table. 2) shows that the constructed production function of the VES-function type allows us to get a more accurate approximation of the values of the final product of the economic system *Y*. The values of the average error of approximation of the source data and its standard deviation for the VES-2-function constructed by the authors are less than for the CES-function. Therefore, the proposed algorithm for constructing production functions of the VES-function type gives a more "stable" approximation of the calculated values of the value *Y* to its original values.

Thus, it can be noted that the proposed and implemented algorithm for constructing σ - homogeneous production function of the VES-type, which meets the requirements for neoclassical production functions, is able to provide the construction of this function with a sufficiently high accuracy of approximation of data that characterize the functioning of the economic system.

REFERENCES

1. Socio-economic development of the Kyrgyz REPUBLIC http://mineconom.kg/index.php?Itemid=159&lang=ru

2. Bessonov V. A. Problems of construction of production functions in the Russian transition economy. - M.: Institute of transition economy, 2002.

3. Gorbunov V. K. Lvov A. G. Construction of two-factor production functions with constant and variable elasticity of substitution. Informats. - mod. technologies in Economics, technology and education. Issue 5. - Yekaterinburg: UGTU-UPI, 2009.

MSC 49M37

OPTIMIZATION MODEL OF URBAN PASSENGER TRANSPORT MANAGEMENT

[1]**Kydyrmaeva S.,** [2]**Nurlanbekov A.,** [3]**Maatov K.**

[1]KNU named after J.Balasagyn, [2,3]Institute of Mathematics of NAS of KR

This article is devoted to the analysis of a complex of economic and mathematical models that take into account various aspects of the functioning of passenger transport. There are considered such concepts as predictive, simulation, dynamic and optimization transport models. The necessity of developing an optimization model for the formation of the route network and its elements, the optimal interaction of the participants in the passenger transport system, the model of technical and economic indicators of the urban passenger transport (UPT) system is presented.

Key words: urban passenger transport, the efficiency of UPT of the city, tasks of the passenger transport system, simulation, optimization models, transport network.

Бул статья пассажирдик транспорттун иштешининар тараптуу аспектерин эске алган экономико-математикалык моделдерди анализдөөгө арналган. Прогноздоодо, имитациалоодо жана оптимизациалоода колдонуучу транспорттук моделдер түшүнүгү каралган. Маршуруттук түйүндөрдү жана анын элементтерин, пассажирдик транспорттук системанын катышуучуларынын бир бирине таасирин оптималдуу моделдөө зарылчылыгы келтирилген. Жана да шаардык пассажирдик транспорттук системанын (ШПТС) технико-экономикалык көрсөткүчтөрүнүн моделин түзүү да каралган.

Урунттуу сөздөр: шаардык пассажирдик транспорттук системасы (ШПТС), ШПТСнын иштешинин эффективтүүлүгү, ШПТСнын милдети, имитациалык моделдештирүү, оптимизациалык моделдештирүү, транспорттук түйүн.

Настоящая статья посвящена анализу комплекса экономико-математических моделей, учитывающих различные аспекты функционирования пассажирского транспорта. Рассматриваются такие понятия как прогнозные, имитационные, динамические и оптимизационные транспортные модели. Представлена необходимость разработки оптимизационной модели формирования маршрутной сети и ее элементов, оптимального взаимодействия участников системы пассажирского транспорта, модели технико-экономических показателей системы городского пассажирского транспорта (ГПТ).

Ключевые слова: городской пассажирский транспорт, эффективность работы городского пассажирского транспорта города, задач системы ГПТ, имитационное моделирование, оптимизационные модели, транспортная сеть.

The urban passenger transport system of a large city is a complex system that includes a large number of interconnected and interacting components. The management of such a large system is becoming more complicated every year due to the growing population of cities, the level of motorization, and therefore there are problems with Parking and transport infrastructure. In the city of Bishkek, the issue of optimizing the route network is currently acute.

Management of urban passenger transport for many years has not been subjected to a serious scientific study of the search for scientifically based solutions. The city is overloaded with duplicate routes and private buses (14-18 local) of small class. To study and objectively assess these problems, it is necessary to build a set of economic and mathematical models that take into account various aspects of the functioning of passenger transport. When building models, three principles must be taken into account: abstraction, multi-modeling, and hierarchical construction. Due to the complexity of the urban transport system, no single model can adequately describe the various aspects of this system.

To solve the problems of the UPT system, various mathematical tools can be used, such as mathematical programming, econometrics, game theory, Queuing systems theory, simulation modeling, etc. In recent years, predictive and simulation transport models have been widely used in research. Classical four-stage predictive models are most common for solving transport planning problems and evaluating the effectiveness of modernization projects for various transport systems [1]. Dynamic models are used to solve problems in the field of traffic management. This class of models is intended for the most accurate simulation of the movement of individual vehicles in a stream. As a result of computational experiments, parameters of traffic flows are obtained: speed, density, delays, transit time for selected sections of the network, and the length of the queue when congestion occurs.

Optimization transport models are used to search for options (scenarios) for the development of the urban agglomeration transport system that are optimal by a certain criterion under specified restrictions. In the formulation of mathematical programming criteria and constraints represent the functional characteristics of the transport system and the parameters of human needs, environmental and other indicators of quality of life. One of the options for building optimization transport models is to analyze costs and effects in order to effectively distribute transport demand across the territory of an urban agglomeration.

Consistent use of optimization, forecast and simulation models is suitable for solving all typical tasks of transport planning, traffic management, and improving the system of public passenger transport.

Optimization models are widely used in research of the urban passenger transport management system. Optimization models for the formation of the route network and its elements, optimal interaction of participants in the passenger transport system, and models of technical and economic indicators of the UPT system are being developed.

In the work of the UPT, the problem of choosing a route scheme for passenger transport is one of the most important and complex among the tasks solved in the process of organizing passenger transportation. The optimal route scheme along the city streets and the extent to which the routes meet the needs of passengers depend on the time spent by the population on movement, as well as on the efficiency of the use of rolling stock. Requirements for the rational use of rolling stock can be expressed by setting the value of the minimum allowable coefficient of use of the capacity of rolling stock on all routes of the city. To solve this problem, we use data that can only be obtained from special surveys, as well as data available in municipalities and transport organizations. The city's transport network is known, consisting of nodes (centers of neighborhoods) and edges (streets where passenger transport is organized or possible) that connect the nodes to each other.

Each node of the transport network corresponds to the value t_{nm}, which is numerically equal to the time spent on transferring at point *M*. Each edge of the network corresponds to the value t_{cij} numerically equal to the time spent on moving the bus between points *i* and *j*. We know the distance between points *i* and *j* - the length of the edge L_{ij}. In the general case $t_{cij} \neq t_{cji}$ и $L_{ij} \neq L_{ji}$. The matrix is set for the number of population movements between all city districts for a certain (calculated) period of the day - $\|K\|$, where the element K_{ij} - corresponds to the number of movements made from point *i* to point *j*. Each combination of routes connecting transport network nodes corresponds to a value *E*, which is numerically equal to the total time spent by all passengers to travel along all routes, including the time spent waiting for the start of the trip, following in rolling stock and transferring from route to route,

As a criterion for the optimal route scheme, the minimum total time spent by the population on movement using transport is taken, including the time-estimated inconveniences experienced by passengers when making transfers and the monetary cost of buying a ticket for travel.

In the process of solving tasks required to minimize the functional *E*:

$$E = \sum_{i=1}^{m}\sum_{j=1}^{m}(t_{cij} + t_{pij})K_{ij} + \sum_{k=1}^{n} t_{\text{ож},k} \cdot p_k + \sum_{v=1}^{w} t_{\text{ож},v} \cdot P_v . \quad (1)$$

where $i = l, 2, ..., m$ – points (neighborhoods) of the beginning of movement;

$j = 1,2,..., m$ – points (neighborhoods) of the end of movement;

$k=1,2,..., n$ – bus routes;

$v=1,2,.., w$ – combined sections of the transport network;

t_{cij}, t_{nij} – the time spent by one passenger when traveling between points *i* and *j*, respectively, for following and transferring;

K_{ij} – the number of movements between points *i* and *j*.

$t_{\text{ож},k}$ – the time spent by one passenger waiting for the start of the trip on route *k*;

P_k – is the number of passengers using route *k* only;

$t_{\text{ож},v}.v$ – time spent by one passenger waiting for the start of the trip when traveling on a combined section of the transport network *V*;

P_v – the number of passengers passing through the combined section *V*.

One of the goals pursued when choosing a route scheme is to provide people with the opportunity to make their movements around the city in a minimum time. This is achieved, first, by choosing the routes between the end points along the shortest path, and, second, by creating a combination of routes that would minimize the total time spent by all passengers on movement.

The goal of solving the problem is to choose the route scheme that best meets the specified optimality criteria. The main task of planning the functioning of the UPT system is to forecast and form a given level of passenger service with minimal costs for their implementation. This allows you to get the maximum profit from the operation of transport at economically reasonable rates. Taking into account its special social significance for the city's population, you can set actual rates lower than the estimated ones, and minimize the amount of subsidies to maintain a given level of service.

It is necessary to determine the strategies for the functioning of the UPT system that ensure the achievement of the optimal value of the level of passenger service, and, consequently, the maximum amount of profit at economically reasonable calculated rates or the minimum amount of loss at low actual rates.

Thus, the UPT system, without having an optimal level of passenger service, must constantly choose strategies related to optimizing the rolling stock by type of transport in accordance with the emerging passenger flows in the city.

Modeling of the process under consideration has shown that the efficiency of passenger transport operation implies achieving the best financial results of the UPT (maximum profit at economically justified calculated rates or minimum loss-subsidies at actual rates less than the calculated ones) while providing a given level of service.

Thus, the main constraints of the optimization problem are selected indicators of service passenger services: the reliability of travel by public transport on schedule, accessibility, safety, comfort, and value indicator of the level of passenger service and rate information service trip. Their level is set in accordance with the proposed method and then adjusted based on the model of choosing a strategy for achieving the optimal level of passenger service.

REFERENCES

1. Kydyrmayeva S.S. Actual problems of economy and management. Sankt-Petersburg state University of aerospace instrumentation. -2018. - No. 3. -P. 31-34.

2. Malikov, R. F. Practicum on simulation of complex systems in the AnyLogic environment 6 [Text]: textbook. manual / R. F. Malikov. - Ufa: BSPU Publishing house, 2013. - 296s. (www.anylogic.ru)

CONTENTS