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FUNCTIONAL ANALYSIS MAIN PRINCIPLES FOR MAX-PLUS SEMIADDITIVE FUNCTIONALS

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We construct a space of normed, max-plus-homogeneous and max-plus-semi-additive functionals and give its description. Variants of Hahn-Banach lemma and Banach-Alaoglu theorem for max-plus-semi-additive functionals. Further, it was shown that every max-plus-semi-additive functional is open and bounded map. And also noted that for a compact metrizable space X the space $IS(X)$ is a metrizable compact.
Keywords: category, normal functor, max-plus-semi-additive functional.

Нормалдаштырылган, max-plus-бир тектүү жана max-plus-жарым-аддитивдик функционалдардын мейкиндиги түзүлгөн жана анын мазмуну келтирилген. Хан-Банах леммасынын варианттары жана Банах-Алаоглу теоремасы дары үчүн түзүлгөн. Андан тышкары, ар бир max-plus-жарым-аддитивдик функционалы ачык жана чектелген чагылдыруу экендиги көрсөтүлгөн. Метризациялануучу X компакты үчүн, max-plus-жарым-аддитивдик функционалдарынын IS (X) мейкиндиги ченелүүчү компакт экендиги белгиленди.
Урунттуу сөздөр: категория, кадимки функционал, max-plus-жарым-аддитивдик функционал.

Построено пространство нормированных, max-plus-однородных и max-plus-полуаддитивных функционалов и дано его описание. Установлено варианты леммы Хана-Банаха и теоремы Банаха-Алаоглу для max-plus-полуаддитивных функционалов. Далее, показано, что каждый max-plus-полуаддитивных функционал является открытым и ограниченным отображением. А также отмечено, что для метризуемого компакта X пространство $IS(X)$ max-plus-полуаддитивных функционалов является метризуемым компактом.
Ключевые слова: категория, нормальный функтор, max-plus-полуаддитивный функционал.

Let X be a compact Hausdorff space, $C(X)$ the algebra of all continuous functions on X with ordinary operations and the sup-norm. We enter a new multiplication and a new addition on $C(X)$ by the rules:

$\odot\,(\lambda, \varphi) = \lambda \odot \varphi = \varphi + \lambda_X$, where $\varphi \in C(X)$ and λ_X is a constant function on X taking everywhere the value $\lambda \in \mathbb{R}$;

$\oplus\,(\varphi, \psi) = \varphi \oplus \psi = \max\{\varphi, \psi\}$, where $\varphi, \psi \in C(X)$.

A functional $\mu: C(X) \to \mathbb{R}$ is said to be [10] an idempotent probability measure on X if it has the following properties:

(i) (normality) $\mu(\lambda_X) = \lambda$ for all $\lambda \in \mathbb{R}$;

(ii) (max-plus-homogeneity) $\mu(\lambda \odot \varphi) = \lambda \odot \mu(\varphi)$ for all $\lambda \in \mathbb{R}$, and $\varphi \in C(X)$;

(iii) (max-plus-additivity) $\mu(\varphi \oplus \psi) = \mu(\varphi) \oplus \mu(\psi)$ for all $\varphi, \psi \in C(X)$.

The set of all idempotent probability measures on X is denoted by $I(X)$. Idempotent measures were observed in [2], [3], [6], [9], [10] and so on.

The following definition was offered by A. Zaitov.

Definition 1. A functional $\mu: C(X) \to \mathbb{R}$ is called *max-plus-semi-additive* if:

(iv) $\mu(\varphi \oplus \psi) \geq \mu(\varphi) \oplus \mu(\psi)$ for any pair $\varphi, \psi \in C(X)$.

The set of all max-plus-semi-additive normalized and max-plus-homogeneous functionals is denoted by $IS(X)$. For brevity, a max-plus-semi-additive, normalized, and homogeneous functional will be called the max-plus-semi-additive functionals. Each max-plus-semi-additive functional $\mu: C(X) \to \mathbb{R}$ is continuous. Consequently, $IS(X) \subset C_p\big(C(X)\big) \subset \mathbb{R}^{C(X)}$. We provide $IS(X)$ with the induction from $\mathbb{R}^{C(X)}$ topology. Sets of the shape

$$\langle \mu;\ \varphi_1, \dots, \varphi_k;\ \varepsilon \rangle = \{\, \mu' \in IS(X): |\mu'(\varphi_i) - \mu(\varphi_i)| < \varepsilon,$$
$$i = 1, \dots, k\}$$

where $\varphi_1, \varphi_2, \dots, \varphi_k \in C(X)$ and $\varepsilon > 0$, form a base of neighborhoods of the functional $\mu \in IS(X)$ with respect to this induced topology. But, on the other hand, these sets form the pointwise convergence topology on $IS(X)$. Note that $IS(X)$ is a compact Hausdorff space with respect to the pointwise convergence topology.

Consider a continuous map $f: X \to Y$ of compact Hausdorff spaces. It induces the following natural map $IS(f): IS(X) \to IS(Y)$ by

$$IS(f)(\mu)(\varphi) = \mu(\varphi \circ f).$$

The map $IS(f)$ is continuous. The functor IS is [4] a normal functor in the category of compact Hausdorff spaces and their continuous maps. Consequently, for each $\mu \in IS(X)$ one may define its support (see [5]):

$$\operatorname{supp} \mu = \cap \{F: F \text{ closed in } X \text{ and } \mu \in IS(F)\}.$$

For a positive integer n we define the following set

$$IS_n(X) = \{\mu \in I(X): |\operatorname{supp} \mu| \leq n\}.$$

Put

$$IS_\omega(X) = \bigcup_{n=1}^{\infty} IS_n(X) .$$

The set $IS_\omega(X)$ is everywhere dense [4] in $IS(X)$. A functional $\mu \in IS_\omega(X)$ is called a max-plus-semi-additive functional with finite support.

Note that for every compact Hausdorff space X we have

$$I(X) \subset IS(X).$$

But, in general, the converse is not true.

Let A be a subset of $I(X)$. For each finite system $\{B_1, \dots, B_n\}$ of subsets $B_i \subset A$ and numbers α_i satisfying the conditions

$$\alpha_i \geq 0, \qquad i = 1,2, \dots, n, \qquad \sum \alpha_i = 1 , \qquad (1)$$

define a functional

$$\nu_{B_1,\dots,B_n}^{\alpha_1,\dots,\alpha_n} = \sum_{i=1}^{n} \alpha_i \nu_{Bi} \,, \qquad (2)$$

where $\nu_{Bi} = \bigoplus_{\mu \in Bi} \mu$.

For each set A, the system $\{B_1, \dots, B_n\}$ of its closed subsets and numbers α_i satisfying conditions (1), the functional $\nu_{B_1,\dots,B_n}^{\alpha_1,\dots,\alpha_n}$ defined equality (2) is a max-plus-semi-additive functional, i. e., $\nu_{B_1,\dots,B_n}^{\alpha_1,\dots,\alpha_n} \in IS(X)$.

We define the following set

$$A_s = \left[\left\{\nu_{B_1,\dots,B_n}^{\alpha_1,\dots,\alpha_n} : B_1, \dots, B_n \text{ closed in } A, \alpha_i \geq 0,\right.\right.$$

$$\left.\left. i = 1,2, \dots, n, \sum_{i=1}^{n} \alpha_i = 1\right\}\right]_{IS(X)}$$

Theorem 1. For every compact Hausdorff space X, it holds the equality

$$\big(I(X)\big)_S = IS(X).$$

Note that Theorem 1 actually describes the space of $IS(X)$ max-plus-semi-additive functionals in the language of idempotent probability measures, i. e., the elements $\mu \in I(X)$.

Below we list main results of the paper. Missing information can be found from [1], [7], [8].

Proposition 1. For every compact Hausdorff space X we have

$$d\big(IS(X)\big) \leq d(X),$$

$$wd\big(IS(X)\big) \leq wd(X),$$

Here $d(X)$ and $wd(X)$ mean the density and weak density of a topological space X, respectively.

Theorem 2. (variant of the Hahn-Banach lemma) For every max-plus linear subspace $L \subset C(X)$ and for any max-plus semi-additive functional $\mu: L \to \mathbb{R}$ there exists a any max-plus semi-additive functional $\tilde{\mu}: C(X) \to \mathbb{R}$ such that $\tilde{\mu}|_L = \mu$.

Proposition 2. For every compact Hausdorff space X a max-plus semi-additive functional $\mu: C(X) \to \mathbb{R}$ is bounded.

Theorem 3. For every compact Hausdorff space X a max-plus semi-additive functional $\mu: C(X) \to \mathbb{R}$ is open map.

Theorem 4. Let X be a compact Hausdorff space. If V is a neighborhood of zero $0_X \in C(X)$ (with respect to uniform topology) the following set is compact in pointwise convergence topology

$$K = \{\mu: \ \mu \text{ satisfies } (ii) - (iv) \text{ and } \mu(\varphi) \leq 1 \text{ for all } \varphi \in V\}.$$

Corollary 1. $IS(X)$ is a compact Hausdorff space.

Theorem 5. Let X be a compact Hausdorff space with second axiom of countability, and K be a compact set of functionals, satisfying conditions $(ii) - (iv)$. Then K is metrizable in pointwise convergence topology.

Corollary 2. Let X be a compact Hausdorff space with second axiom of countability. Then $IS(X)$ is metrizable in pointwise convergence topology.

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MSC 54 I35

ON MULTIMETRIC SPACES

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In this paper, we consider τ -metric spaces and study their properties.
Key words: τ -metric spaces, $\tau-$spectrum, quotient space.

Бул макалада τ-метрикалык мейкиндиктери талкууланып, алардын касиеттери изилденет.
Урунттуу сөздөр: $\tau-$ мейкиндиктери, фактор мейкиндиги, $\tau-$спектр.

В этой работе рассматриваются τ-метрические пространства и изучаются их свойства.
Ключевые слова. $\tau-$метрические пространства, пространства, фактор пространство, $\tau-$спектр.

The class of multimetric or τ -metric spaces is introduced and studied in [1], [2].

In this article we clarify the concept of τ -metric spaces and study its properties.

The most general metrics on topological semifields were discussed in [3], [4] and there are a number of papers devoted to generalization of metrics, in particular metrics on Banach spaces (see, for example, [5]).

Let $\mathbb{R}_\tau = [0,\infty)$, $\mathbb{R} = (-\infty,\infty)$ and let τ be an infinite cardinal number. Let $\mathbb{R}_+^\tau$ and $\mathbb{R}^\tau$ denote the Tichonoff products τ-of copies of the spaces $\mathbb{R}_+$ and $\mathbb{R}$ endowed by the natural topologies, respectively. In the spaces $\mathbb{R}_+^\tau$ and $\mathbb{R}^\tau$ the operations of addition, multiplication and multiplication on scalar, as well as the partial order are defined. The space $\mathbb{R}^\tau$ turns into the so-called "Tichonoff" topological semifield and $\mathbb{R}_+^\tau$ is the positive cone of the topological semifield $\mathbb{R}^\tau$ (see [4], [5]). On the other hand, it was proved that any topological semifield can be embedded into the Tichonoff topological semifield $\mathbb{R}^{\tau}$ (see [4], [5]).

We will use the following important concept (see, for example, [5]).

Let $\{f_a : X \to Y_a,\ a \in A\}$ be a family of the mappings from X into a family of the sets $\{Y_a :\ a \in A\}$. The mapping $f : X \to \prod\{Y_a :\ a \in A\}$ defined by the rule $fx = \{f_a x :\ a \in A\}$ is called the diagonal product of the family of mappings $\{f_a :\ a \in A\}$ and is denoted by $\Delta\{f_a :\ a \in A\}$.

Definition 1. Let X be an infinite set,

$$\mathbb{R}_+^{\tau} = \prod\left\{\mathbb{R}_+^a : a \in A\right\},\ |A| = \tau,\ \mathbb{R}_+^a = \mathbb{R}_+$$

and $q_a : \mathbb{R}_+^{\tau} \to \mathbb{R}_+^a$ the natural projection on $a-$th factor of $\mathbb{R}_+^a$.

A mapping $\rho_\tau : X \times X \to \mathbb{R}_+^{\tau}$ is called a τ-metric or multimetric (if $ τ is not fixed) on X if the following axioms hold:

1. $\rho_\tau(x, y) = \theta$ if and only if $x = y$, where θ is a point in the space $\mathbb{R}_+^{\tau}$ such that coordinates of which consist of zeros;

2. $\rho_\tau(x, y) = \rho_\tau(y, x)$ for all $x, y \in X$;

3. $\rho_\tau(x, y) \le \rho_\tau(x, z) + \rho_\tau(z, y)$ for all $x, y, z \in X$;

4. for any $a, b \in A$, there exists $a_3 \in A$ such that

$\rho_{a_3}(x, y) = \max\left\{\rho_{a_1}\left(\rho_\tau(x, y)\right), \rho_{a_2}\left(\rho_\tau(x, y)\right)\right\}$ for all $x, y \in X$,

A pair of (X, ρ_τ) is called a τ-metric or ultrametric space.

For each $a \in A$, put $\rho_a(x, y) = q_a\left(\rho_\tau(x, y)\right)$. Then $\rho_a(x, y)$ will be a pseudometric on X and definition 1 will be formulated as follows:

The mapping $\rho_\tau : X \times X \to \mathbb{R}_+^{\tau}$ defined by the family of pseudometric $\rho_\tau(x, y) = \{\rho_a(x, y) : a \in A\}$ satisfies the following conditions:

M1) For any $x, y \in X$ there exists $a \in A$ such that $\rho_a(x, y) > 0$.

M2) For any $a_1, a_2 \in A$ there exists $a_3 \in A$ such that $q_{a_3}(x, y) = \max\{q_{a_1}(x, y), q_{a_2}(x, y)\}$ for all $x, y \in X$.

Every multimetric ρ on a set X generates the uniformity U_{ρ_τ} and topology T_{ρ_τ} on X. Let $V_{O(\theta)} = \{(x, y) : \rho_\tau(x, y) \in O(\theta)\}$ for each neighborhood $O(\theta)$ of point θ in the space $\mathbb{R}_+^\tau$.

Then the family $V_{O(\theta)}$, where $O(\theta)$ runs through the base of the neighborhood of the point θ in the space $\mathbb{R}_+^\tau$, forms the base of some uniformity U_{ρ_τ} on X and the uniformity weight of U_{ρ_τ} is equal τ. $G(x) = \{y \in X : \rho_\tau(x, y) \in O(\theta)\}$ of the neighborhood of point $x \in X$ generates some topology T_{ρ_τ} on X and topology generated by the uniformity U_{ρ_τ} coincides to the topology T_{ρ_τ}. Hence the topological space (X, T_{ρ_τ}) is Tichonoff space [5].

Conversely, let (X, U) be a uniform space and $\rho_\tau(x, y) = \{\rho_a(x, y) : a \in A\}$, $|A| = \tau$, the family of all pseudometrics on the set X that are uniform with respect to U. Then the family $\rho_\tau(x, y)$ has the properties M1) and M2). Hence the mapping $\rho_\tau : X \times X \to \mathbb{R}_+^\tau$ defined by the family of pseudometrics $\rho_\tau(x, y) = \{\rho_a(x, y) : a \in A\}$ is a τ-metric.

However the conditions M1) and M2) are not sufficient to define uniformity in terms of pseudometrics. This requires that τ-metric $\rho_\tau(x, y) = \{\rho_a(x, y) : a \in A\}$ satisfies one additional condition:

M3) If $d(x, y)$ is pseudometric on X with the property: for any $\varepsilon > 0$ there are $\rho_a \in \rho_\tau$ and $\delta > 0$ such that $d(x, y) < \varepsilon$ provided $\rho_a(x, y) < \delta$, then

$d \in \rho_\tau$.

Thus τ -metrics on X are "wider" than uniformities on X .

We introduce a partial order on the set A. We assume that $a \le b$ if and only if $\rho_a(x,y) \le \rho_b(x,y)$ for all $x,y \in X$. It follows from the last axiom of the τ -metric that this partial order is directed.

We introduce an equivalence relation on X: $x \overset{a}{\sim} y$ if and only if $\rho_a(x,y)=0$. Let $X_a = X / \overset{a}{\sim}$ be a factorset, $\pi_a^0 : X \to X_a$ a factormapping. By d_a we denote the metric on X which is obtained as the result of factorization of the pseudometric ρ_a.

Let $[x]_a = \left\{ y \in X : x \overset{a}{\sim} y \right\}$. Note that if $a \le b$ then $[x]_b \subseteq [x]_a$.

Remark 1. Let $X_a = \left\{ [x]_a : x \in X \right\}$, $\pi_a^0(x) = [x]_a$. If $a \le b$ then the mapping $\pi_a^b : X_b \to X_a$ is defined by the rule $\pi_a^b\left([x]_b\right) = [x]_a$ for all $x \in X$. If $a \le b \le c$, then $\pi_a^b \cdot \pi_c^b = \pi_a^c$. Since $\rho_b(x,y) \ge \rho_a(x,y)$ for all $x \in X$ that the mapping $\pi_a^b : (X_b, d_b) \to (X_a, d_a)$ is continuous. Such we have obtained the projective spectrum $S = \left\{ (X_a, d_a),\ \pi_a^b,\ A \right\}$ of length $\tau = |A|$ consisting of metric spaces $(X_a, d_a)$and continuous maps π_a^b over a directed set A. The resulting spectrum S will be called a $\tau-$spectrum of $\tau-$metric space (X, ρ_τ).

Let $X' = \underleftarrow{\lim}\left\{ X_a,\ \pi_a^b,\ A \right\}$. If $x \in X$ is an arbitrary element then it will put under the thread $fx = x' = \left\{ \pi_a(x) : a \in A \right\}$, and if $a \le b$ then $\pi_a^b\left(\pi_b(x)\right) = \pi_a(x)$.

From the definition of the mapping $\pi_a^b : X_a \to X_b,\ a \le b,\ \pi_a^b\left([x]_b\right) = [x]_a$ for each $x \in X$ and it follows from remark 1 that every thread of the limit

X' has the form $x' = \{\pi_a(x) = [x]_a : a \in A\}$.

Since the mapping family $\{\pi_a : a \in A\}$ distinguishes elements of the set X the mapping $f : X \to X'$ is injective and from the remark 1, f is a bijection. Let define X' $\tau-$metric by the rule $\rho_\tau^{'}(x', y') = \rho_\tau^{'}(\pi_a(x), \pi_a(y)) = \{\rho_a(x, y) : a \in A\}$ where $x' = \{\pi_a(x) : a \in A\}, .\, y' = \{\pi_a(y) : a \in A\}$. It is easy verified that $\rho'(x', y')$ is a $\tau-$metric on X. Then $\tau-$metric space (X, ρ_τ) is isometric to $\tau-$metric space $(X', \rho_\tau^{'})$.

Thus, we get the following theorem.

Theorem 1. A $\tau-$metric space (X, ρ_τ) is the limit of the projective spectrum $S = \{(X_a, d_a), \pi_a^b, A\}$ of length τ, $|A| = \tau$ consisting of the metric spaces (X_a, ρ_a) and continuous mappings π_a^b.

Let (X, ρ_τ) be a τ-metric space and $x \in X$ arbitrary point. Let $a_1, a_2, ..., a_n \in A$ be some elements and $\varepsilon_1, \varepsilon_2, ..., \varepsilon_n$ some positive numbers. Then every neighborhood of the point x_0 has the form

$$O_{a_1,a_2,...,a_n}^{\varepsilon_1,\varepsilon_2,...,\varepsilon_n} = \{y \in X : \rho_\tau(x_0, y) \in [0, \varepsilon_1] \times [0, \varepsilon_2] \times ... \times [0, \varepsilon_n]\} \times$$

$$\times \prod \{\mathbb{R}_+^a : a \in A / \{a_1, a_2, ..., a_n\}\} \text{ or } O_{a_1,a_2,...,a_n}^{\varepsilon_1,\varepsilon_2,...,\varepsilon_n} = \{y \in X : \rho_\tau(x_0, y) < \varepsilon_n,\ i = 1, 2, ..., n\}.$$

Suppose that $O_{a_0}^{\varepsilon_{a_0}} \subseteq O_{a_1,a_2,...,a_n}^{\varepsilon_1,\varepsilon_2,...,\varepsilon_n}$ where $a_0 = \max\{a_1, a_2, ..., a_n\}$, $\varepsilon_{a_0} = \min\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$. Let $y_0 \in O_a^{\varepsilon_a}$. Then $\rho_a(x_0, y_0) < \varepsilon_{a_0}$. But $\rho_{a_i}(x_0, y_0) \le \rho_{a_0}(x_0, y_0) < \varepsilon_a < \varepsilon_{a_i}$, $i = 1, 2, ..., n.$ Hence, $y \in O_{a_1,a_2,...,a_n}^{\varepsilon_1,\varepsilon_2,...,\varepsilon_n}$. Without limiting generality, we can assume that each neighborhood of the point $x_0 \in X$ has the form $O_{a_0}^{\varepsilon_a}$. Note that $O_{a_0}^{\varepsilon_{a_0}} = \pi_{a_0}^{-1}\left(G_{a_0}^{\varepsilon_{a_0}}\right)$ where $G_{a_0}^{\varepsilon_{a_0}} = \{[y]_a \in X_a : d([y]_a, [x_0]_a) < \varepsilon\}$. If (X, ρ_τ) is homeomorphic to

$\prod\{(X_a,d_a):a\in A\}$ then $O_{a_0}^{\varepsilon_a}=X\cap\prod\{(X_a,d_a):a\in A/\{a_0\}\}$. Thus $\pi_{a_0}\left(O_{a_0}^{\varepsilon_a}\right)=G_{a_0}^{\varepsilon_{a_0}}$ and $\pi_{a_0}\left(O_{a_0}^{\varepsilon_a}\right)=X_a$ for all $a\neq a_0$.

We have obtained the following theorem.

Theorem 2. A mapping $\pi_a:(X,\rho_\tau)\to(X_a,d_a)$ is open continuous map for all $a\in A$.

Let (X,ρ_τ) be a τ -metric space. Above we have shown how τ metric ρ_τ generates the uniformity U_{ρ_τ} on X. Consider other way to determine the uniformity of U_{ρ_τ}.

Let U_a be the uniformity on X_a generated by the metric ρ_a and $\pi_a^{-1}U_a$ its preimage on X, $a\in A$. Consider the set of all finite intersections of elements of the family $\{\pi_a^{-1}U_a:a\in A\}$. It forms a base of some uniformity which coincides with the uniformity U_{ρ_τ}.

Let (X,ρ_τ) be a complete τ metric space. We show that every metric space (X_a,d_a) is also complete. Let F_a be the Cauchy filter of the metric space (X_a,d_a), $a\in A$. Note that $\pi_a^{-1}\Phi_a\cap\pi_a^{-1}\Phi_b\neq\varnothing$ for all $\Phi_a\in F_a$ and $\Phi_b\in F_b$, $a\neq b$.

Then all finite intersections of elements of the family $\{\pi_a^{-1}F_a:a\in A\}$ forms the base of a certain filter F that the Cauchy filter in uniform space $\left(X,U_{\rho_\tau}\right)$.

By properties of the space $\left(X,U_{\rho_\tau}\right)$, the Cauchy filter F converges to some element $x\in X$. Let $x_a=\pi_a x$. Then the Cauchy filter F_a converges to $x_a\in X_a$. Hence the metric space (X_a,d_a) is complete for all $a\in A$.

Thus, the following theorem is true.

Theorem 3. Every complete τ – metric space is a limit of the projective spectrum of length τ consisting of complete metric spaces.

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MSC 54 E15

ABOUT UNIFORM STRUCTURES OF GROUPS

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In this article we study subgroups of strongly paracompact, superparacompact and compactness index $\leq \eta$ of topological groups, and locally μ-compact groups.

Key words: Topological group, uniform group, strong paracompactness, superparacompactness, compactness index $\leq \eta$.

Илимий макалада күчтүү паракомпактуу, суперпаракомпактуу жана компактуулук индекси $\leq \eta$ болгон топологиялык группалардын камтылган группалары, жана локалдуу μ-компактуу группалар изилденет.

Урунттуу сөздөр: Топологиялык группа, бир калыптуу группа, күчтүү паракомпактуулук, суперпаракомпактуулук, компактуулук индекси $\leq \eta$.

В данной статье исследуются подгрупп сильно паракомпактных, суперпаракомпактных и индекса компактности $\leq \eta$ топологических групп и локально μ-компактные группы.

Ключевые слова: Топологическая группа, равномерная группа, сильная паракомпактность, суперпаракомпактность, индекс компактность $\leq \eta$.

A covering α is called finitely additive if $\alpha^{\angle} = \alpha$, $\alpha^{\angle} = \{\cup \alpha_0 : \alpha_0 \subset \alpha - \mathit{finite}\}$. For a covering α of subsets of a set X, we put $St(x,\alpha) = \{A \in \alpha : x \in A\}$, $St(M,\alpha) = \{A \in \alpha : M \cap A \neq \varnothing\}$, $x \in X$, $M \subset X$. Then $\alpha(x) = \cup St(x,\alpha)$ and $\alpha(M) = \cup St(M,\alpha)$.

Let $(G,\cdot,\tau)$ be a topological group and $(G,\cdot,U)$ be a uniform group.

The following theorems give characteristics of subgroups of the index compactness $\leq \eta$, superparacompact and strong paracompact topological groups.

A topological space X is called index compactness $\leq \eta$ space, if every open covering has a finite open covering cardinality $\leq \mu$ refinement; a topological space X is called superparacompact if every open covering has a finite component open refinement; a uniformity U is called η-pre-Lindelöf if any covering α of the set X is such that $\alpha \cap F \neq \varnothing$ for any $F \in \varphi(X)$ belongs to U, and has a base consisting of coverings of cardinality $\leq \eta$; a uniformity U is called pre-Lindelöf if any covering α of the set X is such that $\alpha \cap F \neq \varnothing$ for any $F \in \varphi(X)$ belongs to U, and has a base consisting of countable coverings; a uniformity U is called presuperparacompact if any covering α of the set X is such that $\alpha \cap F \neq \varnothing$ for any $F \in \varphi(X)$ belongs to U, and has a base consisting of finite component coverings; a uniformity U is called strongly preparacompact if any covering α of the set X is such that $\alpha \cap F \neq \varnothing$ for any $F \in \varphi(X)$ belongs to U, and has a base consisting of star finite coverings.

Theorem 1. A topological group $(G,\cdot,\tau)$ is a subgroup of some index compactness $\leq \eta$ topological group if and only if exist uniformity U on (G,τ) satisfying the following conditions:

1) $\tau_U = \tau$;

2) U is a η -pre-Lindelöf uniformity;

3) $(G,\cdot,U)$ is uniform group.

Proof. Necessity. Let the triple $(G,\cdot,\tau)$ be a subgroup of some index compactness $\leq \eta$ of a topological group $(\tilde{G},\cdot\tilde{\tau})$. Let $\tilde{U}$ be a universal uniformity of topological space $(\tilde{G},\tilde{\tau})$. Since the closure of subgroups in topological group is a topological group and a closed subspace of a index compactness $\leq \eta$ topological group is a index compactness $\leq \eta$ topological group, then subgroup $(G,\cdot)$ is an dense subgroup of the topological group $(G,\cdot,\tau)$. Let U be the uniformity on $(G,\cdot,\tau)$ induced by the uniformity $\tilde{U}$. Then by Theorem 1 [4] the uniformity U is η -pre-Lindelöf uniformity and $\tau_U = \tau$. Further from the fact that if $(G,\cdot)$ is a group, U is an arbitrary uniformity on $(G,\cdot,\tau)$, $(\tilde{G},\tilde{U})$ is completion of (G,U) and in order to continue the group operation from $(G,\cdot,\tau_U)$ to $(\tilde{G},\tau_{\tilde{U}})$ with preservation of continuity, it is necessary and sufficient that $(G,\cdot,U)$ be a uniform group (see 1, page 138, Theorem 3.3.6.), it follows that $(G,\cdot,U)$ is a uniform group.

Sufficiency. Let the group G exist uniformity U that satisfies the conditions of the theorem. Then $(\tilde{G},\cdot,\tau_{\tilde{U}})$ is a topological group. By Theorem 1 [4] the compactness index of the topological group $(\tilde{G},\cdot,\tau_{\tilde{U}})$ is $\leq \eta$.

Corollary 1. A topological group $(G,\cdot,\tau)$ is a subgroup of some Lindelöf topological group if and only if exist uniformity U on (G,τ) satisfying the following conditions:

1) $\tau_U = \tau$;
2) U is a pre-Lindelöf uniformity;
3) $(G,\cdot,U)$ is uniform group.

Theorem 2. A topological group $(G,\cdot,\tau)$ is a subgroup of some superparacompact topological group if and only if exist uniformity U on (G,τ) satisfying the following conditions:

1) $\tau_U = \tau$;
2) U is presuperparacompact uniformity;
3) $(G,\cdot,U)$ is uniform group.

The proof follows from Theorem 3.3.6 [see 1, page 138].

Theorem 3. A topological group $(G,\cdot,\tau)$ is a subgroup of some strongly paracompact topological group if and only if exist uniformity U on (G,τ) satisfying the following conditions:

1) $\tau_U = \tau$;
2) U is strongly preparacompact uniformity;
3) $(G,\cdot,U)$ is uniform group.

The proof follows from Theorem 3.3.6 [see 1, page 138] and 3.1.6 [see 1, page 125].

A topological space X is called μ-compact if every open covering cardinality $\leq \mu$ has a finite open refinement: a topological space X is called countable compact if every open countable covering has a finite open refinement.

A uniform space (X,U) is said to be uniformly locally μ-compact, if there exists such uniform covering that the closures of all its elements are μ-compact.

Uniformly locally $\aleph_0$-compact spaces are called uniformly locally countable compact, i.e. a uniform space (X,U) is said to be uniformly locally countable compact, if there exists such uniform covering that the closures of all its elements are countable compact.

A uniform space (X,U) is said to be uniformly locally compact, if there exists such uniform covering that the closures of all its elements are compact.

Every uniformly locally compact spaces is uniformly locally μ-compact.

Every compact spaces is uniformly locally μ-compact.

A topological space X is called μ-paracompact if every open covering cardinality $\leq \mu$ has a locally finite open refinement: a topological space X is called countable paracompact if every open countable covering has a locally finite open refinement.

A uniform space (X,U) is called uniformly μ-paracompact if every open covering cardinality $\leq \mu$ has a uniformly locally finite open refinement [5]. A covering α of a uniform space (X,U) is said to be uniformly locally finite, if there exists such uniform covering $\alpha \in U$, that

$|St(A,\alpha)| < \aleph_0$ for all $A \in \alpha$. A uniform space (X,U) is uniformly μ-paracompact iff $\alpha^{\angle} \in U$ for all open coverings α cardinality $\leq \mu$ of a space (X,U). A uniform space (X,U) is called uniformly countable paracompact if every open countable covering has a uniformly locally finite open refinement [5].

Theorem 4. Any uniformly locally μ-compact space is uniformly μ-paracompact.

Proof. Let a uniform covering $\alpha \in U$ of a uniform space (X,U) be such, that the closures of all its elements are μ-compact. Then for each open covering β cardinality $\leq \mu$ of a uniform space (X,U) the covering α is refined in $\beta^{\angle}$. It means that $\beta^{\angle} \in U$. Therefore, the space (X,U) is uniformly μ-paracompact according to item 2. [see 5, page 320].

Corollary 2. Any uniformly locally countable compact space is uniformly countable paracompact.

The next theorem establishes the μ-paracompactness of any locally μ-compact topological group.

Theorem 5. A locally μ-compact topological group $(G,\cdot,\tau)$ is μ-paracompact.

Proof. Let $(G,\cdot,\tau)$ be a locally μ-compact topological group. Then for each point of the space G, including for a neutral element e there exists a neighborhood O, the closure of which is μ-compact. It is easy to see that the uniform space (G,U_r) is uniformly locally μ-compact. Then according to Theorem 4 and Theorem [see 2, page 120 and 121] the group $(G,\cdot,\tau)$ is μ-paracompact.

Corollary 3. A locally countable compact topological group $(G,\cdot,\tau)$ is countable paracompact.

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MSC 54 E15

ABOUT STRONGLY UNIFORMLY PARACOMPACT SPACES

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In this article we study the strongly uniformly paracompact spaces. Their connection with other properties of the compactness type is studied. The characteristics of these classes of uniform spaces are established using the mappings. The problem is being solved: what are the uniform spaces that have ω-mapping to some strongly paracompact metrizable uniform space for any open covering ω.

Key words: Uniform space, uniformly continuous mapping, strongly uniformly paracompact space, covering, ω-mapping.

Илимий макалада күчтүү паракомпактуу мейкиндиктер изилденет. Алардын башка компактуу типтеги касиеттер менен болгон байланыштары изилденет. Бир калыптуу мейкиндиктердин бул классынын мүнөздөмөсү чагылдыруулардын жардамы аркылуу тургузулат. Каалагандай ω ачык жабдуу үчүн кандайдыр күчтүү паракомпактуу метризацияланууучу мейкиндикке чагылган ω-чагылдырууга ээ болгон бир калыптуу мейкиндиктерди табуу жана изилдөө маселе чечилет.

Урунттуу сөздөр: Бир калыптуу мейкиндик, бир калыптуу үзгүлтүксүз чагылдыруу, күчтүү бир калыптуу паракомпактуу мейкиндик, жабдуу, ω-чагылдыруу.

В данной статье исследуются сильно равномерно паракомпактные пространства. Изучается их связь с другими свойствами типа компактности. Устанавливается характеристики этих классов равномерных пространств при помощи отображений. Решается задача: найти и исследовать те равномерные пространства, которые для любого открытого покрытия ω обладают равномерно непрерывным ω-отображением на некоторое сильно паракомпактное метризуемое пространств.

Ключевые слова: Равномерное пространство, равномерно непрерывное отображение, сильно равномерно паракомпактное пространство, покрытие, ω-отображение.

As we know the paracompactness and strongly uniformly paracompactness playing an important role in the General Topology. In the theory of uniform spaces, there are various variants of uniformly and strongly uniformly paracompact spaces [1 - 9].

Everywhere in this article uniform spaces to be Hausdorff, topological spaces are to be Tychonoff and mappings to be uniformly continuous.

A uniform space (X,U) is said to be strongly uniformly paracompact if it is uniformly paracompact [8] and its topological space (X,τ_U) is a strongly paracompact.

Proposition 1. Every separable metrizable uniform space (X,U) is strongly uniformly paracompact.

Proof. Let (X,U) be a separable merizable uniform space. Then the uniform space (X,U) has a countable base B consisting of countable coverings. It is easy to see that for any open covering β of the space (X,U) the sequence $\{\alpha_n : n \in N\} \subset U$ of countable coverings satisfies the condition условию (BP) [2]. It is clear that the space (X,τ_U) is a strongly paracompact. Consequently, the uniform space (X,U) is strongly uniformly paracompact.

As you know [2], any strongly uniformly R-paracompact space is complete, but non complete separable metrizable uniform spaces are not strongly uniformly R-paracompact. For example, the space of all rational points with natural uniformity is not complete, i.e. is not strongly uniformly R-paracompact, but it is strongly uniformly paracompact.

Proposition 2. Any uniformly Lindelöf space is a strongly uniformly paracompact.

Proof. Any uniformly Lindelöf space is uniformly paracompact and it is space (X,τ_U) is a Lindelöf. Thus, the uniform space (X,U) is a strongly uniformly paracompact.

Proposition 3. If (X,U) is a strongly uniformly paracompact then the topological space (X,τ_U) is strongly paracompact. Conversely, if (X,τ) is strongly paracompact then the uniform space (X,U_X) is strongly uniformly paracompact, where U_X - universal uniformity of the space X.

Proof. It is clear that the space (X,τ_U) is a strongly paracompact. Conversely, let (X,τ)- be a strongly paracompact topological space. Then the set of all open coverings forms a base the universal uniformity U_X of the strongly paracompact space (X,τ). It is easy to see that a uniform space (X,U_X) is a strongly uniformly paracompact space.

Theorem 1. A uniform space (X,U) is a strongly uniformly paracompact if and only if it is for any open covering ω of the space (X,U) there exists a uniformly continuous ω-mapping f of the uniform space (X,U) onto a strongly paracompact metrizable uniform space (Y_ω,V_ω).

Proof. Necessity. Let (X,U) be a strongly uniformly paracompact space and ω be any open covering of the space (X,U). Then for ω there exists a normal sequence $\{\alpha_n\}$ uniformly covering realizing the condition (BP) [2]. For the normal sequence $\{\alpha_n\}$ of uniformly coverings exists such pseudometric ρ on X that the inclusions $\alpha_{n+1}(x)\subset\{y:\rho(x,y)<\frac{1}{2^{n+1}}\}\subset\alpha_n(x)$ are realized for all $x\in X$ and for all $n\in N$. Introduce the relation of equivalence: for all $x,y\in X$ $x\sim y$ if and only if it is $\rho(x,y)=0$. Let Y_ω be a factor set of the set X and $f:X\to Y_\omega$ is the natural mapping of the set X into set Y_ω. On the set Y_ω we define the metric as follows: for any points $y_1,y_2\in Y$ we put $\sigma(y_1,y_2)=\rho(f^{-1}y_1,f^{-1}y_2)$. It is clear that the metric σ generates uniformity V_ω on Y_ω. Obviously, the map $f:(X,U)\to(Y_\omega,V_\omega)$ is uniformly continuous. Let $y\in Y$ be an arbitrary point and x be an arbitrary point in $f^{-1}y$. Then there exist such number $n\in N$ and $L\in\omega$, that $\alpha_n(x)\subset L$. Denote $O_y=\{y\in Y:\sigma(y,y)<\frac{1}{2^{n+2}}\}$. Then $f^{-1}O_y\subset\{x\in X:\sigma(x,x)\le\frac{1}{2^{n+1}}\}\subset\alpha_n(x)\subset L$. Hence, f is a ω-mapping.

Sufficiency. Let for every open cover ω of (X,U) there exist a uniformly continuous ω-mapping f of the uniform space (X,U) onto some strongly paracompact metrizable uniform space (Y_ω, V_ω). We show that the uniform space (X,U) is strongly uniformly paracompact. Let ω be an arbitrary open covering of a uniform space (X,U) and f be a uniformly continuous mapping of the uniform space (X,U) onto strongly paracompact metrizable uniform space (Y_ω, V_ω). Then there exists a sequence of uniform coverings $\{\beta_n\}$ of (Y_ω, V_ω). We put $\{\alpha_n\}$, where $\alpha_n = f^{-1}\beta_n$. It is clear that $\{\alpha_n\}$ is a sequence of uniform coverings of (X,U). We show that for each point $x \in X$ there exists number $n \in N$ and $O \in \omega$, such that $\alpha_n(x) \subset O$. Let $x \in X$ be an arbitrary point. Then there exist $O_x \in \omega$, such that $O_x \ni x$. By virtue of openness of the set O_x exist number $n \in N$, such that $\alpha_n(x) \subset O_x$. Now we prove that (X, τ_U) is a strongly paracompact. For this it suffices to show that a open covering ω can refined a star finite open covering. Since f is a ω - mapping, every point $y \in Y$ has a neighborhood O_y, whose preimage $f^{-1}O_y$ is contained in at least one element of the covering ω. Denote $\lambda = \{O_y : y \in Y\}$. It is clear that it is an open covering of (Y, τ_V). In it we will refined a star finitely covering β. It is easy to see that a star finitely open covering $f^{-1}\beta$ is refined in the covering ω i.e. the space (X, τ_U) is a strongly paracompact. Consequently, (X,U) is strongly uniformly paracompact.

Corollary 1. The uniform space (X,U) is strongly uniformly paracompact if and only if, then every open cover ω of the space (X,U) is uniformly ω-mapping f of (X,U) onto some strongly uniformly paracompact space (Y,V) .

Proof. Necessity. Let a uniform space (X,U) is strongly uniformly paracompact and ω - arbitrary open cover of (X,U). Then identity mapping $i_X:(X,U)\to(X,U)$ satisfies the condition of the corollary, i.e. it is the required uniformly continuous mapping.

Sufficiency. Let ω be an arbitrary open covering of the uniform space (X,U) and $f:(X,U)\to(X,U)$ is ω -mapping. Then for every point $y\in Y$ exist neighborhood $O_y \ni y$, such that $f^{-1}y\subset O$, where $O\in\omega$. Put $\beta=\{O_y : y\in Y\}$. System β is open covering of the space (Y,V). Since a uniform space (Y,V) is strongly uniformly paracompact, than for a covering β there exists a β-mapping $g:(Y,V)\to(Z,W)$ of a uniform space (Y,V) onto a strongly uniformly paracompact metrizable uniform space (Z,W). It is easy to see that the mapping $h=g\circ f:(X,U)\to(Z,W)$ of a uniform space (X,U) onto a strongly paracompact metrizable uniform space (Z,W) is uniformly continuous. Since the $g:(Y,V)\to(Z,W)$ is a β-mapping, than for each point $z\in Z$ there exists a neighborhood $O_z\ni z$ such that $g^{-1}z\subset B$, where $B\in\beta$. Put $\gamma=\{O_z : z\in Z\}$. From $f^{-1}\beta\succ\omega$, $g^{-1}\lambda\succ\beta$ it follows that $h^{-1}\gamma=f^{-1}\circ g^{-1}\gamma\succ f^{-1}\beta\succ\omega$. Consequently, the uniform space (X,U) is strongly uniformly paracompact.

Proposition 4. Each closed subspace (M,U_M) of a strongly uniformly paracompact space (X,U) is strongly uniformly paracompact.

Proof. Let λ_M be an arbitrary open covering of the subspace (M,U_M). Then there exist such open family γ of (X,U), that $\lambda_M=\gamma\wedge\{M\}$. Denote $\mu=\{\gamma, X\setminus M\}$. It‘s clear that μ is open covering of (X,U). Since the space (X,U) is strongly uniformly paracompact, then for μ exists a sequence countable uniform covering $\{\alpha_i\}$, with properties (BP) [2]. Denote $\{\alpha_i^M\}$,

$\alpha_i^M = \alpha_i \wedge \{M\}$. Obviously, $\{\alpha_i^M\}$ is a sequence uniform coverings of the space (M, U_M). It is easy to see that, for each point $x \in M$ exists number $i \in N$ and $L_M \in \lambda_M$, such that $\alpha_i^M(x) = \alpha_i(x) \cap M \subset L \cap M = L_M$. Hence, (M, U_M) is a strongly uniformly paracompact.

Corollary 2. Any compact uniform space is strongly uniformly paracompact.

Proof. Let ω be an arbitrary open covering of the compact uniform space (X, U) and $\{\alpha_n\}$ be a sequence uniform coverings of the space (X, U). We show that for each point $x \in X$ exists such number $n \in N$ and $O \in \omega$, that $\alpha_n(x) \subset O$. Let $x \in X$ be an arbitrary point. Than exist such $O_x \in \omega$ that $O_x \ni x$. Since the set O_x is open, there is a number $n \in N$ such that $\alpha_n(x) \subset O_x$. It is clear that the space (X, τ_U) is strongly paracompact. Consequently, (X, U) is strongly uniformly paracompact.

Theorem 2. The image of a strongly uniformly paracompact space under uniformly open perfect mappings is strongly uniformly paracompact.

Proof. Let a uniform space (X, U) be strongly uniformly paracompact. Let λ be any open covering of the (Y, V). For an open covering $\alpha = f^{-1}\lambda$ there is a sequence $\{\alpha_i\} \subset U$ that satisfies the condition: for any point $x \in X$ there $i \in N$ and $A \in \alpha$ such that $\alpha_i(x) \subset A$. Since the mapping $f : (X, U) \to (Y, V)$ is uniformly open, for each open cover $\alpha_i \in U$ there is such a uniform cover $\lambda_i \in V$, that satisfies the condition: $f(\alpha_i(x)) \supset \lambda_i(f(x))$ for any point $x \in X$. Hence it follows that for any point $y \in Y$ exist $i \in N$ and $L \in \lambda$, such that $\lambda_i(x) \subset L$. Hence, the uniform space (X, U) is uniformly paracompact. As is known, under open perfect mappings strong paracompactness is preserved in the image, therefore, the topological space

(Y, τ_V) is strongly paracompact, so a uniform space (Y, V) is strongly uniformly paracompact.

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MSC 54 F45

CONSTRUCTIONS TO PROVE RECOGNIZABILITY OF TOPOLOGICAL SPACES

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A space together with an everywhere dense set is said to be marked. Two locally homogeneous points of the space are said to be locally distinguishable by marking set if germs of this set in neighborhoods of these points are different. If there exists such a marking that each two points in the space are distinguishable by it then the space is said recognizable. A survey of constructing marking sets in various topological spaces are presented in the paper.

Key words: topological space, everywhere dense set, marking, recognizability, kinematical space, metrical space.

Көптүгү ар жерде коюу болгон мейкиндик белгиленген деп аталат. Эгерде эки бир калыптагы чекиттин айланаларында ал көптүктүн өсмөлөрү ар башка болсо, анда ал чекиттер ажыратылуучу деп аталат. Эгерде мейкиндиктин ар каалаган эки чекиттерин ажыратуучу мындай көптүк бар болсо, анда мындай мейкиндик таанылуучу деп аталат. Макалада белгилеген көптүктөрдүн курууну жалпы көрүнүш берилет.

Урунттуу сөздөр: топологиялык мейкиндик, ар жерде коюу болгон көптүк, белгилөө, таанылуучулук, кинематикалык мейкиндик, метрикалык мейкиндик.

Пространство вместе с всюду плотным множеством называется размеченным. Две локально однородных точки пространства называются различимыми по размечающему множеству, если ростки этого множества в окрестностях этих точек различны. Если существует разметка, различающая любые две точки пространства, то такое пространство называется распознаваемым. В статье представляется обзор построений размечающих множеств в различных топологических пространствах.

Ключевые слова: топологическое пространство, всюду плотное множество, разметка, распознаваемость, кинематическое пространство, метрическое пространство.

Introduction

The first proposal to use active work on computer to present a virtual (four-dimensional Euclidean) space was made [1], but he did not propose any concrete methods of implementation. Controlled motion in various topological spaces was im- plemented and definition of a kinematical space (the metric ρ_K in such space is the minimal time of passing between points) was introduced in [2]. “Painting” of a space constructed by means of coalescence was proposed [3] for its evident presentation. Splitting the set into two everywhere dense sets (white and black points) for this purpose was proposed in [4].

A general definition of marking a space for motion in it was proposed in [5].

Section 1 contains necessary definitions.

Section 2 presents a survey of constructions based on unique distances between points of a marked set.

Section 3 contains results using stretched components of a marked set.

1. Necessary definitions

Definition 1. If two points of *X* have homeomorphic neighborhoods then they are said to be locally homogeneous.

(For subclasses of the class of topological spaces, corresponding morphisms must be involved instead of homeomorphisms).

Definition 2 [5]. A space *X* together with an every-where dense set *W* (and its complement is every-where dense too) is said to be **marked**. Two locally homogeneous points of a space *X* are said to be locally distinguishable by marking *W* if germs of *W* in neighborhoods of these points are different.

Definition 3 [5]. A space *X* is said to be locally distinguishable by a marking *W* if all its locally homogeneous points are locally distinguishable. If there exists such a marking *W* that space *X* is distinguishable by it then the space *X* is said **recognizable**.

Definition 4 [2]. A pair: a set *X* of points and a set *K* of **routes** is said to be a **kinematic space** (each route *M*, in its turn, consists of the positive real number T_M (**time** of route) and the function $m_M : [0, T_M] \to X$ (**trajectory** of route)) if the following conditions are fulfilled:

(K1) For each different $x_0, x_1 \in X$ there exists such $M \in K$, that $m_M(0) = x_0$ and $m_M(T_M) = x_1$, and the set of values of T_M for all such M is bounded with a positive number from below (infinitely fast motion is impossible); the exact lower boundary is the kinematical distance ρ_K.

(K2) If $M = \{T_M, m_M(t)\} \in K$ then the pair $\{T_M, m_M(T_M - t)\}$ is also a route of K (the reverse motion is possible).

(K3) If $M = \{T_M, m_M(t)\} \in K$ and $T^* \in (0, T_M)$ then the pair: T^* and function $m^*(t) = m_M(t)$ $(0 \le t \le T^*)$ is also a route of K (one can stop at any desired moment).

(K4) If $\{T_1, m_1(t)\} \in K$ and $\{T_2, m_2(t)\} \in K$ and $m_1(T_1) = m_2(0)$, then the pair: number $T^* = T_1 + T_2$ and function

$$m^*(t) = \begin{cases} m_1(t) & (0 \le t < T_1) \\ m_2(t - T_1) & (T_1 \le t \le T_1 + T_2) \end{cases}$$

is also a route of K (transitivity).

Definition 5. If the shortest routes between each two points of any set Y of a kinematic space X exist and pass along this set Y only then the set Y is said to be **flat** in the space X; if any point has a flat neighbor then the kinematic space X is said to be **locally flat** [7]. Such trajectories are said to be **straight** (geodesic) lines.

Definition 6. [6] If any set $L \subset X$ can be presented as a trajectory of a route M with time T_M and $T_M < 2\rho_K(m(0), m(T_M))$ then L is said to be a **weakly twisted** line.

2. Survey of constructions based on unique distances between points of a marked set

2.1. [5] $X = [0;1]$ as a metrical space. The set W is constructed as follows: let $\{p_1, p_2, \ldots\} = \{2,3,\ldots\}$ be prime numbers.

Divide the segment $[0;1]$ into p_1 equal segments by means of (p_1-1) points of W;

Divide the first of these segments into p_2 equal segments by means of (p_2-1) points of W and do the second one into p_3 equal segments by means of (p_3-1) points of W … etc.

2.2. X is a separable kinematical space. It is proven that

$$(\forall z_1, z_2 \in X)\,(\forall \varepsilon > 0)(\exists z_3 \in X)$$

$$((\rho_K(z_2, z_3) \le \varepsilon) \wedge (\rho_K(z_1, z_3) \ne \rho_K(z_1, z_2)) \wedge (|\rho_K(z_1, z_3) - \rho_K(z_1, z_2)| \le \varepsilon)).$$

Let $\{z_k : k \in N\} \subset X$ be a countable everywhere dense sequence. Construct an everywhere dense set S as follows. Denote $v_1 = z_1, v_2 = z_2$.

Let all following points are constructed already:

$S_n = \{v_1, v_2, \ldots, v_{n-1}, v_n\}$, and let them meet the following conditions:

I) $\rho_K(z_n, v_n) \le 1/n$; II) in the list D_n of distances between these points all are distinct.

If $z_{n+1} \not\in S_n$ then let $w_{n+1} = z_{n+1}$ else find such w_{n+1} that

$$\rho_K(w_{n+1}, z_{n+1}) < 1/(n+1)/2.$$

Form the list T_n of such points of S_n that the distances from them to the point w_{n+1} coincide with the distances in D_n. Shift the point w_{n+1} to small distances such that at each step one of the distances in T_n changes slightly and because of smallness none new points appear in T_n. By such a way we obtain such point v_{n+1} that the distances from it to the points of S_n differ from all the distances in T_n and $\rho_K(w_{n+1}, v_{n+1}) < 1/(n+1)/2$, hence $\rho_K(z_{n+1},$

$v_{n+1}) \leq \rho_K (z_{n+1}, w_{n+1}) + \rho_K (w_{n+1}, v_{n+1}) < 1/(n+1)$, the properties I and II remain true.

3. Survey of constructions based on stretched objects

3.1. [5] $X = [0;1]^n$, $n>1$ as a metrical space. Let $\{x_k : k \in N\}$ be an every-where dense set with distinct first coordinates in X. To construct the set W draw segments of length $1/k$ along the first coordinate from each x_k, $k \in N$.

3.2. [6] X is a separable kinematical space with additional condition: for any $\varepsilon>0$, weakly twisted line L and $w \in L$: $S_\varepsilon (w) \setminus L \neq \varnothing$ where $S_\varepsilon (w) = \{x \in X \mid \rho_K(x,w) < \varepsilon\}$ is a ball.

The set W is defined as a union of weakly twisted lines L_k with diminishing lengths $\sim 4^{-k}$.

3.3. [7]. X is a separable locally flat kinematical space.

Definition 7. If a set $B \subset X$ is not connected and there exists such countable set $A \subset X$ that the set $A \cup B$ is connected then the set B is said to be **almost connected.**

The set W is constructed as a union of almost connected subsets of straight lines L_k with diminishing lengths and excluding countable sets similar to the set in 2.1.

3.4 [8]. X is a regular topological set with additional conditions. Let an arc be a continuous 1-1-image of the segment $[0;1]$.

Definition 8. A set obtained by means of excluding of finite number of points (of n points) from a connected closed set is said to be a **finitely-(*n*-)almost-connected- closed**.

Let X meet the following property of “more than one dimensionality”: there exists a set L of arcs:

L1) $(\forall x \in X)(\exists M \in L)(x \in M)$;

L2) $(\forall M \in L)(\forall x \in M)(\forall V \text{ being neighbor of } x)(V \setminus M \neq \emptyset)$.

The set *W* is constructed as union of *n*-almost-connected-closed subsets of “sufficiently small” segments of arcs as *n* tends to infinity.

A computer program was demonstrated [9].

Conclusion

The paper demonstrates that there is sufficient difference between “one-dimensional” and "more than one-dimensional" topological spaces from the standpoint of recogni- zability. And the problem remains: is the segment *[0;1]* as a topological space recogni- zable? It seems like the Continuum problem.

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MSC 34 E05, 34 E10, 34 E15

ASYMPTOTIC BEHAVIOR OF THE SOLUTIONS OF BOUNDARY-VALUE PROBLEM FOR A SYSTEM OF LINEAR DIFFERENTIAL EQUATIONS WITH A SMALL PARAMETER

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The article considers a two-point boundary value problem for a system of linear differential equations with a small parameter at two higher derivatives under the condition that the roots of the "additional characteristic equation" are negative. In this work, using the boundary functions and the Cauchy function, an analytical formula for the solutions of the boundary value problem is obtained. A theorem on the asymptotic estimate of the solution of the considered boundary value problem is proved. The asymptotics of the solution with respect to a small parameter and the order of growth of its derivatives are established. It is shown that the solution of the considered boundary value problem at the left end of the segment has the initial jump phenomenon of zero order of the second degree.

Keywords: singularly perturbation, differential equations, small parameter, initial jump, asymptotics

“Кошумча мүнөздөөчү теңдеменин” тамырлары терс болгон шартта, эки жогорку туундудагы кичине параметрлүү сызыктуу дифференциалдык теңдемелер системасы үчүн эки чекиттүү чектик маселеси каралат. Бул макалада чектик жана Коши функцияларын колдонуп, чектик маселесин чыгаруунун аналитикалык формуласы алынган. Каралган чектик маселесин чечүүнүн асимптотикалык баалоосу жөнүндө теорема далилденген. Чыгарылыштын асимптотикалык жүрүшү жана анын туундуларынын өсүү тартиби кичине параметр нөлгө умтулганда мүнөздөлгөн. Ушул сегменттин сол четиндеги каралып жаткан чектик маселесин чечүүдө экинчи даражадагы нөлүнчү тартиптеги баштапкы секирүү кубулушу бар экени көрсөтүлгөн.

Урунттуу сөздөр: сингулярдык толкундануу, дифференциалдык теңдемелер, кичинекей параметр, алгачкы секирүү, асимптотика.

Рассматривается двухточечная краевая задача для системы линейных дифференциальных уравнений с малым параметром при двух старших производных при условии, что корни «дополнительного характеристического уравнения» отрицательны. В данной статье с использованием граничных функций и функций Коши получена аналитическая формула решения краевой задачи. Доказана теорема об асимптотической оценке решения рассматриваемой краевой задачи. Установлено асимптотическое поведение решения и порядок роста его производных при стремлении малого параметра к нулю. Показано, что решение рассматриваемой краевой задачи на левом конце данного отрезка обладает явлением начального скачка нулевого порядка второй степени.

Ключевые слова: сингулярное возмущение, дифференциальные уравнения, малый параметр, начальный скачок, асимптотика.

Consider a system of singularly perturbed linear differential equations of the form

$$\begin{cases} \varepsilon^2 z''' + \varepsilon A_1(t) z'' + B_1(t) z' + C_1 z + D(t) y = F_1(t) \\ \quad y' + A_2(t) z' + B_2(t) z + C_2(t) y = F_2(t) \end{cases} \tag{1}$$

with the following boundary conditions:

$$h_1 z(t,\varepsilon) = z(0,\varepsilon) = \alpha_0, h_2 z(t,\varepsilon) = z'(0,\varepsilon) = \alpha_1, \tag{2}$$

$$h_3 z(t,\varepsilon) = z(1,\varepsilon) = \beta, h_4 y(t,\varepsilon) = y(0,\varepsilon) = \gamma,$$

where $\varepsilon > 0$ - small parameter, and $\alpha_0, \alpha_1, \beta, \gamma$ - known constants.

Assume that the following conditions are satisfied:

I. The functions $A_i(t), B_i(t), C_i(t), D(t), F_i(t), i = 1,2$ on the segment $0 \le t \le 1$ are sufficiently smooth;

II. $A_2(t) \neq 0, \quad 0 \le t \le 1$;

III. The roots of the equation $\mu^2(t) + A_1(t)\mu(t) + B_1(t) = 0$ satisfy the inequalities $\mu_1(t) \neq \mu_2(t), \mu_1(t) < 0, \mu_2(t) < 0$.

From the second equation of the system (1) using the condition $y(0,\varepsilon) = \gamma$ we take the formula

$$y(t,\varepsilon) = \gamma e^{-\int_0^t C_2(x)dx} + \int_0^t (F_2(s) - A_2(s) z'(s,\varepsilon) - B_2(s) z(s,\varepsilon)) e^{-\int_s^t C_2(x)dx} \tag{3}$$

Substituting (3) into the first equation of the system (1), we get the Volterra integrodifferential equation [2]:

$$L_\varepsilon z(t,\varepsilon) \equiv \varepsilon^2 z''' + \varepsilon A_1(t) z'' + B_1(t) z' + C_1(t) z = \tag{4}$$

$$= F(t) + \int_0^t [h_0(t,s) z(s,\varepsilon) + h_1(t,s) z'(s,\varepsilon)] ds$$

where

$$F(t) \equiv F_1(t) - D(t)\gamma e^{-\int_0^t C_1(x)dx} - D(t)\int_0^t F_2(s)e^{-\int_s^t C_1(x)dx}\,ds$$

$$h_1(t,s) = D(t)A_2(s)e^{-\int_s^t C_2(x)dx},\ h_0(t,s) = D(t)B_2(s)e^{-\int_s^t C_2(x)dx}$$

We seek the solution of the problem (4) in the form [1]:

$$z(t,\varepsilon) = \sum_{i=1}^{3} C_i\Phi_i(t,\varepsilon) + \frac{1}{\varepsilon^2}\int_0^t K_3(t,s,\varepsilon)u(s,\varepsilon)ds\ , \qquad (5)$$

where $K_3(t,s,\varepsilon)$ - Cauchy function, and $\Phi_i(t,\varepsilon), i = \overline{1,3}$ - boundary functions [1], $C_i, i = \overline{1,3}$- unknown constants, $u(t,\varepsilon)$- unknown function.

For the function $u(t,\varepsilon)$ we obtain the Volterra integral equation of the second kind:

$$u(t,\varepsilon) = F(t,\varepsilon) + \int_0^t H(t,p,\varepsilon)u(p,\varepsilon)dp, \qquad (6)$$

where

$$F(t,\varepsilon) = F(t) + \sum_{i=1}^{3} C_i\int_0^t [h_1(t,s)\Phi_i'(s,\varepsilon) + h_0(t,s)\Phi_i(s,\varepsilon)]ds$$

$$H(t,p,\varepsilon) = \int_p^t \frac{1}{\varepsilon^2}[h_1(t,s)K_3'(s,p,\varepsilon) + h_0(t,s)K_3(s,p,\varepsilon)]ds.$$

We solve equation (6) using a resolvent of the kernel $H(t,p,\varepsilon)$ [3]. Then, for the solution of the problem (1), (2) from (5), (3) we obtain the following formula:

$$z(t,\varepsilon) = C_1Q_1(t,\varepsilon) + C_2Q_2(t,\varepsilon) + C_3Q_3(t,\varepsilon) + P(t,\varepsilon), \qquad (7)$$

$$y(t,\varepsilon) = \gamma e^{-\int_0^t C_2(x)dx} + \int_0^t [F_2(s) - C_1\bar{Q}_1(s,\varepsilon) - C_2\bar{Q}_2(s,\varepsilon) -$$

$$-C_3\bar{Q}_3(s,\varepsilon) - \bar{P}(s,\varepsilon)]e^{-\int_s^t C_2(x)dx}$$

where

$$Q_i(t,\varepsilon) = \Phi_i(t,\varepsilon) + \frac{1}{\varepsilon^2}\int_0^t K_3(t,s,\varepsilon)\int_0^s \bar{\Phi}_i\,(s,\varepsilon)ds,$$

$$P(t,\varepsilon)=\frac{1}{\varepsilon^2}\int_0^t K_3(t,s,\varepsilon)\Big(F(s)+\int_0^s R_\varepsilon(s,p,1)F(p)dp\Big)ds,$$

$$\bar{\Phi}_i(t,\varepsilon)=\int_0^s\big[\bar{h}_0(s,p,\varepsilon)\Phi_i(p,\varepsilon)+\bar{h}_1(s,p,\varepsilon)\Phi_i'(p,\varepsilon)\big]dp,$$

$$\bar{h}_i(t,s,\varepsilon)=h_i(t,s)+\int_s^t R_\varepsilon(s,p,1)h_i(p,s)dp=\bar{h}_i(t,s)+O(\varepsilon), i=0,1,$$

$$\bar{Q}_i(s,\varepsilon)=A_2(s)Q_i'(s,\varepsilon)+B_2(s)Q_i(s,\varepsilon),\ \ i=1,2,3,$$

$$\bar{P}(t,\varepsilon)=A_2(s)P'(s,\varepsilon)+B_2(s)P(s,\varepsilon)$$

To determine the unknown constants $C_i, i=1,2,3$ taking into account (2) and condition

IV. $$\sigma\equiv 1+\int_0^1\frac{\bar{h}_1(s,0)+\int_0^S(\bar{h}_1(s,0)z_{30}'(p)+\bar{h}_0(s,p)z_{30}(p))dp}{z_{30}(s)\mu_1(s)\mu_2(s)}ds$$

we obtain the following asymptotic representations as $\varepsilon\to 0$:

$$C_1=\alpha_0,\ \ C_2=\alpha_1,\ \ C_3=\omega+O(\varepsilon), \tag{8}$$

where

$$\omega=\frac{1}{\sigma}\left(\beta+\int_0^1\frac{z_{30}(1)\big(\alpha_0\bar{h}_1(s,0)-F(s)-\int_0^s\bar{R}_0(s,p,1)F(p)dp\big)}{z_{30}(s)\mu_1(s)\mu_2(s)}ds\right) \tag{9}$$

Theorem. Under conditions I - IV, for sufficiently small ε the problem (1), (2) has a unique solution, which has the following asymptotic estimations as $\varepsilon\to 0$:

$$\left|z^{(i)}(t,\varepsilon)\right|\le C\left(|\alpha_0|+\varepsilon|\alpha_1|+|\beta|+|\gamma|+\max_{0\le t\le 1}|F_1(t)|+\max_{0\le t\le 1}|F_2(t)|\right)+$$
$$+\frac{C}{\varepsilon^i}\left(|\alpha_0|+\varepsilon|\alpha_1|+|\beta|+|\gamma|+\max_{0\le t\le 1}|F_1(t)|+\max_{0\le t\le 1}|F_2(t)|\right)|\mu_1^i(t)\mu_2(0)$$
$$-\mu_2^i(t)\mu_1(0)|e^{-\gamma\frac{t}{\varepsilon}},\ \ i=\overline{0,2},$$

$$|y(t,\varepsilon)|\le C\left(|\alpha_0|+\varepsilon|\alpha_1|+|\beta|+|\gamma|+\max_{0\le t\le 1}|F_1(t)|+\max_{0\le t\le 1}|F_2(t)|\right)+$$
$$+C\left(|\alpha_0|+\varepsilon|\alpha_1|+|\beta|+|\gamma|+\max_{0\le t\le 1}|F_1(t)|+\max_{0\le t\le 1}|F_2(t)|\right)e^{-\gamma\frac{t}{\varepsilon}},$$

where $C > 0,\ \gamma > 0$ are some constants independent of ε.

The proof of the theorem follows from (7) taking into account (8), (9).

It follows from theorem that the solution of the boundary-value problem (1), (2) at the point $t = o$ have the following orders of growth

$$z(0,\varepsilon) = O(1), \quad z'(0,\varepsilon) = O(1),\ \ z''(0,\varepsilon) = O\left(\frac{1}{\varepsilon^2}\right), \quad \varepsilon \to 0.$$

It means that the solution of problem (1), (2) at the left end of the segment has the initial jump phenomenon of zero order of the second degree.

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MSC 45 D05

ON EIGENVALUES AND EIGENFUNCTIONS OF VOLTERRA INTEGRAL AND INTEGRO-DIFFERENTIAL EQUATIONS WITH A SINGULARITY

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It was found that integral and integro-differential equations of Volterra with a singularity have eigenvalues and eigenfunctions.

Key words: Volterra integral equation of the second kind, Volterra integral equation of the third kind, eigenfunctions, eigenvalues, singular point.

Өзгөчөлөнгөн Вольтерра интегралдык жана интегро-дифференциалдык теңдемелери өздүк маанилерге жана өздүк функцияларына ээ экендиги аныкталды.
Урунттуу сөздөр: II түрдөгү Вольтерра интегралдык теңдемеси, III түрдөгү Вольтерра интегралдык теңдемеси, өздүк функциялар, өздүк маанилер, өзгөчө чекит.

Обнаружено, что интегральные и интегро-дифференциальные уравнения Вольтерра с особенностью имеют собственные значения и собственные функции.
Ключевые слова: интегральное уравнение Вольтерра II рода, интегральное уравнение Вольтерра III рода, собственные функции, собственные значения, особая точка.

The eigenvector of a linear operator *A* in a linear vector space *R* is such a vector $x \in R,$ such that

$$Ax = \lambda x \qquad (x \neq 0), \tag{1}$$

where λ - is some scalar, called the eigenvalue of the operator *A*, corresponding to the eigenvector *x*.

It is known that if *x* is an eigenvector of the operator *A* corresponding to the eigenvalue λ, then the same is true for any vector $x \neq 0$.

If $x_1, x_2, ..., x_k$ - are the eigenvectors of the operator *A* corresponding to the eigenvalue λ, then the same is true for each vector $\alpha_1 x_1 + \alpha_1 x_2 + ... + \alpha_1 x_k \neq 0$, these vectors generate a manifold invariant with respect to *A*.

It is known that a homogeneous Volterra integral equation of the second kind with a continuous kernel does not have eigenfunctions

$$u(t) = \lambda \int_a^t K(t,s) u(s) ds, \qquad K(t,s) \in C(a \leq s \leq t \leq b) \tag{2}$$

that is, for equation (2) a continuous solution other than zero does not exist.

In contrast to them, the Volterra integral equation of the third kind can have nontrivial (both continuous and discontinuous) solutions.

This is a new fact that does not take place in the theory of Volterra integral equations of both the second and the first kind.

For example, consider a simple equation with a singularity in the kernel

$$u(t)=\lambda\int_a^t \frac{u(s)}{s}ds. \tag{3}$$

Any value of the parameter $\lambda\in(0,+\infty)$ is an eigenvalue, and the corresponding eigenfunction will be the function t^{λ}.

It should be noted that this operator (3) has a continuous spectrum. We also note that the eigenfunctions corresponding to different eigenvalues of operator (3) are linearly independent.

Let's look at another example.

Example 1. Consider the integral Volterra equation of the third kind

$$t^2u(t)=-\lambda\int_0^t (2t-6s)u(s)ds. \tag{4}$$

Let $\lambda=-1$. Then the functions $u_1(t)=6t,\quad u_2(t)=2$ are solutions (4), that is, the eigenvalue $\lambda=-1$ has a two-dimensional geometric multiplicity. Obviously, $u(t)=6tc_1+2c_2$ will also be a solution to (4), where c_1, c_2 are arbitrary constants.

Example 2. Consider the Volterra integral equation with a singularity in the kernel

$$u(t)=\lambda\int_0^t \frac{\left(t^2-6s^2\right)}{s^3}u(s)ds\,. \tag{5}$$

Denoting $u(t)/t^3=v(t)$, we have

$$t^3v(t)=\lambda\int_0^t \left(t^2-6s^2\right)v(s)ds\,. \tag{6}$$

Let $\lambda = -1$. Then the functions $u_1(t) = t, \;\; u_2(t) = 1$ are solutions (6), that is, the eigenvalue $\lambda = -1$ has a two-dimensional geometric multiplicity.

It is clear that equation (6) has a two-parameter family of solutions $v(t) = c_1 t + c_2$. And equation (5) has solutions in the form of $u_1(t) = t^4, \;\; u_2(t) = t^3$, as well as a two-parameter family of solutions

$$u(t) = c_1 t^4 + c_2 t^3,$$

where c_1, c_2 are arbitrary constants.

Example 3. Consider the Volterra integro-differential equation with a singularity in the kernel

$$t\frac{du}{dt} + 2\int_0^t \frac{u(s)}{s}\,ds = \lambda u(t)\,. \tag{7}$$

If $\lambda = 3$, then equation (7) has two eigenfunctions: $u_1(t) = t, \;\; u_2(t) = t^2$ and thus also has a two-parameter family of solutions

$$u(t) = c_1 t + c_2 t^2,$$

where c_1, c_2 are arbitrary constants.

Example 4. Consider the Volterra integro-differential equation with the singularity

$$t\frac{du}{dt} + 2\frac{1}{t}\int_0^t u(s)ds = \lambda u(t). \tag{8}$$

If we denote $v(t) \equiv \int_0^t u(s)ds, \; v(0) = 0$, then we obtain the Cauchy problem for the ordinary differential Euler equation

$$t^2 v''(t) - \lambda t v'(t) + 2v(t) = 0, \; v(0) = 0. \tag{9}$$

We will seek a solution to (9) in the form $v(t) = t^\alpha$. We will take into account that $\mathrm{Re}\,\alpha > 0$. From this we obtain a characteristic equation of the form

$$\alpha(\alpha-1)-\lambda\alpha+2=0,\quad \alpha^2-(\lambda+1)\alpha+2=0,\quad \alpha_{1,2}\equiv\frac{(\lambda+1)\pm\sqrt{(\lambda+1)^2-8}}{2}.$$

For those λ for which $\operatorname{Re}\alpha_i>0,\ i=1,2$ the initial problem (9) always has a solution. In particular, for $\lambda=2$, equation (9) has a solution $v_1(t)=t$, $v_2(t)=t^2$. Then equation (8) also has a two-parameter solution of the form $u(t)=c_1t+c_2t^2$,, where c_1, c_2 are arbitrary constants.

For $\lambda=1$, equation (9) has a solution $v_1(t)=t^{1+i},\ v_2(t)=t^{1-i}$. Then equation (8) has a two-parameter solution of the form $u(t)=c_1(1+i)t^i+c_2(1-i)t^{-i}$. Indeed, we take one of these components of the linear combination of solutions $u(t)=(1+i)t^i$. Substitute into equation (8):

$$t(1+i)it^{i-1}+\frac{2}{t}t^{1+i}=1\cdot(1+i)t^i,$$
$$(1+i)i+2=(1+i),\ \ i-1+2=1+i.$$

Which is what was required to be proved.

The second component of the linear combination of solutions $u(t)=(1-i)t^{-i}$ is checked similarly:

$$t(1-i)(-i)t^{-i-1}+\frac{2}{t}t^{1-i}=1\cdot(1-i)t^{-i},$$
$$-(1-i)i+2=(1-i),\ -i-1+2=1-i.$$

Which is what was required to be proved.

The set of all eigenvalues of Volterra integral and integro-differential equations with a singularity will be called its spectrum.

It turns out that the Volterra integral equation with singularity (8) also has a continuous spectrum.

Note that integro-differential equations with constant limits of integration with a fixed singular point also have a number of features.

Example 5. An integro-differential equation

$$t\left(\frac{du}{dt}+8t\int_0^1 su(s)ds\right)=\lambda u \quad (10)$$

is given.

If $\lambda = 4$, then equation (10) has a corresponding eigenfunction $u(t)=t^2$ and thus also has a one-parameter family of solutions

$$u(t)=c\,t^2,$$

where c is an arbitrary constant.

Example 6. Let's solve the integro-differential equation

$$t\left(\frac{du}{dt}+at\int_0^1 su(s)ds\right)=\lambda u. \quad (11)$$

If $a=-8, \lambda=-2$, then equation (11) has the corresponding eigenfunction $u(t)=t$ and thus also has a one-parameter family of solutions

$$u(t)=ct,$$

where c is an arbitrary constant. In addition, for $a\neq -8, \lambda=-2$, equation (11) has a solution with a singularity at $t=0$ in the form

$$u(t)=c\left(t^{-2}-\frac{4a}{8+a}t\right), \quad (12)$$

where c is an arbitrary constant.

Thus, in the case when there is an integro-differential equation with constant limits of integration in some cases, for example, equation (11) has a continuous solution $u(t)=ct,$ and in another case it has a solution of the form (12).

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MSC 34 K20, 45 J05

ON THE BOUNDEDNESS OF SOLUTIONS AND THEIR FIRST DERIVATIVES OF A SINGLE NONLINEAR VOLTERRA INTEGRO-DIFFERENTIAL SECOND-ORDER EQUATION ON THE SEMI-AXIS

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Sufficient conditions are established for boundedness on the semiaxis of all solutions and their first derivatives one class of nonlinear integro-differential equations of the second order of Volterra type. For this, the method of partial cutting is being developed.
Keywords: nonlinear integro-differential equation of the second order, boundedness of solutions, boundedness of the first derivatives of solutions, method of partial cutting.

Вольтерра тибиндеги экинчи тартиптеги бир класстагы сызыктуу эмес интегро-дифференциалдык теңдеменин чыгарылыштарынын жана алардын биринчи туундуларынын жарым окто чектелгендигинин жетиштүү шарттары табылат. Бул үчүн жекече кесүү методу өнүктүрүлөт.
Урунттуу сөздөр: сызыктуу эмес экинчи тартиптеги интегро-дифференциалдык теңдеме, чыгарылыштардын чектелгендиги, чыгарылыштардын биринчи туундуларынын чектелгендиги, жекече кесүү методу.

Устанавливаются достаточные условия ограниченности на полуоси решений и их первых производных одного класса нелинейных интегро-дифференциальных уравнений второго порядка типа Вольтерра. Для этого развивается метод частичного срезывания.

Ключевые слова: нелинейное интегро-дифференциальное уравнение, второго порядка, ограниченность решений, ограниченность первых производных решений, метод частичного срезывания.

All featured functions from $t, (t,\tau), (t,x,y)$ are continuous and the relations take place at $t \ge t_0, t \ge \tau \ge t_0;\ |x|, |y| < \infty; J = [t_0, \infty);$ IDE - integro-differential equation.

Problem. Establish sufficient conditions for boundedness on the half-interval J of all solutions and their first derivatives of a second-order nonlinear IDE of Volterra type of the form

$$x''(t) + h(t, x(t), x'(t)) + a(t)g(x(t)) + \int_{t_0}^{t} K(t,\tau)x'(\tau)d\tau = f(t),\ t \ge t_0. \qquad (1)$$

By the solution of IDE (1) we mean the solution $x(t) \in C^2(J, R)$ with any initial data $x^{(k)}(t_0)$ $(k = 0,1)$. The existence of such a solution is assumed, although it can be established (under additional conditions) by the method of monotone operators [1].

Note that a similar problem was studied earlier in [2] by the method of weightin and cutoff functions [3]. The results of this paper are a supplement to the paper [2], since we consider another class of IDE of the form (1) as a development of the partial cutting method [4].

Let's get down to getting the main result.

Let [4]:

$$K(t,\tau) = \sum_{i=1}^{n} K_i(t,\tau), \qquad (K)$$

$$f(t) = \sum_{i=1}^{n} f_i(t), \qquad (f)$$

$\psi_i(t)$ $(i = 1, ..., n)$ – some cutting functions,

$P_i(t,\tau)\equiv K_i(t,t)\left(\psi_i(t)\right)^{-2}$, $T_i(t,\tau)\equiv K_i(t,\tau)\left(\psi_i(\tau)\right)^{-1}$ – partially cut kernels $(i=1,...,n)$; $E_i(t)\equiv f_i(t)(\psi_i(t))^{-1}$,

$$P_i(t)=A_i(t)+B_i(t)\ (i=1,...,n), \tag{P}$$

$c_i(t)$ $(i=1,...,n)$ – some functions.

For an arbitrarily fixed solution $x(t)$ IDE (1) is multiplied by $x'(t)$ [5, p. 194-217], we integrate within the range of t_0 before t, including in parts, while introducing the conditions $(K),(f),(P)$, function $\psi_i(t), P_i(t), T_i(t,\tau), c_i(t)$, we apply lemma [4]. Then we get the following identity:

$$\begin{gathered}(x'(t))^2+2\int_{t_0}^{t}x'(s)h(s,x(s),x'(s)ds+a(t)G(x(t))-\int_{t_0}^{t}a'(s)G(x(s))ds+\\ \sum_{i=1}^{n}\Big\{A_i(t)(X_i(t,t_0))^2-\int_{t_0}^{t}A_i^{'}(s)(X_i(s,t_0))^2ds+B_i(t)(X_i(t,t_0))^2-\\ -2E_i(t)X_i(t,t_0)+c_i(t)-\int_{t_0}^{t}[B_i^{'}(s)(X_i(s,t_0))^2-2E_i^{'}(s)X_i(s,t_0)+\\ +c_i^{'}(s)]ds-2\int_{t_0}^{t}\int_{t_0}^{s}T_{i\tau}^{'}(s,\tau)X_i(\tau,t_0)x'(s)d\tau ds\Big\}\equiv c_*,\end{gathered} \tag{2}$$

where

$$G(x)\equiv\int_0^x g(u)du,\quad X_i(t,\tau)\equiv\int_\tau^t\psi_i(\eta)x'(\eta)d\eta\quad (i=1,...,n),$$

$$c_*=(x'(t_0))^2+a(t_0)G(x(t_0))+\sum_{i=1}^{n}c_i(t_0).$$

By passing from identity (2) to an integral inequality and applying Lemma 1 [6], we prove the following

Theorem. Let 1) the conditions $(K),(f),(R)$; 2) $x'h(t,x,x')\geq 0$; 3) $G(t)\to\infty$ at $|x|\to\infty$; 4) $a(t)\geq a_0>0$, there is a function $a^*(t)\in L^1(J,R_+)$

such that $a'(t) \le a^*(t)a(t)$; 5) $A_i(t) > 0, B_i(t) \ge 0, B_i'(t) \le 0$, exist a functions $A_i^*(t) \in L^1(J, R_+)$, $c_i(t)$ such that $A_i'(t) \le A_i^*(t)A_i(t)$, $(E_i^{(k)}(t))^2 \le B_i^{(k)}(t)c_i^{(k)}(t)$ $(i = 1...n; k = 0,1)$;

6) $\int_{t_0}^{t} \left|T_{i\tau}'(t,\tau)\right| (A_i(\tau))^{-\frac{1}{2}} d\tau \in L^1(J, R_+)$.

Then any solution $x(t) \in C^2(J, R)$ and its first derivative $x'(t)$ bounded on J.

In this case, we obtain the integral inequality

$$u(t) \equiv (x'(t))^2 + 2\int_{t_0}^{t} x'(s)h(s, x(s), x'(s)ds + a(t)G(x(t)) +$$

$$+\sum_{i=1}^{n} A_i(t)(X_i(t,t_0))^2 \le c_* + \int_{t_0}^{t} a^*(s)G(x(s))ds + \int_{t_0}^{t}\sum_{i=1}^{n}\Big[A_i^*(s)u(s) +$$

$$\left. +(u(s))^{\frac{1}{2}} \int_{t_0}^{s} \left|T_{i\tau}'(s,\tau)\right| (A_i(\tau))^{\frac{1}{2}} (u(\tau))^{\frac{1}{2}} d\tau \right] ds.$$

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ON THE STABILIZATION OF SOLUTIONS OF A LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF THE THIRD ORDER ON THE SEMI-AXIS

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Sufficient conditions are established for all solutions to tend to a finite limit and the first derivatives of all solutions tend to zero, i.e. stabilization of solutions with unbounded growth of the argument of a third-order linear integro-differential equation of Volterra type. For this, a non-standard method of reducing to a system is being developed with the introduction of some three weighting functions. An illustrative example is given.

Keywords: linear integro-differential equation of the third order, stabilization of solutions, non-standard method of reduction to the system, weighting functions.

Вольтерра тибиндеги үчүнчү тартиптеги сызыктуу интегро-дифференциалдык теңдеменин бардык чыгарылыштарынын аргумент чексиз өскөндө горизонталдык асимптотага ээ болушунун жана алардын биринчи туундуларынын нөлгө умтулушунун, б.а. стабилизацияланышынын жетиштүү шарттары табылат. Бул үчүн кандайдыр үч салмактык функцияларды камтыган стандарттык эмес ордуна коюу аркылуу берилген теңдемени системага келтирүү методу өнүктүрүлөт. Иллюстративдик мисал тургузулат.

Урунттуу сөздөр: Үчүнчү тартиптеги сызыктуу интегро-дифференциалдык теңдеме, чыгарылыштардын стабилизацияланышы, системага стандарттык эмес келтирүү методу, салмактык функциялар.

Устанавливаются достаточные условия стремления к конечному пределу всех решений и стремления к нулю первых производных всех решений, т.е. стабилизации решений при неограниченном росте аргумента линейного интегро-дифференциального уравнения третьего порядка типа Вольтерра. Для этого развивается нестандартный метод сведения к системе с введением трех некоторых весовых функций. Приводится иллюстративный пример.
Ключевые слова: линейное интегро-дифференциальное уравнение третьего порядка, стабилизация решений, нестандартный метод сведения к системе, весовые функции.

All appearing functions and their derivatives and relations and take place at $t \geq t_0,\ t \geq \tau \geq t_0;\ [t_0, \infty);$ IDE-integro-differential equation.

Definition. The stabilization of solutions to a third-order linear Volterra IDE is understood as the tendency to the horizontal asymptote of any of its solutions and the tendency to zero of the first derivatives of these solutions as $t \to \infty$.

Note that our definition of stabilization of solutions is practically foreseeable and is given in accordance with the studies of V.V. Rumyantsev [1] on the part of the variables of solutions of the system of differential equations.

In our work, we solve the following

Problem. Establish sufficient conditions for the stabilization of any solution to a third-order linear IDE of Volterra type of the form:

$$x'''(t) + a_2(t)x''(t) + a_1(t)x'(t) + a_0(t)x(t) + \int_{t_0}^{t} [Q_0(t,\tau)x(\tau) +$$

$$+ Q_1(t,\tau)x'(\tau) + Q_2(t,\tau)x''(\tau)]\, d\tau = f(t), \quad t \geq t_0. \qquad (1)$$

We are talking about solutions $x(t) \in C^3(J, R)$ to IDE (1) with any initial data $x^{(k)}(t_0)$ $(k = 0,1,2)$. Each such solution exists and is unique.

To solve this problem, a non-standard method of reduction to the system from [2, 3] is developed, the method of transformation of the equations of V. Volterra [4, pp. 194 - 217.], The method of weight and cutoff functions [5], the method of integral inequalities [6] .

We pass to the presentation of the main result.

In the IDE (1), we will make the following non-standard replacement [2,3]:

$$x''(t) + p(t)x'(t) + q(t)x(t) = W(t)y(t), \tag{2}$$

where $p(t), q(t)$, $W(t)$ - are some weighting functions, and $W(t) > 0$, $y(t)$ - is a new unknown function.

Then, similarly to [2], IDE (1) is reduced to the following equivalent system:

$$\begin{cases} x''(t) + p(t)x'(t) + q(t)x(t) = W(t)y(t), \\ y'(t) + b_2(t)y(t) + b_1(t)x'(t) + b_0(t)x(t) + \\ + \int_{t_0}^{t} [P_0(t,\tau)x(\tau) + P_1(t,\tau)x'(\tau) + K(t,\tau)y(\tau)]\, d\tau = F(t), \ \ t \geq t_0, \end{cases} \tag{3}$$

where $b_2(t) \equiv a_2(t) - p(t) + W'(t)(W(t))^{-1}$,

$b_1(t) \equiv [a_1(t) - a_2(t)p(t) + p^2(t) - p'(t) - q(t)](W(t))^{-1}$,

$b_0(t) \equiv [a_0(t) - a_2(t)q(t) + p(t)q(t) - q'(t)](W(t))^{-1}$,

$P_0(t,\tau) \equiv \left(W(t)\right)^{-1}[Q_0(t,\tau) - Q_2(t,\tau)q(\tau)]$,

$P_1(t,\tau) \equiv \left(W(t)\right)^{-1}[Q_1(t,\tau) - Q_2(t,\tau)p(\tau)]$,

$K(t,\tau) \equiv \left(W(t)\right)^{-1} Q_2(t,\tau)W(\tau)$, $\qquad F(t) \equiv \left(W(t)\right)^{-1} f(t)$.

We apply the method of weighting functions [6] and the method of weighting and cutting functions [5] to system (3).

Let [5]: $0 < \varphi(t)$ - be some weighting function,

$$K(t,\tau) = \sum_{i=0}^{n} K_i(t,\tau), \tag{K}$$

$$F(t) = \sum_{i=0}^{n} F_i(t), \qquad (F)$$

$\psi_i(t)$ $(i = 1 \dots n)$ - some cutting functions,

$R_i(t,\tau) \equiv \varphi(t)K_i(t,\tau)\big(\psi_i(t)\psi_i(\tau)\big)^{-1}$, $E_i(t) \equiv \varphi(t)F_i(t)\big(\psi_i(t)\big)^{-1}$,

$$R_i(t,t_0) = A_i(t) + B_i(t) \quad (i = 1 \dots n), \qquad (R)$$

$c_i(t)$ $(i = 1 \dots n)$ - some functions

For an arbitrarily fixed solution $(x(t), y(t))$ of system (3), we multiply its first equation by $\varphi(t)x'(t)$ [6], the second equation by $y(t)$ [4, p. 194 − 217], then we add the obtained relations and integrate within the range from t_0 to t, including by parts, then, similarly to [5], we introduce conditions (*K*), (*F*), functions $\psi_i(t), R_i(t,\tau), E_i(t)$, condition (*R*), functions $c_i(t)$ $(i = 1 \dots n)$, we apply Lemmas 1.4, 1.5 [7]. As a result, we get the following identity:

$$\varphi(t)\big(x'(t)\big)^2 + \int_{t_0}^{t} \Delta(s)\big(x'(s)\big)^2 ds + \varphi(t)q(t)\big(x(t)\big)^2 + \big(y(t)\big)^2 +$$

$$+2\int_{t_0}^{t} b_2(s)\big(y(s)\big)^2 ds + \sum_{i=1}^{n}\Big\{A_i(t)\big(Y_i(t,t_0)\big)^2 + B_i(t)\big(Y_i(t,t_0)\big)^2 -$$

$$-2E_i(t)Y_i(t,t_0) + c_i(t) - \int_{t_0}^{t} [B_i'(s)\big(Y_i(s,t_0)\big)^2 - 2E_i'(s)Y_i(s,t_0) +$$

$$+c_i'(s)]ds + \int_{t_0}^{t} R_{i\tau}'(t,\tau)\big(Y_i(t,\tau)\big)^2 d\tau\} \equiv$$

$$\equiv c_* + 2\int_{t_0}^{t} \Big\{\frac{1}{2}(\varphi(s)q(s))'\big(x(s)\big)^2 +$$

$$+\varphi(s)W(s)y(s)x'(s)+\frac{1}{2}\sum_{i=1}^{n}[\,A_i'(s)\big(Y_i(s,t_0)\big)^2+$$

$$+\int_{t_0}^{s}R_{is\tau}''(s,\tau)\,\big(Y_i(s,\tau)\big)^2d\tau]ds\}+$$

$$+2\int_{t_0}^{t}y(s)\{F_0(s)-b_1(s)x'(s)-b_0(s)x(s)-$$

$$-\int_{t_0}^{s}P_0(s,\tau)\,x(\tau)+P_1(s,\tau)x'(\tau)+K_0(s,\tau)y(\tau)]d\tau\}ds, \tag{4}$$

where $\Delta(t)\equiv 2\varphi(t)p(t)-\varphi'(t)$,

$$Y_i(t,\tau)\equiv\int_{\tau}^{t}\psi_i(\eta)y(\eta)\,d\eta\quad(i=1\dots n),\qquad c_*=\varphi(t_0)\big(x'(t_0)\big)^2+$$

$$+\varphi(t_0)q(t_0)\big(x(t_0)\big)^2+\big(y(t_0)\big)^2+\sum_{i=1}^{n}c_i(t_0).$$

Theorem. Let conditions 1) $\varphi(t)>0$, $W(t)>0$, (*K*), (*F*), (*R*) are satisfied; 2) $\Delta(t)\geq 0$; 3) $\varphi(t)q(t)\geq q_0>0$, there is a function $0\leq q^*(t)\in L^1(J,R_+)$ such that $(\varphi(t)q(t))'\leq q^*(t)\varphi(t)q(t)$;
4) $b_2(t)\geq 0$; 5) $A_i(t)\geq 0, B_i(t)\geq 0, B_i'(t)\leq 0, R_{i\tau}'(t,\tau)\geq 0$, there is a function $A_i^*(t)\epsilon L^1(J,R_+)$, $c_i(t)$, $R_i^*(t)\epsilon L^1(J,R_+)$ such that $A_i'(t)\leq A_i^*(t)A_i(t), \left(E_i^{(k)}(t)\right)^2\leq B_i^{(k)}(t)c_i^{(k)}(t), R_{i\tau}''(t,\tau)\leq R_i^*(t)R_{i\tau}'(t,\tau)$ $(i=1\ \dots n;\ k=0{,}1)$;

6) $\big(\varphi(t)\big)^{\frac{1}{2}}W(t)+|F_0(t)|+|b_1(t)|\big(\varphi(t)\big)^{-\frac{1}{2}}+|b_0(t)|+$

$$+\int_{t_0}^{t}[\,|P_0(t,\tau)|+|P_1(t,\tau)|\big(\varphi(\tau)\big)^{-\frac{1}{2}}+|K_0(t,\tau)|\,]d\tau\in L^1(J,R_+\backslash\{0\}).$$

Then, for any solution $(x(t), y(t))$ of system (3), the following statements are true:

$$|x'(t)| \le \sqrt{M(c_*)}\big(\varphi(t)\big)^{-\frac{1}{2}}, \tag{5}$$

$$\int_{t_0}^{t} \Delta(s)(x'(s))^2 ds \le M(c_*), \tag{6}$$

$$|x(t)| \le q_0^{-\frac{1}{2}}\sqrt{M(c_*)}, \tag{7}$$

$$|y(t)| \le \sqrt{M(c_*)}, \tag{8}$$

$$\int_{t_0}^{t} b_2(s)(y(s))^2 ds \le M(c_*), \tag{9}$$

$$A_i(t)\big(Y_i(t,t_0)\big)^2 \le M(c_*), \tag{10}$$

Where $M(c_*) = [\sqrt{c_*} + \int_{t_0}^{\infty} e^{-\int_{t_0}^{s} V(s)ds} F_0(s)ds]^2 \exp(2\int_{t_0}^{\infty} \{\frac{1}{2} q^*(s) +$

$+\big(\varphi(s)\big)^{\frac{1}{2}} W(s) + \frac{1}{2}\sum_{i=1}^{n}[A_i^*(s) + R_i^*(s)] + |b_1(s)|\big(\varphi(s)\big)^{-\frac{1}{2}} +$

$+|b_0(s)|\, q_0^{-\frac{1}{2}} + \int_{t_0}^{s}[|P_0(s,\tau)|\, q_0^{-\frac{1}{2}} + |P_1(s,\tau)|\big(\varphi(\tau)\big)^{-\frac{1}{2}} +$

$+|K_0(s,\tau)|]d\tau\}ds) < \infty,$

$$V(t) \equiv \frac{1}{2} q^*(t) + \big(\varphi(t)\big)^{\frac{1}{2}} W(t) +$$

$$+\frac{1}{2}\sum_{i=1}^{n}[A_i^*(t) + R_i^*(t)] + |b_1(t)|\big(\varphi(t)\big)^{-\frac{1}{2}} +$$

$$+|b_0(t)| q_0^{-\frac{1}{2}} + \int_{t_0}^{t} [|P_0(t,\tau)|\, q_0^{-\frac{1}{2}} + |P_1(t,\tau)|\big(\varphi(\tau)\big)^{-\frac{1}{2}} + |K_0(t,\tau)|]d\tau.$$

By virtue of conditions 1) – 5) of the theorem, we pass from identity (4) to the following integral inequality:

$$0 \le u(t) \le \varphi(t)\big(x'(t)\big)^2 + \int_{t_0}^{t} \Delta(s)\big(x'(s)\big)^2 ds +$$

$$+q_0\big(x(t)\big)^2+\big(y(t)\big)^2+2\int_{t_0}^{t} b_2(s)\big(y(s)\big)^2 ds+\sum_{i=1}^{n}[A_i(t)\big(Y_i(t,t_0)\big)^2+$$

$$+\int_{t_0}^{t} R'_{i\tau}(t,\tau)\,\big(Y_i(t,\tau)\big)^2\,d\tau]\le c_*+2\int_{t_0}^{t}\Big\{\frac{1}{2}q^*(s)+\big(\varphi(s)\big)^{\frac{1}{2}}W(s)+$$

$$+\frac{1}{2}\sum_{i=1}^{n}[A_i^*(s)+R_i^*(s)]\}u(s)ds+2\int_{t_0}^{t}\big(u(s)\big)^{\frac{1}{2}}\{\,|F_0(s)|+$$

$$+|b_1(s)|\big(\varphi(s)\big)^{-\frac{1}{2}}\big(u(s)\big)^{\frac{1}{2}}+|b_0(s)|q_0^{-\frac{1}{2}}\big(u(s)\big)^{\frac{1}{2}}+\int_{t_0}^{s}[|P_0(s,\tau)|\,q_0^{-\frac{1}{2}}+$$

$$+|P_1(s,\tau)|\big(\varphi(\tau)\big)^{-\frac{1}{2}}+|K_0(s,\tau)|]\big(u(\tau)\big)^{\frac{1}{2}}d\tau\}ds. \qquad (11)$$

We apply Lemma 1[6] to integral inequality (11) and, based on conditions 3), 5), 6), theorem, we have

$$u(t)\le M(c_*). \qquad (12)$$

Inequality (12) implies statements (5) – (10) of the theorem.

Corollary 1. If all conditions of the theorem are satisfied and

1) $(\varphi(t))^{-\frac{1}{2}}\in L^1(J,R_+\backslash\{0\})$, 2) $\lim_{t\to\infty}\varphi(t)=\infty$, then any solution x (t) to IDE (1) is stabilizable.

This follows from assertion 5) of the theorem, namely, based on condition 1) of Corollary 1, we have $x'(t)\in L^1(J,R)$, whence it follows that there is a finite limit: $|\lim_{t\to\infty}x(t)|<\infty,$ and condition 2) of this corollary gives: $\lim_{t\to\infty}x'(t)=0$.

Corollary 2. If all conditions of the theorem are satisfied and $\Delta(t)>0$, $(\Delta(t))^{-1}\in L^1(J,R_+\backslash\{0\})$, then any solution x (t) to IDE (1) tends to a finite limit as t → ∞.

The statement of this corollary is obtained from:

$$|x'(t)| = (\Delta(t))^{-\frac{1}{2}} (\Delta(t))^{\frac{1}{2}} |x'(t)| \leq \frac{1}{2}[(\Delta(t))^{-1} + \Delta(t)(x'(t))^2]$$

integration within the limits from t_0 to $t,$ taking into account statement (6) of the theorem, similarly to [8].

Corollary 3. If all conditions of corollary 2 are satisfied and condition 2) of corollary 1 is satisfied, then any solution $x(t)$ to IDE (1) is stabilizable.

Example. For IDE (1) with $a_2(t) \equiv 3e^t$, $a_1(t) \equiv 2e^{2t} + e^t - \frac{1}{t+1} +$ $+sine^{-2t}$, $a_0(t) \equiv \frac{2e^t}{t+2} - \frac{1}{(t+2)^2} - 10e^{-3t}$, $Q_0(t,\tau) \equiv \frac{Q_2(t,\tau)}{\tau+2} - 15e^{-2t-\tau}$, $Q_1(t,\tau) \equiv Q_2(t,\tau)e^\tau + 23e^{-5t}\sin(t\tau+1)$, $Q_2(t,\tau) \equiv e^{-t+\tau}\left[\frac{1}{t-\tau+2} + \right.$ $\left.+\exp\left(\frac{t+\tau+3}{t+\tau+4}\right)\right] e^{4t+4\tau}(sint\ sin\tau)^{\frac{1}{7}} - \frac{e^{-t+\tau}}{(t+\tau+1)^5}$, $f(t) \equiv \frac{e^{3t}(sint)^{\frac{1}{7}}}{t+5} - \frac{31e^{-t}}{t^2+8}$ all conditions of the theorem, conditions 2) of corollary 1, all conditions of corollary 2 are satisfied for $p(t) \equiv e^t$, $q(t) \equiv \frac{1}{t+2}$, $W(t) \equiv e^{-t}$, $\varphi(t) \equiv t+1$, here $t_0 = 0$, $b_2(t) \equiv 2e^t - 1$, $b_1(t) \equiv e^t sine^{-2t}$, $b_0(t) \equiv -10e^{-2t}$,

$P_0(t,\tau) \equiv -15e^{-t-\tau}$, $P_1(t,\tau) \equiv 23e^{-4t}\sin(t\tau+1)$, $K(t,\tau) \equiv [\frac{1}{t-\tau+2} +$ $\exp(\frac{t+\tau+3}{t+\tau+4})]e^{4t+4\tau}(sint\ sin\tau)^{\frac{1}{7}} - \frac{1}{(t+\tau+1)^5}$, $F(t) \equiv \frac{e^{4t}(sint\ )^{\frac{1}{7}}}{t+5} - \frac{31}{t^2+8}$,

$K_0(t,\tau) \equiv -\frac{1}{(t+\tau+1)^5}$, $F_0(t) \equiv -\frac{31}{t^2+8}$, $n = 1$, $\psi_1(t) \equiv e^{4t}(sint)^{\frac{1}{7}}$,

$R_1(t,\tau) \equiv \frac{1}{t-\tau+2} + \exp(\frac{t+\tau+3}{t+\tau+4})$, $A_1(t) \equiv \exp\left(\frac{t+3}{t+4}\right)$,

$A_1^*(t) \equiv \frac{1}{(t+4)^2}$, $R_1^*(t) \equiv \frac{1}{(t+4)^2}$, $B_1(t) \equiv \frac{1}{t+2}$,

$$F_1(t) \equiv \frac{e^{4t}(sint)^{\frac{1}{7}}}{t+5}, \quad E_1(t) \equiv \frac{1}{t+5}\, , c_1(t) \equiv \frac{1}{t+2}\, .$$

Therefore, any solution $x(t)$ of such an IDE (1) is stabilizable.

Thus, we managed to find a class of third-order IDEs of the form (1), for which the problem posed above can be solved.

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MSC 34 K20, 45 J05

ESTIMATES OF SOLUTIONS AND THEIR FIRST DERIVATIVES OF A WEAKLY NONLINEAR SECOND-ORDER VOLTERRA INTEGRO-DIFFERENTIAL EQUATION WITH DELAYS

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Sufficient conditions are established for estimating the boundedness, power-law absolute integrability on the semi-axis, and the tendency to zero for unlimited growth of the argument of a weakly nonlinear second-order integro-differential equation of the Volterra type with delays. An illustrative example is provided.
Keywords: second-order integro-differential equation, argument delays, asymptotic properties of solutions, asymptotic properties of first derivative solutions.

Экинчи тартиптеги Вольтерра тибиндеги сызыктуу сымал кечигүүчү аргументтери бар интегро-дифференциальдык теңдеменин бардык чыгарылыштарынын жана алардын биринчи туундуларынын жарым октогу асимптотикалык касиеттерин (баалоолорун, чектелгендигин, даражалуу абсолюттук интегралданышын, аргумент чексиз өскөндө нөлгө умтулуусун) камсыз кылуучу жетиштүү шарттар табылат. Иллюстративдик мисал тургузулат.
Урунтттуу сөздөр: экинчи тартиптеги интегро-дифференциалдык теңдеме, кечигүүчү аргументтер, чыгарылыштардын асимптотикалык касиеттери, чыгарылыштардын биринчи туундуларынын асимптотикалык касиеттери.

Устанавливаются достаточные условия для оценки, ограниченности, степенной абсолютной интегрируемости на полуоси, стремления к нулю при неограниченном росте аргумента слабо нелинейного интегро-дифференциального уравнения второго порядка типа Вольтерра с запаздываниями. Приводится иллюстративный пример.

Ключевые слова: интегро-дифференциальное уравнение второго порядка, запаздывания аргументов, асимптотические свойства решений, асимптотические свойства первых производных решений.

All functions that appear in the work are continuous when $t \geq t_0, t \geq \tau \geq t_0, |x|,|y|,|z| < \infty; J = [t_0, \infty)$; IDE - integro-differential equation.

Problem. Establish sufficient conditions for estimating power-law absolute integrability on a half-interval *J*, tending to zero when $t \to \infty$ all solutions and their first derivatives of a weakly nonlinear second order IDE of the Volterra type with a delay of the form:

$$x''(t) + a_1(t)x'(t) + a_0(t)x(t) =$$

$$= F(t, x(\gamma_1(t)), x'(\gamma_2(t)), \int_{t_0}^{t} H(t,\tau, x(\gamma_3(\tau)), x'(\gamma_4(\tau))d\tau), \quad t \geq t_0, \qquad (1)$$

where are the functions $F(t,x,y,z)$, $H(t,\tau,x,y)$ satisfies the conditions of weak nonlinearity

$$\begin{cases} |F(t,x,y,z)| \leq F_0(t) + g_0(t)|x| + g_1(t)|y| + g_2(t)|z|, \\ |H(t,\tau,x,y)| \leq g_3(t,\tau)|x| + g_4 t,\tau)|y| \end{cases} \qquad (F,H)$$

with non-negative functions $F_0(t)$, $g_k(t)$ $(k=0,1,2)$, $g_3(t,\tau)$, $g_4(t,\tau)$;

delays $\gamma_k(t)$ $(k=1,2,3,4)$ meet the conditions:

$t_0 \leq \gamma_k(t) \leq t$ $(k=1,2,3,4)$,

the initial set E_{t_0} consists of a single point $\{t_0\}$.

To solve this problem, the following methods are used: the method of weight functions [1], the squaring method [2, p. 28], the method of integral inequalities with delays [3], and the Lyusternik – Sobolev Lemma [4, 393-394; 5].

The solution of IDE (1) is understood as the solution $x(t) \in C^2(J,R)$ with any initial data $x^{(k)}(t_0)$ (k=0,1). Due to conditions (F,H), (γ) such solutions exist.

For an arbitrarily fixed solution x (t), both parts of the IDE (1) are multiplied by a certain weight function $0 < \varphi(t)$ [1], then both parts of the resulting expression are squared [2, c.28], and integrated in the range from t_0 to t, including in parts. Then we get the following identity:

$$u(t) \equiv \int_{t_0}^{t} [(\varphi(s))^2 (x''(s))^2 + D_1(s)(x'(s))^2 + D_0(s)(x(s))^2]ds + A(t)(x(t))^2 +$$

$$+2(\varphi(t))^2 a_0(t)x(t)x'(t) + (\varphi(t))^2 a_1(t)(x'(t))^2 \equiv$$

$$\equiv u(t_0) + \int_{t_0}^{t} (\varphi(s)F(s;x))^2 ds, \qquad (2)$$

where $D_1(t) \equiv (\varphi(t)a_1(t))^2 - 2(\varphi(t))^2 a_0(t) - [(\varphi(t))^2 a_1(t)]'$,

$D_0(t) \equiv (\varphi(t)a_0(t))^2 - A'(t)$, $A(t) \equiv (\varphi(t))^2 a_0(t)a_1(t) - [(\varphi(t))^2 a_0(t)]'$,

Let

$$A(t) \equiv A_1(t) + A_2(t), \quad A_1(t) > 0, \quad A_2(t) \geq 0, \qquad (A)$$

$$B(t) \equiv (\varphi(t))^2 a_1(t) = B_1(t) + B_2(t), \quad B_1(t) > 0, \quad B_2(t) \geq 0. \qquad (B)$$

We pass from identity (2) to integral inequality and apply lemmas about integral inequality with delays [3], we prove

Theorem. Let 1) $\varphi(t) > 0$, the conditions (F,H), (γ), (A), (B);
2) $D_k(t) \geq 0$; 3) $(\varphi(t))^4 (a_0(t))^2 \leq A_2(t)B_2(t)$; 4) $(\varphi(t))^2 (F_0(t))^2 +$

$$+ (\varphi(t))^2 \{g_0(t)(A_1(\gamma_1(t)))^{-1/2} + g_1(t)(B_1(\gamma_2(t)))^{-1/2} + g_2(t)\int_{t_0}^{t} [g_3(t,\tau)((A_1(\gamma_1(\tau)))^{-1/2} +$$
$$+ g_4(t,\tau)((B_1(\gamma_1(\tau)))^{-1/2}]d\tau\}^2 \in L^1(J,R_+).$$

Then for any solution x (t) the statements are true:

$$\int_{t_0}^{t} [(\varphi(s))^2 (x''(s))^2 + D_1(s)(x'(s))^2 + D_0(s)(x(s))^2]ds = O(1), \qquad (3)$$

$$x(t) = (A_1(t))^{-1/2} O(1), \qquad (4)$$

$$x'(t) = (B_1(t))^{-1/2} O(1), \qquad (5)$$

From estimates (4), (5), we can deduce consequences about the asymptotic properties of $x^{(k)}(t)$ (k=0,1) of IDE (1), similar to consequences 3.1-3.3 [2, p.117].

From statement (3), it immediately turns out

Corollary 1. If all the conditions of the theorem are met and $\varphi(t) \geq \varphi_0 > 0$, $D_k(t) \geq D_{k0} > 0$ (k-0,1), then any solution of $x(t)$ and its first derivative $x'(t)$ tends to zero at $t \to \infty$.

In this case, we have $x^{(k)}(t) \in L^2(J,R)$ (k=0,1,2) and, based on the Lyusternik – Sobolev Lemma [4, p.393-394;5], we obtain that $x^{(k)}(t) \to 0$ (k=0,1) by $t \to \infty$.

From the theorem $g_0(t) \equiv g_1(t) \equiv g_2(t) \equiv 0$ when we have

Corollary 2. If the conditions $|F(t,x,y,z)| \leq F_0(t)$; (A), (B), conditions 2), 3) of the theorem and $(\varphi(t))^2 (F_0(t))^2 \in L^1(J, R_+)$, $\varphi(t) \geq \varphi_0 > 0$, $D_k(t) \geq D_{k0} > 0$ (k-0,1), are satisfied, then for any solution $x(t)$ the statements of corollary 1 are true.

Example. To IDE:

$$x''(t) + (t+5)x'(t) + (t+4)x(t) = \frac{1}{t+6}\sin\left(x\left(\frac{t}{2}\right) + \int_0^t \frac{|x(\tau/6)|}{t+\tau+2} d\tau - \right.$$

$$\left. - \int_0^t \frac{|x'(\tau/3)|}{t-\tau+4} d\tau \right), \ t \geq 0$$

all conditions of corollary 2 are satisfied for $\varphi(t) \equiv 1$, here $t_0 = 0$, $F_0(t) = \dfrac{1}{t+6}$.

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UNIQUENESS AND STABILITY OF SOLUTIONS FOR CERTAIN LINEAR EQUATIONS OF THE THIRD KIND WITH TWO VARIABLES

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In the present article the theorem about uniqueness and stability of the linear integral equations of the third kind two independent variables, with method of nonnegative quadratic forms and functional analysis methods.
Key words: linear inteqral equations, third kind, two variables, uniqueness, stability.

Бул макалада терс эмес квадраттык формалар усулунун, функционалдык анализдин усулдарынын жардамы менен үчүнчү түрдөгү эки өзгөрүлмөлүү сызыктуу интегралдык тендемелердин чечимдеринин жалгыздыгы жана туруктуулугу далилденди.
Урунттуу сөздөр: үчүнчү түрдөгү эки өзгөрүлмөлүү сызыктуу интегралдык тендемелер, жалгыздык, туруктуулук.

В данной работе, с помощью методом неотрицательных квадратичных форм, методам функционального анализа доказывается единственность и устойчивость решений линейных интегральных уравнений третьего рода с двумя независимыми переменными.
Ключевые слова: Линейные интегральные уравнения, третьего рода, с двумя независимыми переменными, единственность, устойчивость.

The relevance of the problem is due to the needs in development of new approaches for the regularization and uniqueness of the solution of linear integral equations of the third kind with two independent variables. Integral and operator equations of the first kind with two independent variables arise in theoretical and applied problems.Works of A.N. Tikhonov, M.M. Lavrentyev and B.K. Ivanov, in which a new concept of correctness of setting such targets is given, different from the classical, shown tool for research of ill-posed problems, which stimulated the interest to the integral equations that are of great practical importance. At the present time has been rapidly developing theory and applications of ill-posed problems. One of the classes of such ill-posed problems are integral equations of the third kind with two independent variables.As of approximate solutions of such problems, stable to small variations of the initial data, we use the solutions derived by the method of regularization. In this article we prove uniqueness theorem and obtain estimates of stability for such equations in families of sets of correctness's.For the decision of tasks of the used methods of functional analysis and method of nonnegative quadratic forms. The results of the work are new.

1. Introduction

The integral equations of the first and thrid kind were studied in [1–8]. More specifically, fundanental results for Fredholm integral equations of the first kind were obtained in [6], where regularizing operators in the sense of $M.M.Lavrentyev$ were constructed for solutions of linear Fredholm inteqral equations of the first kind. For linear Volterra integral equations of the first kind and third kinds with smooth kernels, the existence of a multiparameter family of solution was proved in [7]. The regularization and uniqueness of solutions to systems of nonlinear Volterra integral equations of the first kind were investigated in [4].In this work we shall study the problems of uniqueness and stability of solution of the integral equation

$$Ku = f(t,x), (t,x) \in G = \{(t,x) \in R^2 : t_0 \le t \le T, a \le x \le b\}, \tag{1}$$

where

$$Ku \equiv m(t,x)u(t,x) + \int_a^b K(t,x,y)u(t,y)dy + \int_{t_0}^t H(t,x,s)u(s,x)dx + \int_{t_0}^T \int_a^b C(t,x,s,y)u(s,y)dy, \tag{2}$$

$P(t,x,y)$ and $Q(t,x,s)$ are given functions, respectively on the domains

$$G_1 = \{(t,x,y) : t_0 \le t \le T,\ a \le y \le x \le b\},$$
$$G_2 = \{(t,x,s) : t_0 \le s \le t \le T,\ a \le x \le b\},$$

$C(t,x,s,y),\ m(t,x),\ f(t,x)$ are given functions is a unknown function.

2. Uniqueness and Stability of solutions of integral equations

Assume that the following conditions are satisfied:

(i). $P(t,b,a)\geq 0 \quad t\in[t_0,T], P(t,b,a)\in C[t_0,T],$

$m(t,x)\geq 0 \; for \; all \; (t,x)\in G$

$P'_y(t,y,a)\leq 0 \; for \; all\,(t,y)\in G, \; P'_y(t,y,a)\in C(G),$

$P'_z(s,b,z)\geq 0 \; for \; all \; (s,z)\in G, \; P'_z(s,b,z)\in C(G),$

$P''_{zy}(s,y,z)\leq 0 \; for \; all \; (s,y,z)\in G_1, \; P''_{zy}(s,y,z)\in C(G_1).$

(ii). $Q(T,y,t_0)\geq 0 \; for \; all \; y\in[a,b], \; Q(T,y,t_0)\in C[a,b],$

$Q'_s(s,y,t_0)\leq 0 \; for \; all \; (s,y)\in G, Q'_s(s,y,t_0)\in C(G),$

$Q'_\tau(T,y,\tau)\geq 0 \; for \; all \; (y,\tau)\in G, \; Q'_{\psi(\tau)}(T,y,\tau)\in C(G),$

$Q''_{\tau s}(s,y,\tau)\leq 0 \; for \; all \; (s,y,\tau)\in G_2, Q''_{\tau s}(s,y,\tau)\in C(G_2).$

(iii). At least one of the following conditions holds:

$(a) P'_y(s,y,a)<0 \; for \; almost \; all \; (s,y)\in G;$

$(b) P'_z(s,b,z)>0 \; for \; almost \; all \; (s,z)\in G;$

$(c) Q'_s(s,y,t_0)<0 \; for \; almost \; all \; (s,y)\in G;$

$(d) Q'_\tau(T,y,\tau)>0 \; for \; almost \; all \; (y,\tau)\in G;$

$(e) \; P''_{zy}(s,y,z)<0 \; for \; almost \; all \; (s,y,z)\in G_1;$

$(f) \; Q''_{\tau s}(s,y,\tau)<0 \; for \; almost \; all \; (s,y,\tau)\in G_2;$

$(h) \; m(t,x)>0 \; for \; almost \; all\,(t,x)\in G.$

(iv). $C(t,x,s,y)\in L_2(G^2)$ and

$$\frac{1}{2}\left[C(t,x,s,y)+C(s,y,t,x)\right]=\sum_{i=1}^{m}\lambda_i\varphi_i(t,x)\varphi_i(s,y),$$

$$C(t,x,s,y)=\sum_{i=1}^{m}\lambda_i\varphi_i(t,x)\varphi_i(s,y), \; m\leq\infty, \; 0\leq\lambda_i, \; i=1,2,...,m \qquad (3)$$

$m\leq\infty, \; 0\leq\lambda_i, \; i=1,2,...,m$

where $\{\varphi_i(t,x)\}$ is an orthonormal sequence of eigen functions from $L_2(G)$

and $\{\lambda_i\}$ is the sequence of corresponding nonzero eigenvalues of the Fredholm integral operator C generated by the kernel $\frac{1}{2}\left[C(t,x,s,y)+C(s,y,t,x)\right]$ with the elements $\{\lambda_i\}$ arranged in decreasing order of their absolute values. If $C(t,x,s,y)=0$ for all $(t,x,s,y)\in G^2$, we assume that $\lambda_1=\lambda_2=...=\lambda_m=0.$

Theorem 1. Let conditions (i)-(iv) be satisfied. Then the solution of the equation (1) is unique in $L_2(G)$.

Proof. Taking the multiplication of both sides of the equation (1) with u(t, x), integrating the results on G, we obtain

$$\int_a^b\int_{t_0}^T m(s,y)u^2(s,y)dyds+\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)u(s,z)u(s,y)dzdyds+$$
$$+\int_a^b\int_{t_0}^T\int_{t_0}^s Q(s,y,\tau)u(\tau,y)u(s,y)d\tau dsdy+$$
$$+\int_a^b\int_{t_0}^T\int_{t_0}^T\int_a^b C(s,y,\tau,z)u(\tau,z)u(s,y)dzd\tau dsdy=\int_a^b\int_{t_0}^T f(s,y)u(s,y)dsdy. \quad (5)$$

Integrating by parts and using the Dirichlet formula

$$\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)u(s,z)u(s,y)dzdyds=$$
$$=-\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)\frac{\partial}{\partial z}\left(\int_z^y u(s,v)dv\right)dz\, u(s,y)dyds=$$
$$=\frac{1}{2}\int_{t_0}^T\int_a^b\left[P(s,y,a)\frac{\partial}{\partial y}\left(\int_a^y u(s,v)dv\right)^2\right]dyds+$$
$$+\frac{1}{2}\int_{t_0}^T\int_a^b\int_z^b P'_z(s,y,z)\frac{\partial}{\partial y}\left(\int_z^y u(s,v)dv\right)^2\varphi ydzds=$$

$$=\frac{1}{2}\int_{t_0}^{T}P(s,b,a)\left(\int_a^b u(s,\nu)\,d\nu\right)^2 ds-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_a^b P'_y(s,y,a)\left(\int_a^y u(s,\nu)\,d\nu\right)^2 dyds+$$

$$+\frac{1}{2}\int_{t_0}^{T}\int_a^b P'_z(s,b,z)\left(\int_z^b u(s,\nu)\,d\nu\right)^2 dzds-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_a^b\int_a^y P''_{zy}(s,y,z)\left(\int_z^y u(s,\nu)\,d\nu\right)^2 dzdyds. \qquad (5)$$

Similarly integrating by parts and using the Dirichlet formula analogically we have

$$\int_a^b\int_{t_0}^{T}\int_{t_0}^{s}Q(s,y,\tau)u(\tau,y)u(s,y)\,d\nu dsdy=$$

$$=\frac{1}{2}\int_a^b Q(T,y,t_0)\left(\int_{t_0}^{T}u(\xi,y)\,d\xi\right)^2 dy-$$

$$-\frac{1}{2}\int_a^b\int_{t_0}^{T}Q'_s(s,y,t_0)\left(\int_{t_0}^{s}u(\xi,y)\,d\xi\right)^2 dsdy+$$

$$+\frac{1}{2}\int_a^b\int_{t_0}^{T}Q'_\tau(T,y,\tau)\left(\int_\tau^T u(\xi,y)\,d\xi\right)^2 d\tau dy-$$

$$-\frac{1}{2}\int_a^b\int_{t_0}^{T}\int_{t_0}^{s}Q''_{\tau s}(s,y,\tau)\left(\int_\tau^s u(\xi,y)\,d\xi\right)^2 d\tau dsdy. \qquad (6)$$

Using the Dirichlet formula we have

$$\int_a^b\int_{t_0}^{T}\int_a^b\int_{t_0}^{T}C(t,x,s,y)u(s,y)u(t,x)\,dsdydtdx=$$

$$=\int_a^b\int_{t_0}^{T}\int_a^b\int_{t_0}^{t}C(t,x,s,y)u(s,y)u(t,x)\,dsdydtdx=$$

$$=\int_a^b\int_{t_0}^T\int_a^b\int_t^T C(t,x,s,y)u(s,y)u(t,x)\,dsdydtdx=$$

$$=\int_a^b\int_{t_0}^T\int_a^b\int_{t_0}^T\left[C(t,x,s,y)+C(s,y,t,x)\right]u(s,y)u(t,x)\,dsdydtdx. \qquad (7)$$

Taking into account (5), (6), (7) and (4) from (5) we obtain

$$\int_{t_0}^T\int_a^b m(s,y)u^2(s,y)dyds+\frac{1}{2}\int_{t_0}^T P(s,b,a)\left(\int_a^b u(s,\nu)d\nu\right)^2 ds-$$

$$-\frac{1}{2}\int_{t_0}^T\int_a^b P'_y(s,y,a)\left(\int_a^y u(s,\nu)d\nu\right)^2 dyds+$$

$$+\frac{1}{2}\int_{t_0}^T\int_a^b P'_z(s,b,z)\left(\int_z^b u(s,\nu)d\nu\right)^2 dzds-$$

$$-\frac{1}{2}\int_{t_0}^T\int_a^b\int_a^y P''_{zy}(s,y,z)\left(\int_z^y u(s,\nu)d\nu\right)^2 dzdyds+$$

$$+\frac{1}{2}\int_a^b Q(T,y,t_0)\left(\int_{t_0}^T u(\xi,y)d\xi\right)^2 dy-$$

$$-\frac{1}{2}\int_a^b\int_{t_0}^T Q'_s(s,y,t_0)\left(\int_{t_0}^s u(\xi,y)d\xi\right)^2 dsdy+$$

$$+\frac{1}{2}\int_a^b\int_{t_0}^T Q'_\tau(T,y,\tau)\left(\int_\tau^T u(\xi,y)d\xi\right)^2 d\tau dy-$$

$$-\frac{1}{2}\int_a^b\int_{t_0}^T\int_{t_0}^s Q''_{\tau s}(s,y,\tau)\left(\int_\tau^s u(\xi,y)d\xi\right)^2 d\tau dsdy+.$$

$$+\frac{1}{2}\int_a^b Q(T,y,t_0)\left(\int_{t_0}^T u(\xi,y)d\psi(\xi)\right)^2 d\varphi(y)-$$

$$-\frac{1}{2}\int_a^b\int_{t_0}^T Q'_{\psi(s)}(s,y,t_0)\left(\int_{t_0}^s u(\xi,y)d\psi(\xi)\right)^2 d\psi(s)d\varphi(y)+$$

$$+\sum_{i=1}^m \lambda_i\left(\int_a^b\int_{t_0}^T \varphi_i(s,y)u(s,y)dsdy\right)^2 = \int_a^b\int_{t_0}^T f(s,y)u(s,y)dsdy. \qquad (8)$$

Example 1. We consider the equation (1) for

$m(t,x)=(t-t_0)(T-t)^2(x-a)^3(b-x)^4,\ (t,x)\in G,$

Рассмотрим уравнение (1) при

$$P(t,x,y)=\alpha_0(t)\beta_0(x)\gamma_0(y),\ (t,x,y)\in G_1,$$

$$Q(t,x,s)=\alpha_2(t)\beta_1(x)\left[\alpha_2(s)+\gamma_2(s)\right],\ (t,x,s)\in G_2,$$

$$C(t,x,s,y)=\sum_{i=1}^m\left[c_i(t,x)c_i(s,y)+d_i(t,x)-d_i(s,y)\right],\ (t,x,s,y)\in G^2$$

where $\alpha_0(t),\alpha_1(t),\alpha'_1(t),\alpha_2(t),\alpha'_2(t),\gamma_2(t),\gamma'_2(t)\in C[t_0,T],$
$\beta_0(x),\beta'_0(x),\gamma_0(x),\gamma'_0(x),\gamma_1(x),\gamma'_1(x)\in C[a,b],$
$c_i(t,x),\ d_i(t,x)\in C(G)\ (i=1,2,...,m),\ \alpha'_1(t)\le 0$ *and* $\gamma_2(t)+\alpha'_2(t)\ge 0$
for all $t\in[t_0,T],\ \beta'_0(x)<0$ *for almost all* $x\in[a,b],\ \gamma'_0(x)+\gamma'_1(x)\ge 0,$
for all $x\in[a,b],\gamma'_0(a)+\gamma'_1(a)>0,\ \alpha_0(t)>0$ *for almost all* $t\in[t_0,T],$
$\beta_1(x)>0$ *for almost all* $x\in[a,b],\gamma_2(t_0)+\alpha_2(t_0)\ge 0.$

In this case the conditions (i)-(iv) be satisfied. The following condition is assumed to hold in what follows.

v). The Fredholm operator C generated by the kernel $\frac{1}{2}\left[C(t,x,s,y)+C(s,y,t,x)\right]$ defined by (3) is positive, i.e. all the eigenvalues $\{\lambda_i\}$ of $\frac{1}{2}\left[C(t,x,s,y)+C(s,y,t,x)\right]$ are positive (i = 1, 2, ..., m, m = ∞) and $\varphi(x)\in C^1[a,b],\ \psi(t)\in C^1[t_0,T].$

The family of well-posedness depending on the parameter α is defined as

$$M_\alpha = \left\{ u(t,x) \in L_2(G) : \sum_{\nu=1}^{\infty} \lambda_i^{-\alpha} \left| u^{(\nu)} \right|^2 \le c \right\},$$

were $c > 0, \quad 0 < \alpha < \infty,$

$$u^{(\nu)} = \int_{t_0}^{T} \int_{a}^{b} u(t,x) \varphi_\nu(t,x) dxdt, \ \nu = 1, 2, ..., \infty.$$

Theorem 2. Let conditions (i)-(ii) and (v) be satisfied. Then the solution $u(t,x)$ of the equation (1) is unuque in $L_2(G)$. Moreover, on the set $K(M_\alpha) \subset L_2(G)$ is the image of M_α under the action of the operator K defined by formula (2)), the inverse K^{-1} of operator K is uniformly continuous with the Holder exponent $\frac{\alpha}{2+\alpha}$, i.e.

$$\left\| u(t,x) \right\|_{L_2} \le c^{\frac{1}{2+\alpha}} \left\| f(t,x) \right\|_{L_2}^{\frac{\alpha}{2+\alpha}}, \ 0 < \alpha < \infty. \quad (9)$$

were

$$\left\| u(t,x) \right\|_{L_2} = \int_{t_0}^{T} \int_{a}^{b} \left\| u(t,x) \right\|^2 dxdt.$$

Proof. a) In this case, the orthonormal sequence of eigenfunctions $u(t,x) \in M_\alpha$ is complete in $L_2(G)$.There fore (8) implies the uniqueness of the solution to equation (1) in $L_2(G)$. Let $f(t,x) \in K(M_\alpha)$. Then the equation (1) has a solution $u(t,x) \in M_\alpha$ and it follows from (8) that

$$\sum_{\nu=1}^{\infty} \lambda_\nu \left| u^{(\nu)} \right|^2 \le \left\| f(t,x) \right\|_{L_2} \left\| u(t,x) \right\|_{L_2}. \quad (10)$$

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|^2 \le \left[\sum_{\nu=1}^{\infty}\frac{\left|u^{(\nu)}\right|^2}{\lambda_\nu^{-1}}\right]^{\frac{\alpha}{2+\alpha}} \cdot \left[\sum_{\nu=1}^{\infty}\lambda_\nu^{-\alpha}\left|u^{(\nu)}\right|^2\right]^{\frac{1}{\alpha+1}}, \tag{11}$$

On the other hand,

$$\sum_{\nu=1}^{\infty}\left|u^{(\nu)}\right|^2 \le \left[\left\| f(t,x)\right\|\left\|u(t,x)\right\|\right]^{\frac{\alpha}{1+\alpha}} c^{\frac{1}{1+\alpha}} \tag{12}$$

Combining the last two inequalities gives estimate (9). The theorem 2 is proved.

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ASYMPTOTICAL QUOTIENT SPACES IN THEORY OF DELAY-DIFFERENTIAL EQUATIONS

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Supra, the author introduced the following equivalence relation is in the space of solutions of initial value problems for dynamical systems: distance between two solutions tends to zero while time increases. The phenomenon "the dimension of the quotient space is less than one of the initial space" was called "asymptotical reduction of dimension of space of solutions". In this paper the Hausdorff asymptotical equivalence relation is introduced: distance between two solutions with invertible transformation of argument tends to zero. The corresponding quotient spaces generate new mathematical objects.

Keywords: equivalence relation, asymptotical equivalence, delay-differential equation, initial value problem, Hausdorff metric.

Мурда автор динамикалык системалардын чыгарылыштарынын мейкиндигинде төмөнкүдөй асимптотикалык эквиваленттүүлүктүн түшүнүгү киргизилди: убактыт өскөндө эки чыгарылыштын арасында аралык нөлгө умтулат. Фактор-мейкиндиктин ченеми баштапкы мейкиндиктин ченеминен кичүү болгон кубулуш, «чыгарылыштар мейкиндигинин ченемин асимптотикалык төмөндөтүү» деп айтылган. Бул макалада хаусдорфтук асимптотикалык эквиваленттүүлүктүн түшүнүгү киргизилди: убактыт өскөндө аргументин кайра калыбына келтирүүчү өзгөртүү менен эки чыгарылыштын арасында аралык нөлгө умтулат. Дал келген фактор-мейкиндик жаңы математикалык объекттерди тууйт.

Урунттуу сөздөр: кечигүүчү аргументтүү дифференциалдык теңдеме, баштапкы маселе, монотондук, интегралдык норма.

Ранее автор ввела следующее отношение асимптотической эквивалентности в пространстве решений начальных задач для динамических систем: расстояние между двумя решениями стремится к нулю при увеличении времени. Явление «размерность фактор-пространства меньше, чем размерность исходного пространства» было названо «асимптотическое уменьшение размерности пространства решений». В статье введено понятие хаусдорфовой асимптотической эквивалентности: неограниченное сближение решений с обратимым преобразованием аргумента с увеличением времени. Соответствующее фактор-пространство порождает новые математические объекты.

Ключевые слова: отношение эквивалентности, асимптотическая эквивалентность, дифференциальное уравнение с запаздывающим аргументом, начальная задача, хаусдорфова метрика.

Introduction

The problem of behavior of solutions of initial value problems as time tends to infinity is one of the main in the theory of dynamical systems. Many mathematical methods were developed for this purpose including the theory of stability [1]-[2], method of characteristic equations for autonomous and periodical dynamical systems, method of special solutions for delay-differential equations [3]-[4]. Various sufficient conditions were obtained to provide some kinds of behavior of solutions. Various definitions and denotations were introduced for each kind.

Supra, the author [5] introduced the following equivalence relation is in the space of solutions of initial value problems for dynamical systems: distance between two solutions tends to zero while time increases. The phenomenon "the dimension of the quotient space is less than one of the initial space" was called "asymptotical reduction of dimension of space of solutions". In this paper the Hausdorff asymptotical equivalence relation is introduced: distance between two solutions with invertible transformation of argument tends to zero. The corresponding quotient spaces generate new mathematical objects.

Section 1 contains definitions of asymptotical equivalence and λ-exponential asymptotical equivalence and the phenomenon of asymptotical reduction of dimension.

Section 2 proposes definitions of Hausdorff asymptotical equivalence and Hausdorff asymptotical quotient space.

Section 3 contains examples of Hausdorff asymptotical quotient space for various types of differential equations.

1. Review of preceding results

We suppose dynamical systems as equations for functions of time fitting the property "the present depends on the past only" (differential equations, delay-differential eq-uations, Volterra integral equations of the second kind, difference equations etc.).

"Ordinary" equations and systems of equations in more general form can be presented as follows (we are restricted with existence and uniqueness of solution of an initial value problem).

Denote $\boldsymbol{R}_{+} := [0,\infty)$; $\boldsymbol{R}_{++} := (0,\infty)$.

Definition 1. A dynamical system is a tuple of a number $h \geq 0$ [delay], a totally ordered set Λ of real numbers with the least element but without the greatest one [domain of functions]: $\Lambda = \boldsymbol{R}_{\boldsymbol{h}} := [-h,\infty)$ or $\Lambda = \boldsymbol{N}_0 := \{0,1,2,3,\ldots\}$, a topological space Z [range of functions]; a set Φ of functions $[-h,0] \to Z$ [initial conditions]; if $h=0$ then $\Phi = Z$; a function $W(t,\varphi)\colon \Lambda \times \Phi \to Z$ such that its restriction on $[-h,0]$ equals φ [solutions of initial value problems]. If $\Lambda = \boldsymbol{R}_{\boldsymbol{h}}$ then $W(t,\varphi)$ is supposed to be continuous with respect to t.

We will consider the following classes of spaces with their dimensions:

1-spaces: Z = $\boldsymbol{R}$; dimension =1;

d-spaces: $Z = \boldsymbol{R}^d$, $d \in N:=\{1,2,...\}$; dimension = d;

N-spaces: Z is a normed linear space with norm $||\cdot||_Z$; dimension (finite or infinite) is the number of elements in the basis;

M-spaces: Z is a metric space with metric ρ_Z; the inductive Ind-dimension is used;

U-spaces: Z is a uniform space with set of entourages Y_Z; Ind-dimension is used.

Definition 1. (The most general are U-Spaces).

The following equivalence is said to be asymptotical equivalence (λ-exponential asymptotical equivalence):

For N-spaces

$$(\varphi_1 \sim \varphi_2) \Leftrightarrow (\ lim\{\ ||W(t,\varphi_1) - W(t,\varphi_2)||_Z : t \to \infty\} = 0);$$

$$((\varphi_1 \sim_\lambda \varphi_2) \Leftrightarrow (\ sup\{\ ||W(t,\varphi_1) - W(t,\varphi_2)||_Z\, exp(\lambda t) : t \in \Lambda\} < \infty));$$

For M-spaces

$$(\varphi_1 \sim \varphi_2\,) \Leftrightarrow (\ lim\{\ \rho_Z(W(t,\varphi_1),\, W(t,\varphi_2)) : t \to \infty\} = 0).$$

$$(\ (\varphi_1 \sim_\lambda \varphi_2\,) \Leftrightarrow (\ sup\{\ \rho_Z(W(t,\varphi_1),\, W(t,\varphi_2))\ exp(\lambda t) : t \in \Lambda\} < \infty)).$$

For U-spaces:

$$(\varphi_1 \sim_\lambda \varphi_2) \Leftrightarrow (\ \forall V \in Y_Z)(\exists\, t_1 \in \Lambda)\ (\ \forall t > t_1)((W(t,\varphi_1),\, W(t,\varphi_2))\ \in V).$$

(Exponential asymptotical equivalence cannot be defined in such general spaces).

Definition 2. The factor-space $\Phi^*:=\Phi\ /\sim$ of the space Φ by the asymptotical equivalence is said to be an asymptotical quotient space; respectively, the quotient space $\Phi^*_\lambda:=\Phi\ /\sim_\lambda$ of the space Φ by the λ-exponential asymptotical equivalence is said to be λ-exponential asymptotical quotient space.

Example 1. (The Floquet-Lyapunov theory).

Some types of linear autonomous delay-differential equations have countable sets of characteristic values $\{\mu_1, \mu_2,...\}$ which can be semi-ordered: $Re(\mu_1)\geq Re(\mu_2) \geq ...;$
$lim\{Re (\mu_k): k \to\infty\} = -\infty$ such that functions $exp(\mu_k t)$ (and for multiple values also $exp(\mu_k t+\nu_k ln t), \nu_k \in \mathbf{N}),$ are (components of) particular solutions.

If $W(t, \varphi)$ can be presented as $\Sigma\{c_k (\varphi)exp(\mu_k t+\nu_k ln t): k\in \mathbf{N}\}$ where $c_k (\varphi)$ are linear operators then the phenomenon "asymptotical reduction of dimension of space of solutions" takes place, the infinite-dimensional space Φ reduces to the space with basis $\{ exp(\mu_k t+\nu_k ln t): Re \mu_k \geq 0\}$.

2. Definitions of Hausdorff asymptotical equivalence and asymptotical quotient space

$\Lambda=\mathbf{R}_h$ in this section.

Definition 2. Let $s\in\mathbf{R}_+, \vartheta:[s,\infty)\to \mathbf{R}_+$ be of the class Θ of strictly increasing continuous functions, $lim\{\vartheta(t):t\to\infty\}=\infty$.

The following equivalence is said to be Hausdorff asymptotical equivalence:

For N-spaces

$$(\varphi_1 \cong \varphi_2) \Leftrightarrow (\forall\varepsilon\in \mathbf{R}_{++})(\exists s, \vartheta)(\forall t\in[s,\infty)) (||W(t,\varphi_1)-W(t,\varphi_2)||_Z <\varepsilon);$$

For M-spaces

$$(\varphi_1 \cong \varphi_2) \Leftrightarrow (\forall\varepsilon\in \mathbf{R}_{++})(\exists s, \vartheta) (\forall t\in[s,\infty))(\rho_Z(W(t,\varphi_1), W(\vartheta(t),\varphi_2))<\varepsilon);$$

For U-spaces

$$(\varphi_1 \cong \varphi_2) \Leftrightarrow (\forall\varepsilon\in\Gamma_Z) (\forall t\in[s,\infty))((W(t,\varphi_1), W(\vartheta(t),\varphi_2)) \in\varepsilon). \quad (1)$$

Lemma 1. The introduced relation is a correct relation of equivalence.

Proof. Reflexivity of the relation $\cong$ is obvious (let $\vartheta(t) \equiv t$).

Prove the symmetricity. Let $\varphi_1 \cong \varphi_2$. There exists the inverse function $\zeta(t) \in \Theta$ to the function $\vartheta(t)$.

Substituting $\zeta(t)$ instead of t into (1), we obtain:

$$(\forall \zeta(t) \in [s,\infty))((W(\zeta((t)),\varphi_1), W(\vartheta(\zeta(t)),\varphi_2)) \in \varepsilon).$$

The condition $\zeta(t) \geq s$ is equivalent to the condition $\vartheta(\zeta(t)) \geq \vartheta(s)$. Hence

$$(\forall t \in [\vartheta(s),\infty))(W(t),\varphi_2), (W(\zeta(t),\varphi_1)) \in \varepsilon); \ \varphi_2 \cong \varphi_1.$$

Prove the transitivity. For given $\varepsilon \in \Gamma_Z$ find such $\varepsilon_1 \in \Gamma_Z$ that $\varepsilon_1 \circ \varepsilon_1 \subset \varepsilon$. There exist such s_{12}, s_{23}, $\vartheta_{12}(t)$, $\vartheta_{23}(t)$, that

$$(\forall t \in [s_{12},\infty))((W(t,\varphi_1), W(\vartheta_{12}(t),\varphi_2)) \in \varepsilon_1), \qquad (2)$$

$$(\forall t \in [s_{23},\infty))((W(t,\varphi_2), W(\vartheta_{23}(t),\varphi_3)) \in \varepsilon_1). \qquad (3)$$

Substituting $\vartheta_{12}(t)$ instead of t into (3), we obtain

$$(\forall \vartheta_{12}(t) \in [s_{23},\infty))((W(\vartheta_{12}(t),\varphi_2), W(\vartheta_{23}(\vartheta_{12}(t)),\varphi_3)) \in \varepsilon_1). \qquad (4)$$

The condition $\vartheta_{12}(t) \geq s_{23}$ is equivalent to the condition $t \geq \zeta_{12}(s_{23})$. Hence (4) can be written as

$$(\forall t \in [\zeta_{12}(s_{23}),\infty))(W(\vartheta_{12}(t),\varphi_2), (W((\vartheta_{23}\vartheta_{12})(t),\varphi_3)) \in \varepsilon_1). \qquad (5)$$

If we choose $s_{13}=max\{s_{12}, \zeta_{12}(s_{23})\}$ then (2) and (5) imply

$$(\forall t \in [s_{13},\infty))(((W(t,\varphi_1), W(\vartheta_{12}(t),\varphi_2)) \in \varepsilon_1) \wedge (W(\vartheta_{12}(t),\varphi_2),$$
$$(W((\vartheta_{23}\vartheta_{12})(t),\varphi_3)) \in \varepsilon_1)).$$

Hence $(\forall t \in [s_{13},\infty))((W(t,\varphi_1), W((\vartheta_{23}\vartheta_{12})(t),\varphi_3)) \in \varepsilon_1 \circ \varepsilon_1)$.

The transitivity is proven. Lemma is proven.

A Hausdorff asymptotical quotient space will be denoted as $\Phi^{*=}$.

3. Examples of new objects

Solutions of scalar differential equations ($\Lambda=Z=\boldsymbol{R}$).

Example 2. All continuous and increasing to infinity functions are Hausdorff asymptotically equivalent. All continuous, increasing and tending to any number fun-ctions are Hausdorff asymptotically equivalent.

Example 3. All solutions of the equation $z'(t)=az(t),\ a>0$ form three classes of Hausdorff quotient space $\Phi^{*=}$.

Solutions of vector differential equations.

Example 4. ($\Phi = Z = \boldsymbol{R}^2$). The system $x'(t)=y(t),\ y'(t)=x(t),\ x(0)=\varphi_1,$ $y(0)=\varphi_2$.

$(x(t),y(t))=W(t,\varphi_1,\ \varphi_2)=$

$= (\ (\varphi_1+\varphi_2)/2\cdot \exp t+(\varphi_1-\varphi_2)/2\cdot \exp(-t),\ (\varphi_1+\varphi_2)/2\cdot \exp t+(\varphi_2-\varphi_1)/2\cdot$ $\exp(-t))$.

The Hausdorff quotient space $\Phi^{*=}$ contains three elements represented by:

$W(t,0,0)=(0,0)$ (saddle point); $W(t,1,0);\ W(t,-1,0)$.

We propose to call the last two elements "trajectories without beginning"

$\{(x, y)=(t,t):t \in \boldsymbol{R}_{++}\};\ \{(x, y)=(-t,-t):t \in \boldsymbol{R}_{++}\}$.

Example 5. ($\Phi = Z = \boldsymbol{R}^3$). A strange attractor with attracting set of two touching cycles. The Hausdorff quotient space $\Phi^{*=}$ contains three elements:

permanent alternation of cycles until infinity;

final winding on the first cycle;

final winding on the second cycle.

Conclusion

We hope that consecutive revealing of functions being Hausdorff asymptotically eq-uivalent for various types of differential equations would yield new mathematical objects and it would be interesting for investigation of equations.

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MSC 34 A26, 35 A16

CATEGORY OF OBJECTS WITH FUNCTIONAL RELATIONS FOR DIFFERENTIAL EQUATIONS

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Category of differential equations with functional relations is considered in the paper. Objects of this category are ranges of solutions as domains of multiplace predicates; morphisms are transformations of such sets preserving truth of predicates. The main condition: a set can be replenished by one element in such a way that the

predicate becomes truth. A classification of them is proposed: ordinary differential equations; partial differential equations. Examples are given. An application of such relations to investigate some equations is demonstrated.
Keywords: category, functional relation, differential equation, set, object, morphism, multiplace predicate, classification, function.

Бул макалада өз ара функционалдык байланыштары болгон дифференциалдык теңдемелердин категориясы каралат. Бул категориядагы объектилер – чыгарылыштардын маанилеринин мейкиндиктеринде аныкталган көп орундуу предикаттардын көптүгү, морфизмдер – мындай көптүктөрдү предикаттардын чындыгын сактаган өзгөртүүлөр. Негизги шарты: көптүктөрдү, предикат чын болуп тургандай, бир элемент менен толуктоо мүмкүнчүлүгү. Төмөнкү классификация сунушталат: кадимки дифференциалдык теңдемелер; жеке туундулуу дифференциалдык теңдемелер. Мисалдар келтирилген. Мындай өз ара функционалдык байланыштарды кээ бир дифференциалдык теңдемелерди изилдөө үчүн колдонуу көрсөтүлгөн.
Урунттуу сөздөр: категория, функционалдык байланыш, дифференциалдык теңдеме, көптүк, объект, морфизм, көп орундуу предикат, классификациялоо, функция.

В статье вводится определение категории дифференциальных уравнений с функциональными соотношениями. Объектами в этой категории являются множества значений решений с определенными на них многоместными предикатами, морфизмами - такие преобразования этих множеств, которые сохраняют истинность предикатов. Основное условие: возможность такого пополнения множеств одним элементом, что предикат становится истинным. Предлагается классификация: обыкновенные дифференциальные уравнения; дифференциальные уравнения в частных производных. Приведены примеры. Показано использование таких соотношений для исследования некоторых дифференциальных уравнений.
Ключевые слова: категория, функциональное соотношение, дифференциальное уравнение множество, объект, морфизм, многоместный предикат, классификация, функция.

Introduction

The notion of “categories” being more general than sets and families of sets was introduced in [1]. In Kyrgyzstan the first works on the category theory were [2] and [3]. We introduced the principle of preservation of solution while transformations (supra it was meant implicitly). We proposed to introduce the category of equations and its subcategories including the category of correct equations ([4], [5], [6], [7]). The aim of this paper is to connect this notion with other categories.

To investigate differential equations of various types we propose to use the following fact. Solutions of some types of differential equations have functional relations connecting their values in different points. By given values of solutions in several points one can find their values in other points.

For examples in the first section, even, odd and periodical solutions, Vallée-Poussin's assertion, Lagrange interpolation polynomial, Hermite interpolation polynomial, spline-functions are considered for ordinary differential equations are considered. Their approximations are in the second section. Asgeirsson's identity and its generalizations for partial differential equations of hyperbolic type are described in the third section. In this paper we will use functional denotations of type *x[n]* instead of x_n.

1. Definitions

A category is defined by its objects and morphisms.

The main well-known categories are the following:

The category of sets *Set*. *Ob(Set)* are non-empty sets, *Mor(Set)* are functions.

We proposed to consider the category of functions *Func* (it used in mathematics implicitly). *Ob(Func)* = *Mor(Set)*, *Mor(Func)* are transformations of functions.

The category of topological spaces *Top*. *Ob(Top)* are topological spaces, *Mor(Top)* are continuous functions.

We proposed the category *Equa* of equations:

Definition 1 [4], [5]. *Ob(Equa)* contains tuples

{Non-empty sets X, Y, predicate $P(x)$ on X, transformation $B: X \to Y$}.

If $(\exists\, x \in X)(P(x) \wedge (y=B(x)))$ then $y \in Y$ is said to be a solution of the equation $\{X, Y, P, B\}$.

Particularly, if B is the identity operator I, then we obtain the equation *“P(x)”* only.

Mor(Equa) are such transformations of tuples $\{X, Y, P, B\}$ that solutions (or their absence) preserve.

Among subcategories for the category *Equa,* we also proposed the category of equations for functions *Equa-Func*.

Definition 2. *Ob(Equa-Func)* contains tuples
$\{X \in Ob(Func), Y \in Ob(Func)$, predicate $P(x)$ on X, transformation $B{:}X \to Y\}$.

Mor(Equa-Func) contains invertible transformations of functions inherited from *Mor(Equa)* and specific transformations.

We considered functional relations in [8]. We propose the category *Func-Diff-Equa.*

Definition 3. *Ob(Func-Diff-Equa)* contains tuples
{F being a space of functions-solutions of differential equations $f{:}X \to Y$; *P being a multiplace predicate defined on values of functions of F}.*

The predicate has the form $P(x_1, y_1, x_2, y_2, \ldots, x_n, y_n)$ where all $x_1, x_2, \ldots, x_n$ are distinct. It meets the condition: for any $x_1, y_1, x_2, y_2, \ldots, x_{n-1}, y_{n-1}, x_n$ such that $Q(x_1, y_1, x_2, y_2, \ldots, x_{n-1}, y_{n-1}, x_n)$ there exists such y_n that $P(x_1, y_1, x_2, y_2, \ldots, x_n, y_n)=true$, where Q is an additional predicate. *Mor(Func-Diff-Equa)* are bijections preserving values of the predicates P and Q.

2. Subcategory of ordinary differential equations (ODE)

$Q \equiv true$ in this section.

Let ζ be the minimal number of connected points for a differential equation as (if it exists). Some examples of elements of *Func-Diff-Equa:*

2.1. The ODE of the first order with initial condition $y'(x)=a$, $y(0)=u>0$, $a>0$: $F=\{u+ax: x\in R_+\}$; $\zeta=3$:

$$P(x_1, y_1, x_2, y_2, x_3, y_3)=\text{"}(y_1-y_3)(x_1-x_2)-(y_1-y_2)(x_1-x_3)=0\text{"}.$$

A morphism: substitute $y(x)=z^2(x)$. The transformed ODE $z'(x)=a/(2z(x))$, with initial condition $z(0)=\sqrt{u}$;

$$P_1(x_1, z_1, x_2, z_2, x_3, z_3)=\text{"}(z_1^2-z_3^2)(x_1-x_2)-(z_1^2-z_2^2)(x_1-x_3)=0\text{"}.$$

2.2. The linear ODE of the k-th order $y^{(k)}(x)=0$. F is the space of polynomials of $(k-1)$-th order: $F=\{a_0+a_1x+\ldots+a_{k+1}x^{k-1}: x\in R_+\}$; $\zeta=k+1$.

Let $L(x_1, y_1, x_2, y_2, \ldots, x_k, y_k, x)$ be the Lagrange interpolation polynomial of the $(k-1)$-th order. Then

$$P(x_1, y_1, x_2, y_2, \ldots, x_k, y_k, x_{k+1}, y_{k+1})=\text{"}L(x_1, y_1, x_2, y_2, \ldots, x_k, y_k, x_{k+1})=y_{k+1}\text{"}.$$

2.3. The general linear ODE of the k-th order

$$y^{(k)}(x)+p_1(x)\,y^{(k-1)}(x)+\ldots+p_k(x)\,y(x)=0,\quad a\le x\le b,\quad p_i(x)\in C[a,b],\ i=1,\ldots,k, \tag{1}$$

with the multipoint value problem

$$y(x_i)=y_i,\ i=1,\ldots,k. \tag{2}$$

C. J. de la Vallée Poussin (for instance see [1]) proved that this problem has a unique solution if

$$\|p_1\|_{[a,b]}(b-a)+\|p_2\|_{[a,b]}(b-a)^2/2!+\ldots+\|p_n\|_{[a,b]}(b-a)^n/n! < 1. \tag{3}$$

Here F is the space of solutions of (1); $\zeta=k+1$. Denote $G(x_1, y_1, x_2, y_2, \ldots, x_k, y_k, x)$ as the solution of (1)-(2) with the additional condition (3). Then

$$P(x_1, y_1, x_2, y_2, \ldots, x_k, y_k, x_{k+1}, y_{k+1})=\text{"}G(x_1, y_1, x_2, y_2, \ldots, x_k, y_k, x_{k+1})=y_{k+1}\text{"}.$$

3. Subcategory of partial differential equations (PDE)

Let $x=(\xi,\eta)\in R^2$. Consider the hyperbolic equation $y_{\xi\eta}''(\xi,\eta)=0$. Let F be the space of solutions of it. They meet the Asgeirsson's identity:

$$y(w_1, v_1)+y(w_2, v_2)-y(w_1, v_2)-y(w_2, v_1)\equiv 0. \qquad (4)$$

Hence, $\zeta=4$;

the predicate *Q="three points are in vertices of a coordinate rectangle";*

the predicate *P="sums of values of* $y(\xi,\eta)=0$ *in end-points of diagonals of rectangle are equal".* $y_{\xi\eta}''(\xi,\eta)=0$.

Morphism: by means of linear substituting of variables (ξ,η) the equation $y_{\xi\eta}''(\xi,\eta)=0$ transforms into the wave equation

$$y_{\xi\xi}''(\xi,\eta)= y_{\eta\eta}''(\xi,\eta),$$

the predicate *Q ="three points are in vertices of a rectangle with angle* 45^0*to coordi-nate axes";* the predicate *P* in this form remains.

4. Conclusion

The paper demonstrates that the notion of category yields possibility to present facts of theory of ordinary and partial differential equations uniformly.

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MSC 37 K20

SYSTEMS OF DIFFERENTIAL EQUATIONS AND COMPUTER PHENOMENA

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Some phenomena were discovered by physical, chemical and technical experiments and further were explained and substituted by mathematical models, especially by means of differential equations, or by computational experiments. Also, some phenomena were discovered by computer experi-ments and further substituted by

other methods. Meanwhile, the computer is a self-standing real object and phenomena on it are specific ones. This paper contains a survey on this matter.
Keywords: phenomenon, computer, attractor, smooth surface, differential equation, difference equation, system of equations, implementation

Кээ бир кубулуш физикалык, химиялык жана техникалык эксперименттер аркылуу ачылды, андан кийин математикалык моделдер (алардын ичинде дифференциалдык теңдемелер) аркылуу же эсептөөчү эксперименттер аркылуу түшүндүрүлдү жана негизделди. Ошондой эле, кээ бир кубулуш компьютердеги эксперименттер аркылуу табылды, андан кийин башка ыкмалар аркылуу негизделди. Аны менен бирге, компьютер өз алдынча чыныгы объект, андагы кубулуш айрыкча. Бул макалада ал предмет каралат.
Урунттуу сөздөр: кубулуш, компьютер, тартып жакындаткыч, жылмакай бет, кубулуш, дифференциалдык теңдеме, айырмалык теңдеме, теңдемелер системасы, жүзөгө ашыруу.

Некоторые явления были открыты с помощью физических, химических и технических экспериментов, а потом были объяснены и обоснованы с помощью математических моделей (в особенности - дифференциальных уравнений). Также, некоторые явления были обнаружены с помощью компьютерных экспериментов, а потом обоснованы другими методами. Вместе с тем, компьютер - самостоятельный реальный объект, явления на нем - специфические. В этой статье производится обзор по данному вопросу.
Ключевые слова: явление, компьютер, аттрактор, гладкая поверхность, явление, дифференциальное уравнение, разностное уравнение, система уравнений, реализация.

1. Introduction

Searching effects and phenomena, their investigation and substantiation are one of main sources of development of science.

Framework definitions and systematical investigation of effects and pheno-mena in mathematics were initiated in [1], [2].

Some phenomena were discovered by physical, chemical and technical experi-ments and further were explained and substituted by mathematical models, espe-cially by means of differential equations, or by computational experiments. Also, some phenomena were discovered by computer experiments and further substi-tuted by other methods.

Meanwhile, the computer is a self-standing real object and phenomena on it are to be considered especially. Formally they may be considered as electronic ones. But there was the hypothesis [3] that the computer presents a new form of motion (separately from traditional mechanical, physical, chemical, biological, social ones). Hence, phenomena on computer are specific ones.

Remark. Approximate solving of differential equations on computer is considered traditionally as substantiation of some phenomena of them. But actually some systems of difference, not differential equations are used to be solved. From the standpoint of this paper, all three phenomena: for differential equations; for systems of difference equations and implementation of the last on computer are distinct. We call such phenomena “analogous”.

This paper contains a survey on this matter.

2. Examples of phenomena

2.1. The idea of creating order (cosmos) out of chaos is well-known from ancient times. The Kyrgyz word irgöö means: discrete optimization by means of synergetic, or (I1)"random vibration of balls of different sizes of same material in a wide symmetrical vessel yields migration of the biggest one to the center of their surface." (Mechanical form of motion). Until XIX century Kyrgyz language was unwritten, so it is impossible to conclude how many centuries ago this phenomenon was discovered and this term appeared.

A system of stochastic difference equations (I2) was built in [4]. Their computer implementation (I3) demonstrated that (I2) is an analog of (I1) and substituted the hypothesis: for a large number of balls in a vessel, in a certain class of processes described by random difference equations, the probability of the event "the biggest ball is close to the center of surface of heap of balls" is 1 as time tends to infinity.

2.2. The second example of synergetic was “Rayleigh-Benard convection cells” (1900). (B1) A plane horizontal layer of fluid heated from below fluid develops a regular pattern of convection (hexagon) cells. (B2) There was built a system of nonlinear differential Oberbeck-Boussinesq equations [5]. (B3) “The system is solved using the finite element method”. From the standpoint of this paper, another system of equations using the finite element method was built and (B4) was implemented on a computer.

2.3. We consider “strange attractors”. One of definitions: An attractor is called strange if it is locally unstable yet globally stable: once some objects have entered the attractor, nearby points diverge from one another but never depart from the attractor. If a strange attractor is chaotic, exhibiting sensitive dependence on initial conditions, then any two arbitrarily close alternative initial points on the attractor, after any of various numbers of iterations, will lead to points that are arbitrarily far apart (subject to the confines of the attractor), and after any of various other numbers of iterations will lead to points that are arbitrarily close together.”

The first example was Lorenz attractor:

(L1) $x'(t)=10(y(t)-x(t)); y'(t)=x(t)(28-z(t))-y(t); z'(t)=x(t)y(t)-8/3\cdot z(t).$

Real processes: (L2) So-called Malkus waterwheel, a constant flow of water pours in at the top bucket of a simple circular symmetrical waterwheel and the base of the waterwheel has perforations to allow the outflow of water.

(L3) Reaction in chemical mixture of three special components.

(L4) is a system of difference equations; (L5) is its implementation on computer.

In preceding papers, for instance, [6], implementations of strange attractors we-re too complicated. In [7] we presented such phenomenon mechanically as rolling of a ball along a smooth surface with three juts by gravity effect. The motion of the ball proves to be unpredictable.

Also such surface was made of iron and tested.

(**M1**) System of differential equations

$$x''(t) = Z_x'(x(t), y(t)) \,/\, \left(\left(Z_x'(x(t), y(t)) \right)^2 + (Z_x'(x(t), y(t)))^2 + 1 \right),$$

$$y''(t) = Z_x'(x(t), y(t)) \,/\, \left(\left(Z_x'(x(t), y(t)) \right)^2 + (Z_x'(x(t), y(t)))^2 + 1 \right).$$

Z(x,y) is the surface defined by the formula

$$Z(x, y) = \sum_{j=1}^{3} \left(\left(x - \cos(2\pi j/3) \right)^2 + \left(y - \sin(2\pi j/3) \right)^2 + 0.01 \right)^{-1} + x^2 + y^2.$$

If the initial conditions is $(-a;0)$, $0 < a < 1$ then the point moves along the li-ne $(-a < x < 0; y = 0)$, further does along the line $(0 < x < \varepsilon; y = 0)$, $\varepsilon < a$, and further motion is unpredicable.

M2. Definition. If an algorithm treating rational numbers and elaborating "suffice-ently long" sequence $\{x_n: n=1,2,3, \ldots\}$ fulfills the following conditions:

1) $\exists(q>0)(\forall n \in N)(|x_n| < q)$;

2) $\exists(p>0)(\forall m \in N)(\exists n>m)(|x_n| > p)$;

3) a sequence $\{x_n: n \in N\}$ is not close to a periodic one;

4) small perturbation of initial data (passing to neighbor machine numbers the sequ-ence $\{x_n: n \in N\}$ changes "sufficiently" (i.d. computational instability takes place),

then such algorithm is said to be a discrete strange attractor.

Corresponding program in pascal:

```
PROGRAM sab_att; USES CRT, graph;
var x,y,xn,yn,vx,vy,vxn,vyn,dx,dy,dxy2,ht,z,ffx,ffy,
xn1,yn1,vxn1,vyn1,ffx1,ffy1: double; i,j,nxy,it,nt,np,ihand,n_time,ik:
longint; var drv, mode,f,n,xg,yg,zg: integer; xf,yf:array[1..3] of double;
xfg,yfg:array[1..3] of integer;
procedure grad(var fx,fy,xx,yy:double); var fxx, fyy, fxy, a: double;
begin  fxx:=0.; fyy:=0.; a:=1.0; for j:=1 to 3 do begin dxy2:=sqr(xx-
xf[j])+sqr(yy-yf[j])+0.01; fxx:=fxx+2.0*(xx-xf[j])/sqr(dxy2);
fyy:=fyy+2.0*(yy-yf[j])/sqr(dxy2) end; fxx:=fxx-2.0*a*xx; fyy:=fyy-
2.0*a*yy; fxy:=sqr(fxx)+sqr(fyy)+1.0;
fxx:=fxx/fxy; fyy:=fyy/fxy; fx:=fxx; fy:=fyy; end;
begin {main} drv:=0; mode:=VgaHi; InitGraph(drv,mode,'c:\tp\bgi');
randomize; SetTextStyle(0,0,2); OutTextXY(30,20,'Pankov, Tagaeva,
2018. Strange attractor'); z:=300.; zg:=round(z)+30; xf[1]:=-1.0;
yf[1]:=0.0; xf[2]:=1.0/2.0; yf[2]:=sqrt(3.0)/2.0; xf[3]:=1.0/2.0; yf[3]:=-
sqrt(3.0)/2.0;
for j:=1 to 3 do begin xfg[j]:=round(z*xf[j]); yfg[j]:=round(z*yf[j]);
Setcolor(green); circle(xfg[j]+zg,yfg[j]+zg,8); end;
x:=0.3; y:=0.1; vx:=0.; vy:=0.; ht:=0.1; nt:=400;
   for it:=0 to nt do begin {it}  grad(ffx,ffy,x,y); vxn1:=vx+ffx*ht;
vyn1:=vy+ffy*ht;
 xn1:=x+(vx+vxn1)*ht/2.0; yn1:=y+(vy+vyn1)*ht/2.0;
  grad(ffx1,ffy1,xn1,yn1); vxn:=vx+(ffx+ffx1)*ht/2.0;
vyn:=vy+(ffy+ffy1)*ht/2.0;
 xn:=x+(vx+vxn)*ht/2.0; yn:=y+(vy+vyn)*ht/2.0;
```

```
 xg:=round(z*xn); yg:=round(z*yn);
Setcolor(white); circle(xg+zg,yg+zg,2+(it div 100)); delay(50);
x:=xn; y:=yn; vx:=vxn; vy:=vyn; end {it}; END.
```

M3. Mechanical implementation

1) Cut a hexagon of diameter 60-100 cm from tin. Number its vertices as 1-2-3-4-5-6 and its center as 0.

2) Arrange the hexagon horizontally and flex it in such a way that segments 1-0, 3-0, 5-0 have large slope down to point 0 and segments 2-0, 4-0, 6-0 have slight slope down to point 0.

3) Release a little steel ball from point 2. It should roll down to point 0, lift (a little) up to point 5, roll down (unpredictable) along segment 4-0 or segment 6-0, lift (a little) etc.

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MSC 35 M33

SOLVING OF A SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS OF THE FIRST ORDER WITH INITIAL-BOUNDARY CONDITIONS BY THE METHOD OF AN ADDITIONAL ARGUMENT

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This article uses the method of additional argument for some first-order differential equations with initial-boundary conditions. A purpose of the article: the reduction of initial-boundary value problems for systems of nonlinear partial differential equations to equivalent systems of integral equations using the method of an additional argument.

Keywords: differential equation, additional argument, initial condition, integral equation.

Бул макала кошумча аргумент ыкмасын колдонуу менен жекече туундулуу дифференциалдык теңдемелер үчүн коюлган баштапкы-чектик маселелерди изилдөө. Макаланын максаты: кошумча аргумент ыкмасын колдонуу менен жекече туундулуу сызыктуу эмес дифференциалдык теңдемелер системасы үчүн коюлган баштапкы-чектик маселелерди эквиваленттик интегралдык теңдемелер системасына келтирүү.

Урунттуу сөздөр: кошумча аргумент ыкмасы, баштапкы-чектик маселелер, интегралдык теңдеме.

Статья применяет метод дополнительного аргумента к системам дифференциальных уравнений в частных производных с начально-краевыми условиями. Цель статьи: с помощью метода дополнительного аргумента свести начально-краевую задачу для системы нелинейных уравнений в частных производных к эквивалентной системе интегральных уравнений.

Ключевые слова: метод дополнительного аргумента, начально-краевая задача, интегральные уравнение.

1. Introduction

There are various methods for studying solvability of systems of partial differential equations. For instance, there are classical methods of characteristics, Galerkin's method, flows methods. The additional argument method was invented in Kyrgyzstan (see, for instance, [1]) for studying solvability of systems of first order partial differential equations when other methods are not applicable. In some cases the application of this method allows one to find more effectively and specifically the conditions of local solvability in original coordinates for systems of nonlinear and quasi-linear first order differential equations.

2. Statement of problem

In [2, 3, 4] it was shown that a system of differential equations with various initial-boundary conditions can be explored by using the method of an additional argument.

The present paper is devoted to an investigation of the following system of nonlinear partial differential equations:

$$\frac{\partial v(t,x)}{\partial t} + v(t,x)\frac{\partial v(t,x)}{\partial x} = -\frac{\partial p(t,x)}{\partial x} + f(t,x,v(t,x)),\ t\in[0,\mathrm{T}] \qquad (1)$$

$$-\frac{\partial^2 p(t,x)}{\partial x^2} = \left(\frac{\partial v(t,x)}{\partial x}\right)^2, \qquad x\in[0,\mathrm{X}] \qquad (2)$$

with initial–boundary conditions:

$v(0,x)=\varphi(x), \quad v(t,0)=\psi(t),\ v(t,X)=\mu(t)$, (3)

$p(t, X)=\alpha(t), \qquad \frac{\partial p(t,0)}{\partial x}=\gamma(t)$. (4)

It is assumed, that the functions $f(t, x, v), \varphi(x), \ \psi(t), \mu(t), \ \alpha(t), \ \gamma(t)$ are continuously differentiable with respect to all their arguments and that the following compatibility condition holds:

$$\varphi(0)=\psi(0), \tag{5}$$

with $\psi(t) \geq \Psi=const>0,\ t\in [0,T]$.

It is required to find functions $v(t,x), p(t,x)$, which satisfy the equations (1)-(2) and the initial–boundary conditions (3)-(4).

3. Solving of problem

The boundary-value problem (2), (4) has the solution

$$p(t,x)= \alpha(t)+ \gamma(t)(x-X)+(x-X)\int_0^x \partial_s v(t,s)ds+\int_x^X (s-X)\partial_s v(t,s)ds.$$

Differentiating (1) with respect to x and taking advantage of equality (2), for the function $w(t,x):= \frac{\partial v(t,x)}{\partial x}$ we get the equation

$$\frac{\partial w(t,x)}{\partial t}+v(t,x)\frac{\partial w(t,x)}{\partial x}=-\frac{\partial f(t,x,v(t,x))}{\partial x}+\frac{\partial f(t,x,v(t,x))}{\partial v}w(t,x), \tag{6}$$

where

$$v(t,x)= \mu(t) + \int_x^X w(t,s)ds .$$

From (3) it follows that $w(0,x)=\varphi'(x)$, and we determine the values for $w(t,0)$ and $w(t,X)$ from the equality:

$$\frac{\partial v(t,0)}{\partial t}+v(t,0)w(t,0)=-\frac{\partial p(t,0)}{\partial x}+f(t,0,v(t,0)),$$

$$\frac{\partial v(t,X)}{\partial t}+v(t,X)w(t,X)=-\frac{\partial p(t,X)}{\partial x}+f(t,X,v(t,X)).$$

It yields the following relations:

$$w(t,0)=(1/\psi(t))[f(t,0,\psi(t))-\gamma(t)-\psi'(t),$$

$$w(t,X)=(1/\mu(t))[f(t,X,\mu(t))-\alpha'(t)-\mu'(t)].$$

Denoting the right side of these equations by $\beta(t)$ and $\tau(t)$, we can write down initial– boundary conditions for the equations (6) as

$$w(0,x)=\varphi'(x),\ \ w(t,0)=\beta(t),\ w(t,X)=\tau(t). \tag{7}$$

then compatibility condition of the problem (6)-(7) are

$$\varphi'(0)=\beta(0). \tag{8}$$

The problem (6)-(7) can be reduced to the following system of equations (their equivalence can be proven), i.e.

$$w(t,x)=\beta(z(t,x))+\varphi'(q(z(t,x),t,x)-\varphi'(0)+$$

$$+\int_{z(t,x)}^{t}[\partial_s f(\rho,q(\rho,t,x),\psi(\rho)+\int_{r(t,\rho)}^{x}w(\rho,q(\rho,t,\tau)\exp(-\int_{\rho}^{t}w(\eta,q(\eta,t,\tau))d\eta)d\tau)d\rho+$$

$$+\int_{z(t,x)}^{t}[\partial_v f(\dots)w(\rho,q(\rho,t,x))d\rho, \tag{9}$$

with $(t,x)\in G$, where the domain

$G=\{(t,x):\ 0\le z(t,x)\le t\le T,$

$0\le x\le X+r(t,0)-\int_{r(t,0)}^{x}(1-\exp(-\int_{0}^{t}w(\eta,q(\eta,t,\tau))d\eta))d\tau\};$

$$r(t,s)=\int_{s}^{t}\psi(\tau)\exp(\int_{\tau}^{t}w(\eta,q(\eta,t,r(t,\tau)))d\eta)d\tau, \tag{10}$$

with $(t,x)\in G_0$, where the domain $G_0=\{(s,t):\ 0\le s\le t\le T\};$

$$q(s,t,x)=\int_{r(t,s)}^{x}\exp(-\int_{s}^{t}w(\eta,q(\eta,t,\tau))d\eta)d\tau, \tag{11}$$

with $(s,t,x)\in P$, where the domain

$P=\{(s,t,x):\ z(t,x)\le s\le t\le T,$

$r(t,s) \le x \le X + r(t,0) - \int_{r(t,0)}^{x} (1-\exp(-\int_0^t w(\eta, q(\eta,t,\tau))d\eta))d\tau \};$

$$x = \int_{z(t,x)}^{t} \psi(\tau) \exp(\int_{\tau}^{t} w(\eta, q(\eta, t, r(t,\tau)))d\eta)d\tau, \quad (12)$$

with $(t,x) \in G_1$, where the domain $G_1 = \{(t,x) : 0 \le z(t,x) \le t \le T, \ 0 \le x \le r(t,0)\}$, with the conditions

$$z(t,0)=t, \text{ for } t \in [0,T], \quad (13)$$

$$z(0,x) \equiv 0, \text{ for } (t,x) \in G_2, \quad (14)$$

where the domain $G_2 = \{(t,x) : t \in [0,T],$

$r(t,0) \le x \le X + r(t,0) - \int_{r(t,0)}^{x} (1-\exp(-\int_0^t w(\eta, q(\eta,t,\tau))d\eta))d\tau \}.$

The following theorem can be proven.

Theorem 1. If functions $w(t,x)$, $z(t,x)$, $r(t,x)$, $q(s,t,x)$ satisfy the system of relations (9) - (14), and $w(t,x) \in C^{1,1}(G)$, $z(t,x) \in C^{1,1}(G_1)$, $r(t,s) \in C^{1,1}(G_0)$, $q(s,t,x) \in C^{1,1,1}(P)$ and matching condition (8), then $w(t,x)$ will satisfy differential equation (6) and initial conditions (7).

Conversely, if problem (6) - (7) has a continuously differentiable solution $w(t,x)$, the consistency condition (8) is satisfied, the functions $w(t,x)$, $z(t,x)$, $r(t,s)$, $q(s,t,x)$ satisfy relations (10) - (14), then the function $w(t,x)$ is a solution to the integral equation (9).

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MSC 00 B55

PECULIARITIES OF TRANSLATION OF MATHEMATICAL TEXTS (ENGLISH-KYRGYZ-RUSSIAN)

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Nowadays most of scientific texts in Kyrgyzstan must be written in two or three languages. Hence, two or three such texts are to be equivalent. This is a necessary task for all researchers and teachers of high schools. Various difficulties arising in such process are considered in this paper. A table of differences in denotations is also given.

Keywords: mathematical text, translation, Kyrgyz language, Russian language, English language, difference

Азыркы учурда Кыргызстандагы илимий тексттердин көпчүлүгү эки же үч тилде жазылышы абзел. Демек, эки же үч текст эквиваленттүү болушу зарыл. Бул

баардык изилдөөчүлөр жана жогорку окуу жайлардын мугалимдери үчүн зарыл маселерден болуп саналат. Бул процессте келип чыккан ар кандай кыйынчылыктар бул макалада каралат. Ошондой эле белгилөө айырмачылыктары таблицада берилет.

Урунттуу сөздөр: математикалык текст, котормо, кыргыз тили, орус тили, англис тили, айырмачылык.

В настоящее время многие научные тексты в Кыргызстане должны быть написаны на двух или на трех языках. Следовательно, два или три текста должны быть эквивалентны. Это - задача для всех исследователей и преподавателей вузов. Различные трудности, возникающие в этом процессе, рассмотрены в статье. Также приведена таблица различий в обозначениях.

Ключевые слова: математический текст, перевод, кыргызский язык, русский язык, английский язык, различие.

1. Introduction

Nowadays most of scientific texts (or their parts) in Kyrgyzstan must be written in three languages. Hence, two or three such texts are to be equivalent. This is a necessary task for all researchers and teachers of high schools, not for philologists only. Moreover, a philologist who does not know the subject area cannot prepare an adequate translation.

There are a lot of publications on this subject. For instance, [1] stresses difficulties of translations: «Mathematical texts raise particular dilemmas for the translator. With its arm's-length relation to verbal expression and long-standing "mathematics is written for mathematicians" ethos, mathematics lends it self awkwardly to textually centered analysis. Mathematics has an uneasy relationship with language. Geometry is intrinsically visual rather than verbal, while algebraic texts often use more symbols than words.»

Nevertheless, there were not distinguished peculiarities of translation of mathe-matical texts in very different languages such as English, Kyrgyz and Russian, toge-ther with capacities of Kyrgyz language [2] for presentation of mathematical objects.

Also, we did not find systematical description of differences in denotations.

2. Definition of language and related topics

We proposed

Definition 1 [3]. If low energetic outer influences can cause sufficiently various reactions and changing of the inner state of the object (by means of inner energy of the object or of outer energy entering into object besides of commands) at any time then such (permanently unstable) object is an *affectable object*, or a *subject*, and such outer influences are *commands*. In Kyrgyz: «таасир этилүүчү» объект.

Definition 2 [3]. A system of commands such that any subject can achieve desired efficiently various consequences from other one is a *language*.

These definitions unite human and computer languages.

Hypothesis 1. A human's genuine understanding of a text in a natural language can be clarified by means of observing the human's actions in real life situations corresponding to the text.

These definitions lead to the following definition of one kind of information.

Definition 3. Command information is the quotient set of the set of commands by the following relation of equivalence: two commands yield same actions by an affectable object.

We have used this definition for competitions of translators (see below).

3. Examples of difficulties in translation

A whole list of such differences is too vast, so we give examples.

3.1. Differences in common meaning of words and one in mathematical terminology.

The Russian words «острый, тупой» correspond Kyrgyz ones «курч, мокок» and English ones “sharp”, “blunt” but the expressions «острый угол, тупой угол» are to be translated as «тар бурч», «кең бурч» (literally “narrow angle”, “wide angle”); “acute angle”, “obtuse angle”.

3.2. Different ranges of meanings of words.

The Russian word «угол» and the Kyrgyz one «бурч» correspond to two Eng-lish ones: «corner» and «angle».

«квадрат имеет четыре угла» - «a square has four corners» - «квадрат төрт бурчка ээ»;

«угол в квадрате равен 90^0» - «the angle of the square is 90^0» - «квадраттагы бурч 90^0ка барабар».

The Russian word «решение» corresponds to four Kyrgyz words «чыгаруу, чыгарылыш, чечүү, чечим» and to six English ones: «to solve (solving), solution, to decide, decision, to resolve, resolution».

3.3. Differences in definitions. The Kyrgyz expression «натуралдык сандар» is to be translated into English “natural numbers without zero”, and vice versa the English expression “natural numbers” is to be translated into Kyrgyz «натуралдык сандар жана (мүмкүн) нөл».

3.4. Any language has notions which are absent in other languages. In Kyrgyz language: «иргөө» - “discrete optimization by means of synergetic”;

«сырт(ы)» - “exterior”, «ич(и)» - “exterior” (exists in English);

«үст(ү)» – “upper-space”, «аст(ы)» – “before-and-lower-(observed)-space”, «чек(чеги)» – “boundary-strip”, «сол» – “left-space”, «оң» – “right-space”, «орто(су)» – “middle-spot”, «жан(ы)» – “near-space”, «ара(сы)» – “between-space”, «алд(ы)» – “before-forward-space”, «арт(ы)» – “behind-space”, «каршы
(сы)» – “opposite-space”.

For instance, «куш үстөлдүн үстүнөн учуп кетти» can be translated “the bird flew out of the upper-space of the table” only.

«предел(и)» is a mathematical term in Kyrgyz. The Russian «предел» and the English “limit” are incorrect extensions of a common Latin word “limit” (boundary).

In English: “upload” means « Интернет аркылуу компүтерге жүктөө».

3.5. Any language has specific expressions. For instance the phrase “the room was nine feet in square” is usually understood and translated as

“the room was nine (foot in square)” = nine square feet?

while the genuine sense is

“the room was (nine feet) in square”= nine feet × nine feet.

3.6. While translating the aim is not the term but the meaning. For instance, translation of the (feeble, but standard) Russian «датчик» into Kyrgyz is «сезгич» (from Latin “sensor”). Translation of Russian «пятиконечная звезда» into Kyrgyz is “беш учтуу жылдыз”.

Remark. While investigations new notions can arise. They need to be named in all three languages. A new relation was detected in [4]: two points on different sheets of a Riemann surface have same coordinates.

4. Table of differences in denotations

Notion	English	Kyrgyz, Russian
	natural numbers 1, 2, 3… or 0, 1, 2, 3…	натуралдык сандар натуральные числа 1, 2, 3…
Large natural numbers	*2,020 2020* *10,000,000* *10000000*	*2020* *10000000*
Decimal fractions	*5.2 (decimal point)*	*5,2 (үтүр, позиционная запятая)*
n!/(k!(n–k)!)	$C(n,k)$, $\binom{n}{k}$	C_n^k
n!/(n–k)!	$P(n,k)$	A_n^k
Division of numbers	÷ /	: /
Tangent of *x*	*tan*	*tg*
Cotangent of *x*	*cot*	*ctg*
Hyperbolic sine of *x*	*sinh*	*sh*
Hyperbolic cosine of *x*	*cosh*	*ch*
Hyperbolic tangent of *x*	*tanh*	*th*
Hyperbolic cotangent of *x*	*coth*	*cth*
Inverse sine	sin^{-1}	*arcsin*
Inverse cosine	cos^{-1}	*arccos*
Inverse tangent	tan^{-1}	*arctg*
Inverse cotangent	cot^{-1}	*arcctg*
Inverse hyperbolic sine	*arsinh*, $sinh^{-1}$	*arsh*
Inverse hyperbolic cosine	*arcosh*, $cosh^{-1}$	*arch*

Inverse hyperbolic tangent	*artanh, tanh*$^{-1}$	*arth*
Inverse hyperbolic cotangent	*arcoth, coth*$^{-1}$	*arcth*
Curl of vector	*curl*	*rot*
Scalar product of vectors	*has many different denotations in all languages:* ab; $a \cdot b$; (a,b); $\langle a,b \rangle$ ...	
Vector product of vectors	*has various denotations in all languages:* $a \times b$; $[a,b]$ ...	

5. Translators' competition with objective quality estimation [5]

See Definition 3.

A team consists of three persons.

1st step. The first teammate ("watcher") is shown a simple drawing and must describe it (what s/he see) in English. If all English words are correct (their correlation is not checked by the computer) then this text is sent to the second class to the second teammate.

2nd step. The second teammate ("translator") is shown this English text and translates it into Kyrgyz. If all Kyrgyz words are correct then the Kyrgyz text is sent to the third class to the third teammate.

3rd step. The third teammate ("painter") draws by then the Kyrgyz text.

If his/her drawing coincides with the initial one then the translation is adequate.

6. Conclusion

We hope that this paper would draw attention to difficulties and improve quality of translations of mathematical texts by our colleagues.

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MSC 49M37

DETERMINATION OF THE OPTIMAL VOLUME OF LIVESTOCK PRODUCTION BY THE CRITERION OF MAXIMUM INCOME

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The paper develops a mathematical model and algorithm for solving the problem of choosing the optimal breed of animals without restrictions on the volume of production according to the criterion of maximum net income. The performance of the model is shown in a numerical example.

Keywords: mathematical model, acreage, production, net income, agriculture, livestock, consumption, solution algorithm.

Бул жумушта өндүрүлүчү мал чарба азыктарынын көлөмү чектелбеген учурда малдын өндүрүмдүүлүк асылдуулугунун оптималдуу тандоо маселесине

математикалык модель жана чыгаруу алгоритмасы иштелип чыккан. Математикалык моделдин иштемдүүлүгү сандык мисал менен көрсөтүлгөн.
Урунттуу сөздөр: математикалык модель, айдоо аянты, өндүрүш, түшүмдүлүк, чыгым, киреше, чарба.

В работе разработана математическая модель и алгоритм решения задачи выбора оптимальной породы животных без ограничения на объемы производства продукции животноводства. Работоспособность модели показаны на числовом примере.
Ключевые слова: Математическая модель, посевная площадь, производство, урожайность, расход, доход, хозяйство.

Problem statement. Let a livestock farm that has sufficient financial resources and acreage of various categories (irrigated, rainfed, etc.) in the amount of s_k, $k \in K$ planned to get the maximum income from production by updating his farm with more productive breeds of cattle. It is assumed that for each type of cattle breed, its productivity and the corresponding daily feeding ration are known. In addition, the yield of feed crops for each category of sown area is known.

It is required to determine the optimal composition of productive animals to ensure the maximum net income for the farm from the production and sale of livestock products. The mathematical model of the problem is represented as.

Find the maximum

$$L(x,y,z) = \sum_{h \in H} a^h z^h - \Big(\sum_{k \in K} \sum_{j \in J_0} c_{kj} x_{kj} + \sum_{h \in H} \sum_{l \in L} c_l^h y_l^h\Big) \qquad (1)$$

under conditions

$$\sum_{j \in J_0} x_{kj} \le s_k, \quad k \in K, \qquad (2)$$

$$\sum_{k \in K} a_{kj} x_{kj} = \sum_{h \in H} \sum_{l \in L} \alpha_{jl}^h y_l^h, \quad j \in J_0, \qquad (3)$$

$$\sum_{l\in L}\theta_l^h y_l^h = z^h, \quad h\in H, \qquad (4)$$

$$x_{kj} \geq 0, \quad k\in K, \ j\in J_0, \qquad (5)$$

$$z^h \geq 0, \quad h\in H, \qquad (6)$$

$$y_l^h \geq 0, \quad l\in L, \quad h\in H \text{ - is an integer }, \qquad (7)$$

where $z = \{ z^h: h\in H\}$, $x = \{ x_{kj}: k\in K, \ j\in J_0\}$, $y = \{ y_l^h : h\in H, \ l\in L$ - is an integer},

j – is an index of the type of agricultural crop production used in the daily diet of animal feeding, $j\in J_0$;

J_0 – is population of types of crop production aimed at animal feed, $J_0=\{1,2,\dots,n\}$;

k – is an index of the type category of acreage in the farm, $k\in K$;

K – is population of types of acreage categories, $K=\{1,2,\dots,p\}$;

h – is an index of the type of livestock products produced on the farm, $h\in H$;

H – is population of the types of products animal husbandry, $H=\{1,2,\dots,\bar{H}\}$;

l – is an index of the type of animal breed in the farm, $l\in L$;

L– is population of the types of animal breeds, $L=\{1,2,\dots,\bar{L}\}$;

The known parameters are:

s_k – is the size of cultivated area of k category in the economy, $k\in K$;

a_{kj} – yielding capacity of j type of cropper on k category of farm acreage, $k\in K$, $j\in J_0$;

α_{jl}^h – annual demand for j-th type of crop production per animal of l-th breed in the production of h-type of product, where

$$\alpha_{jl}^{h} = \beta_{jl}^{h}\gamma_{jl}^{h}, \quad j \in J_0, \quad l \in L, \quad h \in H; \tag{8}$$

β_{jl}^{h} –is a fraction of j-th crop production in the daily diet per animal of l-breed on the farm for production of h-type of product, $j \in J_0$, $l \in L$, $h \in H$;

γ_{jl}^{h} – is the number of days in the diet of feeding crop products of the j-th type for the l–th breed of animal in the production of the h-th type of product, $j \in J_0$, $l \in L$, $h \in H$;

θ_{l}^{h} – is the the volume of production of the h -th type received by the farm from one animal of the l-th breed, $l \in L$, $h \in H$;

a^h – is the sale price of the h-th type of livestock products on the farme, $h \in H$;

c_{kj} – is the costs per unit size of the k -th category of sown area for the j-th type of crop, $j \in J_0$, $k \in K$;

c_{l}^{h} – is the annual expenditure per animal of the l-th breed in the production of the h – th type of animal products, $h \in H$, $l \in L$;

Unknown variables are:

x_{kj} – the size of the k-th category of sown area allocated for the j-th type of crop, $j \in J_0$, $k \in K$;

y_{l}^{h} – number of animals of the l-th breed in the farm for the production of the h-th type of product, $h \in H$, $l \in L$;

z^h – volume of h - type livestock products sold, $h \in H$.

Objective function (1) determines the maximum net income received by the farm from the production and sale of livestock products;

Assumption (2) requires that the total size of the sown area of the farm allocated for forage crops for each category should not be greater than the size of the sown area of this category;

Assumption (3) shows that the volume of agricultural products produced for each type of feed must be equal to the volume of the farm's needs for internal needs (for feed);

Assumption (4) requires that the volume of livestock products produced for each type must be sold;

Assumption s (5), (6) require non-negativity of variables;

Assumption (7) requires that the value of variables must be an integer. The Mathematical Model (1)-(7) can be presented in the form of Table 1.

Table 1

Record of the problem condition (1)-(7) as a Table

x_{11}	x_{12}	…	x_{1n}	x_{21}	x_{22}	…	x_{2n}	…	x_{p1}	x_{p2}	…	x_{pn}	z^1	z^2	…	z^h
1	1	…	1													
				1	1	…	1									
								…								
									1	1	…	1				
a_{11}				a_{21}					a_{p1}							
	a_{12}				a_{22}					a_{p2}						
		…				…		…			…					
			a_{1n}				a_{2n}					a_{pn}				
													-1			
														-1		
															…	
																-1
c_{11}	c_{12}	…	c_{1n}	c_{21}	c_{22}	…	c_{2n}	…	c_{p1}	c_{p2}	…	c_{pn}	a^1	a^2	…	a^h

y_1^1	y_2^1	$\dots$	y_l^1	y_1^2	y_2^2	$\dots$	y_l^2	$\dots$	y_1^h	y_2^h	$\dots$	y_l^h		
													$\le$	S_1
													$\le$	S_2
													$\dots$	$\dots$
													$\le$	S_p
$-\alpha_{11}^1$	$-\alpha_{12}^1$	$\dots$	$-\alpha_{1l}^1$	$-\alpha_{11}^2$	$-\alpha_{12}^2$	$\dots$	$-\alpha_{1l}^2$	$\dots$	$-\alpha_{11}^h$	$-\alpha_{12}^h$	$\dots$	$-\alpha_{1l}^h$	=	0
$-\alpha_{21}^1$	$-\alpha_{22}^1$	$\dots$	$-\alpha_{2l}^1$	$-\alpha_{21}^2$	$-\alpha_{22}^2$	$\dots$	$-\alpha_{2l}^2$	$\dots$	$-\alpha_{21}^h$	$-\alpha_{22}^h$	$\dots$	$-\alpha_{2l}^h$	=	0
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$-\alpha_{n1}^1$	$-\alpha_{n2}^1$	$\dots$	$-\alpha_{nl}^1$	$-\alpha_{n1}^2$	$-\alpha_{n2}^2$	$\dots$	$-\alpha_{nl}^2$	$\dots$	$-\alpha_{n1}^h$	$-\alpha_{n2}^h$	$\dots$	$-\alpha_{nl}^h$	=	0
θ_1^1	θ_2^1	$\dots$	θ_l^1										=	0
				θ_1^2	θ_2^2	$\dots$	θ_l^2						=	0
								$\dots$					$\dots$	$\dots$
									θ_1^h	θ_2^h	$\dots$	θ_l^h	=	0
c_1^1	c_2^1	$\dots$	c_l^1	c_1^2	c_2^2	$\dots$	c_l^2	$\dots$	c_1^h	c_2^h	$\dots$	c_l^h	$\to$	max

Solution Algorithm. The algorithm for solving problem (1)-(7) differs from the algorithm described in [1] with minor changes as in [1], calculations begin with calculating the values of the parameters α_{jl}^h, $j \in J_0$, $l \in L$, $h \in H$ by the formula (8).

Using the values of the parameters a_{kj}, c_{kj}, s_k, $k \in K$, $j \in J_0$ и θ_l^h, c_l^h, a^h, $h \in H$, $l \in L$, a numerical model of the problem is formulated according to (1)-(7).

From the solution of the problem, the quantitative composition of cattle in the farm is determined $y=\{ y_l^h :, h \in H, l \in L\}$ and the corresponding size of the acreage for feed crops $x=\{ x_{kj}: k \in K, j \in J_0\}$, as well as the volume of livestock products sold $z=\{ z^h: h \in H\}$, which allows to ensure the maximum net income for the farm. The Solution Algorithm ends.

Let's check the performance of the mathematical model and the algorithm for solving the problem using a numerical example.

Example. Let the main activity of the farm is the production of livestock products: milk and beef meat. The farm knows two breeds of dairy cattle and two breeds of meat cows. At the same time, the farm has acreage of S = 366 the farm has acreage of S_1 = 280 hectares are irrigated, and S_2 = 86 hectares are rainfed.

We believe that the first type of breed is a cash cow with a milk yield of 3600 kg of milk and a corresponding feeding ration (Table 2), and the second type of breed - a cash cow with a yield of 4500 kg of milk and a feeding ration (Table 3). Similarly, the first and second types of breeds for the production of meat are bulls (heifers) with a live feeding diet weighing 300 kg and 450 kg, respectively (meat Tables 4 and 5).

Table 2

Daily feeding ration dairy cows of the first type of breed with a milk yield of 3600 kg of milk

	Feed name	Daily ration kg (1 animal)	Fodder unit	Common unit	Total for 1 animal	Number of days
1	Medick (dry fodder)	4	0,5	2,0	720,0	180

2	Chaff	Wheat	3	1	0,2	0,6	180.0	180
		Barley		2			360.0	
3	Haylage		6		0,3	1,8	1080,0	180
4	The concentration of the feed	Wheat	2,4	0,3	1	2,4	109,5	365
		Barley		1,5			547,5	
		Seed		0,6			219,0	
5	Silage (Corn)		10		0,3	3,0	1800,0	180
6	Mineral feed		0,010		-	-	-	365
7	Salt		0,030		-	-	-	365
8	Pasture Forage		40		-	-	7200,0	180
	Green Fodder							
	Total		-		-	9,8	-	-

Table 3

Daily feeding ration for dairy cows of the second type of breed with a milk yield of 4500 kg of milk

	The name of the Fodder		**Daily ration (1 animal)**		**Number of days**	**For a year (1 animal)**
1.	Medick (dry fodder)		10 kg		180	1800 kg
2.	Chaff	Wheat	3 kg	1 kg	180	180 kg
		Barley		2 kg		360 kg
3.	Haylage		8 kg		180	1440 kg
4.	The concentration of the feed	Wheat	3 kg	2 kg	365	730 kg
		Barley		0,5 kg		182,5 kg
		Seed (Corn)		0,5 kg		182,5 kg
5.	Silage (Corn)		12 kg		180	2160 kg
6.	Mineral feed		-		365	3,6 kg
7.	Salt		-		365	10,8 kg
8.	Pasture Forage		50 kg		180	9000 kg
	Green Fodder					

Table 4

Daily feeding ration for bulls and heifers of the first type with a live weight of 300 kg

<table>
<tr><th></th><th colspan="2">The name of the Fodder</th><th colspan="2">Daily ration (1 animal)</th><th>Fodder unit</th><th>Comm on unit</th><th>Total for 1 animal y</th><th>Number of days</th></tr>
<tr><td>1.</td><td colspan="2">Medick (dry fodder)</td><td colspan="2">3</td><td>0,5</td><td>1,5</td><td>540,0</td><td>180</td></tr>
<tr><td rowspan="2">2.</td><td rowspan="2">Chaff</td><td>Wheat</td><td rowspan="2">2</td><td>0,5</td><td rowspan="2">0,2</td><td rowspan="2">0,4</td><td>90,0</td><td rowspan="2">180</td></tr>
<tr><td>Barley</td><td>1,5</td><td>270,0</td></tr>
<tr><td>3.</td><td colspan="2">Hayalage</td><td colspan="2">6</td><td>0,3</td><td>1,8</td><td>1080,0</td><td>180</td></tr>
<tr><td rowspan="3">4.</td><td rowspan="3">The concent ration of the feed</td><td>Wheat</td><td rowspan="3">1,5</td><td>0,5</td><td rowspan="3">1</td><td rowspan="3">1,5</td><td>182,5</td><td rowspan="3">365</td></tr>
<tr><td>Barley</td><td>0,5</td><td>182,5</td></tr>
<tr><td>Seed (Corn)</td><td>0,5</td><td>182,5</td></tr>
<tr><td>5.</td><td colspan="2">Silage (Corn)</td><td colspan="2">5,0</td><td>0,3</td><td>1,5</td><td>900,0</td><td>180</td></tr>
<tr><td>6.</td><td colspan="2">Mineral feed</td><td colspan="2">0,010</td><td>-</td><td>-</td><td>-</td><td>365</td></tr>
<tr><td>7.</td><td colspan="2">Salt</td><td colspan="2">0,030</td><td>-</td><td>-</td><td>-</td><td>365</td></tr>
<tr><td rowspan="2">8.</td><td colspan="2">Pasture Forage</td><td colspan="2" rowspan="2">30</td><td rowspan="2">-</td><td rowspan="2">-</td><td rowspan="2">5400,0</td><td rowspan="2">180</td></tr>
<tr><td colspan="2">Green Fodder</td></tr>
<tr><td></td><td colspan="2">Total</td><td colspan="2">-</td><td>-</td><td>6,7</td><td>-</td><td>-</td></tr>
</table>

Table 5

Daily ration of feeding bulls and heifers of the second type for meat with a live weight of 450 kg

<table>
<tr><th></th><th colspan="2">The name of the Fodder</th><th colspan="2">Daily ration (1 animal)</th><th>Number of days</th><th>For a year (1 animal)</th></tr>
<tr><td>1.</td><td colspan="2">Medick (dry fodder)</td><td colspan="2">5 kg</td><td>365</td><td>1825 kg</td></tr>
<tr><td>2.</td><td colspan="2">Chaff</td><td colspan="2">1 kg</td><td>365</td><td>365 kg</td></tr>
<tr><td>3.</td><td colspan="2">Hayalage</td><td colspan="2">10 kg кг</td><td>180</td><td>1800 kg</td></tr>
<tr><td rowspan="3">4.</td><td rowspan="3">The concentration of the feed</td><td>Barley</td><td rowspan="3">3 kg</td><td>1 kg</td><td rowspan="3">365</td><td>365 kg</td></tr>
<tr><td>Seed (Corn)</td><td>1 kg</td><td>365 kg</td></tr>
<tr><td>Barley</td><td>1 kg</td><td>365 kg</td></tr>
<tr><td>5.</td><td colspan="2">Silage (Corn)</td><td colspan="2">10 kg</td><td>180</td><td>1800 kg</td></tr>
<tr><td>6.</td><td colspan="2">Mineral feed</td><td colspan="2">-</td><td>365</td><td>-</td></tr>
</table>

7.	Salt	-	365	-
8.	Pasture Forage	30 kg	180	5400 kg
	Green Fodder			

In addition, the following are known: - sales price per unit volume of milk a^1 =25 soms, meat a^2 = 140 soms;

- crop yields in irrigated fields (I) and rainfed fields (II) included in the feeding ration,, a_{kj}, k=1,2, j=1,2,…,7, Table 6;

Table 6

	Wheat	Barley	Medick (dry fodder)	Hayalage	Green Fodder	Silage (Corn)	Seed (Corn)
	1	2	3	4	5	6	7
I	2070.0	1962.2	2380.0	6281.0	5730.0	12340.0	20280.0
II	1500.0	0	1700.0	0	0	0	0

- these are expenses for growing agricultural crops per unit of size (I) and (II) fields, $|c_{kj}|_{2,7}$, Table 7.

Table 7

	1	2	3	4	5	6	7
I	2279.0	1096.0	2618.0	5071.0	9225.0	13574.0	7743.0
II	2000.0	1000.0	2000.0	5000.0	9000.0	13000.0	7000.0

In addition, the consumption for the maintenance of one cattle of each breed in the production of milk and meat, respectively, is known $c_{l=1}^{h=1}$ =50040.0 soms, $c_{l=2}^{h=1}$ =55080.0 soms, $c_{l=1}^{h=2}$ =18030.0 soms, $c_{l=2}^{h=2}$ =25020.0 soms.

It is required to determine the optimal quantitative composition of cattle, which allows to ensure the maximum net income to the farm from production and sale of products.

For Mathematical Formalization of the problem we will determine the annual need for feed per animal of each type of breed in the production of products α_{jl}^{h}, $j \in J_0, l \in L, h \in H.$

For Mathematical Formalization of the task will determine the annual requirement of feed for one animal of each breed in production using the daily feeding ration, determine the annual demand of each type of agricultural products included in the composition of the feed for one dairy cow with milk yield of 3600 kg of milk and one cow with milk yield of 4500 kg. We will also determine the annual demand of each type of agricultural products for feed for one bull and heifer of the first and second types of breed for meat (refer to Table 8).

Table 8

Annual feed requirement per animal depending on breed and productivity

The name of the Fodder	Consumption of feed for one dairy cow		Feed requirement per cow for meat	
	1 type of breed with a yield of 3600 kg	2 types of breed with a yield of 3600 kg	1 type of breed with a live weight of 300 kg	2 types of breed with a live weight of 450 kg
1. Wheat	289,5	362,5	272,5	365,0
2. Barley	907,5	1090,0	452,5	730,0
3. perennial grass 3.1. Medick (dry fodder)	720,0	1800,0	540,0	1825,0
3.2. Hayalage	1080,0	1440,0	1080,0	1800,0
3.3. Green Fodder	7200,0	9000,0	5400,0	5400,0
4. Corn 4.1. Silage	1800,0	2160,0	900,0	1800,0
4.2. Seed	219,0	182,5	182,5	365,0

We formulate a numerical model of the problem.

To find the minimum of

$L(x,y,z)=25z^1 +140z^2 – (2279.0x_{11}+1096.0x_{12}+2618.0x_{13}+5071.0x_{14}+$

$+9225.0x_{15}+13574.0x_{16}+7743.0x_{17}+2000.0x_{21}+1000.0x_{22}+2000.0x_{23}+$

$+5000.0x_{24}+9000.0x_{25}+ +13000.0x_{26}+7000.0x_{27}+50040.0\, y_1^1+55080.0\, y_2^1$

$+18030.0\, y_1^2 +25020.0\, y_2^2\,)$ (9)

under conditions

$$\sum_{j=1}^{7} x_{1j} \le 280, \qquad \sum_{j=1}^{7} x_{2j} \le 86, \qquad (10)$$

$2070,0x_{11}+1500,0x_{21}=289,5\, y_1^1+362,5\, y_2^1+272,5\, y_1^2 +365,0\, y_2^2$,

$1962,0x_{12}+0x_{22}=907,5\, y_1^1+1090,0\, y_2^1+452,5\, y_1^2 +730,0\, y_2^2$,

$2380,0x_{13}+1700,0x_{23}=720,0\, y_1^1+1800,0\, y_2^1+540,0\, y_1^2 +1825,0\, y_2^2$,

$6281,0x_{14}+0x_{24}=1080,0\, y_1^1+1440,0\, y_2^1+1080,0\, y_1^2 +1800,0\, y_2^2$,

$5730,0x_{15}+0x_{25}=7200,0\, y_1^1+9000,0\, y_2^1+5400,0\, y_1^2 +5400,0\, y_2^2$,

$12340,0x_{16}+0x_{26}=1800,0\, y_1^1+2160,0\, y_2^1+900,0\, y_1^2 +1800,0\, y_2^2$,

$20280,0x_{17}+0x_{27}=219,0\, y_1^1+182,5\, y_2^1+182,5\, y_1^2 +265,0\, y_2^2$, (11)

$3600\, y_1^1+4500\, y_2^1= z^1, \qquad 300\, y_1^2 +450\, y_2^2 = z^2,$ (12)

$x_{kj}\ge 0, \quad k=1,2, \quad j=1,2,\dots,7,$ (13)

$z^h \ge 0, \; h=1,2,$ (14)

$y_l^h \ge 0, \quad l=1,2, \quad h=1,2$ - целое. - is an Integer (15)

Let's write the numerical model of the problem (9) - (15) in the form of Table 9.

Table 9

Representation of the problem condition (9)-(15) as a Table

x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{21}	x_{22}	x_{23}	x_{24}
1	1	1	1	1	1	1				
							1	1	1	1
2070.0							1500.0			
	1962.0							0		
		2380.0							1700.0	
			6281.0							0
				5730.0						

					12340.0					
						20280.0				
-2279.0	-1096.0	-2618.0	-5071.0	-9225.0	-13574.0	-7743.0	-2000.0	-1000.0	-2000.0	-5000.0

Continuation of Table 9

x_{25}	x_{26}	x_{27}	y_1^1	y_2^1	y_1^2	y_2^2	z^1	z^2	0	
									$\le$	S_1
1	1	1							$\le$	S_2
			-289,5	-362,5	-272,5	-365,0			=	0
			-907,5	-1090,0	-452,5	-730,0			=	0
			-720,0	-1800,0	-540,0	-1825,0			=	0
			-1080,0	-1440,0	-1080,0	-1800,0			=	0
0			-7200,0	-9000,0	-5400,0	-5400,0			=	0
	0		-1800,0	-2160,0	-900,0	-1800,0			=	0
		0	-219,0	-182,5	-182,5	-265,0			=	0
			3600,0	4500,0			*-1*		=	0
					300,0	450,0		*-1*	=	0
-9000,0	-13000,0	-7000,0	-50040,0	-55080,0	-18030,0	-25020,0	25,0	140,0	$\rightarrow$	max

Let's solve problem (9)-(15) using the MS EXCEL spreadsheet [2].

Get the optimal plan for the distribution of acreage for forage crops (refer tp Appendix)

x={ $x_{11}=0{,}83$; $x_{12}=54{,}81$; $x_{13}= 28{,}74$; $x_{14}=22{,}58$; $x_{15}=154{,}87$; $x_{16}=17{,}27$; $x_{17}=0{,}89$, $x_{21}=22{,}69$, $x_{22}=0$, $x_{23}=63{,}31$, $x_{24}=0$, $x_{25}=0$, $x_{26}=0$, $x_{27}=0$}, composition of cattle breeds in the dairy and meat sector

$$y=\{\ y_1^1=2;\quad y_2^1=97;\quad y_1^2=0;\quad y_2^2=0\ \}$$

and the volume of products sold by the farm z={ z^1 =443,7 т.}, as well as the size of the household's net income *L(x,y,*z*)* =4356053.0 soms.

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OPEN THREE-SECTOR MODEL

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The article offers a macro-economic open three-sector model. The main task of developing a three-sector model is to maximize the volume of consumption. The criterion of maximum specific consumption is used as a criterion for the dynamic problem.

As a result of the study, based on the open three-sector model of the economy and using the Pontryagin maximum principle, the optimal dynamic rule for the distribution of labor and investment resources between the sectors of the open three-sector economy was found.

Keywords: open three-sector model, maximization of consumption, distribution of labor and investment resources.

Бул статьяда ачык үч секторлуу макроэкономикалык модель снушталат. Үч сеторлуу моделди түзүүнүн башкы маселеси болуп керектөөнүн көлөмүн максималдаштыруу болуп эсептелет. Динамикалык маселенин критерийи катары керекөөнүн үлүшүнүн максимуму кабыл алынат.

Ачык үч секторлуу моделдин негизинде жүргүзүлгөн изилдөөнүн жыйынтыгы жана Понтрягиндин максимум принцибинин жардамы менен эмгек жана инвестициялык ресурстардын ачык үч сектордун арасында бөлүштүрүүнүн оптималдуу динамикалык эрежеси табылды.

Урунттуу сөздөр: ачык үч секторлуу модели, керектөөнүн көлөмүн максималдаштыруу, эмгек жана инветициялык ресурстарды бөлүштүрүү.

В статье предлагается макроэкономическая открытая трехсекторная модель. Основная задача разработки трехсекторной модели является максимизация объема потребления. В качестве критерия динамической задачи принят критерий максимума удельного потребления.

В результате проведенного исследования на основе открытой трехсекторной модели экономики и с помощью принципа максимума Понтрягина было найдено оптимальное динамическое правило распределения трудовых и инвестиционных ресурсов между секторами открытой трехсекторной экономики.

Ключевые слова: открытая трехсекторная модель, максимизация объема потребления, распределение трудовых и инвестиционных ресурсов.

Currently, the economy of the Kyrgyz Republic (KR) still has a number of structural imbalances in the economic sector due to the fact that the level of consumption has been declining for many years, while investment has continued to grow. Without structural changes, the KR economy will gradually lose momentum due to insufficient domestic demand. Insufficient domestic consumer demand is mainly due to an unbalanced distribution of income.

These challenges are closely linked to a number of institutional problems, among many others. Currently, the government is promoting reform of the organizational structure to resolve these problems.

Kyrgyzstan does not have sufficient energy resources and industry to drive economic growth. Because of this, after gaining independence, Kyrgyzstan has not been able to achieve stable economic growth and is the second-poorest country in the CIS, after Tajikistan.

According to preliminary data from the National Bank of the Kyrgyz Republic, by the end of this year (2020), due to well-known events (the coronavirus pandemic, October political events and the scale of corruption processes), economic growth will decrease by 5.9%.

The problems that have arisen in the country require specialists to develop new methods and methodology for analyzing the country's economy. The development of a new macroeconomic cross-industry model should take into account the problem of the domestic market's availability of domestic goods (especially the needs of the food market), as well as the ratio of imports and exports.

At present, we can conclude that due to the acute confrontation between various political forces, reaching a compromise and implementing the agreed option seems hopeless, and the "shock" option is the only possible and feasible one.

To solve the accumulated problems, you can use a subset of the open three-sector model. The open three-sector model is presented in relative terms. For construction, we use the following notation:

$\nu -$ is the growth rate of the number of employees (assuming its constancy, it follows, as in the Solow model, that $L = L(t) = L(0) \cdot e^{\nu t}$);

$\theta_i = \frac{L_i}{L} -$ share of the i −th sector in the distribution of labor resources;

$s_i -$ share of the i −th sector in the distribution of investment resources;

$k_i = \frac{K_i}{K}$ – stock-to-weight ratio of the i −th sector;

$\frac{X_i}{L_i} = F_i(k_i, 1)$ – industry productivity of the i −th sector;

$y_2 = \frac{Y_2}{L}$ – specific import of consumer goods;

γ_1 – quota coefficient for import of investment goods;

z_0 – specific export of materials;

$q_0 -$ is the world price of the exported materials;

$q_1^+, q_2^+ -$ world prices of imported investment and consumer goods;

$a_1 -$ direct costs of materials per unit of output of the $i -$ th sector;

μ – physical capital depreciation rate (same for all sectors);

λ=μ+ ν – coefficient of reduction of capital stock due to depreciation of physical capital and growth in the number of employees.

Specific weight of sectors

$x_i -$ labor productivity of the i −th sector.

$$x_i = \frac{X_i}{L} = \theta_i f_i(k_i), i = 0,1,2, \quad (1)$$

Differential equations for the stock-to-weight ratio of sectors

$$\frac{dk_i}{dt} = -\lambda k_i + \frac{s_i}{\theta_i}(x_1 + y_1), k_i(0) = k_i^0, \quad i = 0,1,2. \quad (2)$$

Natural balances

- labour

$$\theta_0 + \theta_1 + \theta_2 = 1, 0 \le \theta_i < 1; \quad (3)$$

- investment

$$s_0 + s_1 + s_2 = 1, 0 \le s_i < 1; \quad (4)$$

- material

$$(1 - \alpha_0)x_0 = \alpha_1 x_1 + \alpha_2 x_2 + z_0. \quad (5)$$

Trade balance

$$q_0 z_0 = q_1^+ y_1 + q_2^+ y_2. \quad (6)$$

Industrial balance

$$y_0 \le \gamma_1 x_1. \quad (7)$$

Thus, the reduced subset of the open three-sector model differs from the full model in the following aspects:

1) out of the three foreign trade balances of the sectors, the most important balance (6) of the material sector was selected;
2) omitted internal cost balances of sectors;
3) the specific import of consumer goods is considered fixed, so the consumer security condition is omitted.

The simplification of the model is caused by difficulties in solving the full problem of optimal balanced growth of an open three-sector economy. As these difficulties are overcome, it is hoped that the full task will be solved.

Optimal balanced growth refers to the growth in the capital-to-capital ratio of all sectors, balanced in terms of labor, investment, and material resources, and optimal in terms of the "maximum discounted specific consumption" criterion.

The main task of developing a three-sector model is to maximize the volume of consumption. As a criterion for the dynamic problem, we take the criterion of the maximum specific consumption:

$$\delta \int_{0}^{\infty} e^{-\delta t} c(t) dt \rightarrow max, \qquad (8)$$

since $c(t) = x_2(t) + y_2$, then for $y_2 - const$, the problem reduces to maximizing the specific output of the consumer sector:

$$\delta \int_{0}^{\infty} e^{-\delta t} x_2(t) dt \rightarrow max, \qquad (9)$$

This problem can be solved using the Pontryagin principle [Pontryagin et al. (1969)]. The phase variables are the stock-weight ratio of sectors k_0, k_1, k_2, and the equations of motion are equations (2) for the stock-weight ratio of sectors.

The transition economy is characterized by the following features in the distribution of labor and investment resources between sectors:

- insufficient resource provision of the Fund-creating sector, which is partially compensated by equal shares of labor and investment resources;
- oversupply of labor resources in the consumer sector due to more favorable working conditions compared to the sectors that produce the means of production;
- lack of labor resources in the material sector, which is compensated by greater capital intensity (consequently, large capital investments are required).

Formally, these features are reflected in the following relations:

$$\begin{cases} s_1^0 < s_0^0,\ s_1^0 < s_2^0,\ s_0^0 > s_2^0, \\ \theta_1^0 < \theta_0^0,\ \theta_1^0 < \theta_2^0,\ \theta_0^0 < \theta_2^0, \\ s_1^0 \approx \theta_1^0 \end{cases} \tag{10}$$

This initial allocation of resources differs significantly from the stationary one and will shift in its direction as a result of implementing the optimal rule for the dynamic problem.

We use the Cobb-Douglas production function to solve the stationary problem for the stock-building capacity of the Fund-creating sector

$$F_i(K_i, L_i) = A_i K_i^{\alpha_i} L_i^{1-\alpha_i}, \qquad i = 0,1,2,$$

then

$$f_i(k_i) = A_i k_i^{\alpha_i}, \qquad i = 0,1,2.$$

Considering that the stationary stock-to-weight ratio of sectors

$$k_i^E = \left(\frac{(1+\gamma_1)A_1}{\lambda}\right)^{\frac{1}{1-\alpha_i}} \frac{s_i}{\theta_i} \theta_1 s_1^{\frac{1}{1-\alpha_i}}, i = 0,1,2.$$

fixed specific issues of the sector are as follows:

$$\begin{cases} X_0 = \theta_0 f_0(k_0^E) = B_0 s_0^{\alpha_0} \theta_0^{1-\alpha_0} \theta_1^{\alpha_0} s_1^{\frac{\alpha_1}{1-\alpha_1}}, \\ X_1 = \theta_1 f_1(k_1^E) = B_1 \theta_1 s_1^{\frac{\alpha_0\alpha_1}{1-\alpha_1}} \\ X_2 = \theta_2 f_2(k_2^E) = B_2 s_2^{\alpha_2} \theta_2^{1-\alpha_2} \theta_1^{\alpha_2} s_1^{\frac{\alpha_2\alpha_1}{1-\alpha_1}} \end{cases} \tag{11}$$

where

$$B_i = A_i \left(\frac{(1+\gamma_i)}{\lambda}\right)^{\frac{\alpha_i}{1-\alpha_i}}, \quad i = 0,1,2. \tag{12}$$

Thus, in the case of Cobb – Douglas functions, the stationary problem has the following form:

$$B_2 s_2^{\alpha_2} \theta_2^{1-\alpha_2} \theta_1^{\alpha_2} s_1^{\frac{\alpha_2\alpha_1}{1-\alpha_1}} \to max, \tag{13}$$

$$\theta_0 + \theta_1 + \theta_2 = 1, \qquad \theta_i \geq 0, \tag{14}$$

$$s_0 + s_1 + s_2 = 1,\ s_i \geq 0, \qquad (15)$$

$$(1-\alpha_0)B_0 s_0^{\alpha_0}\theta_2^{1-\alpha_0}\theta_1^{\alpha_0}s_1^{\frac{\alpha_0\alpha_1}{1-\alpha_1}} =$$

$$(a_1+b_1)B_1\theta_1 s_1^{\frac{\alpha_1}{1-\alpha_1}} + a_2 B_2 s_2^{\alpha_2}\theta_2^{1-\alpha_2}\theta_1^{\alpha_2}s_1^{\frac{\alpha_2\alpha_1}{1-\alpha_1}} + z_{02}^0 \quad (16)$$

The problem formed in this way is a classical problem for a conditional extremum. Labor and investment balances (14), (15) will be taken into account directly ($\theta_1, \theta_2, s_1, s_2 -$ are variable variables), and the material balance (11) will be included in the Lagrange function

$$L = x_2 + \lambda[(1-a_0)x_0 - (a_1+b_1)x_1 - a_2x_2 - z_{02}^0],$$

Where x_0, x_1, x_2 are their expressions (66) and

$$\theta_0 = 1 - \theta_1 - \theta_2, \qquad s_0 = 1 - s_1 - s_2.$$

As a result of the study, based on the open three-sector model of the economy, the optimal dynamic rule for the distribution of labor and investment resources between the sectors of the open three-sector economy was found.

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MSC 49M37

MODELS FOR SELECTING THE OPTIMAL ROLLING STOCK

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The article considers an optimization two-level mathematical model for selecting rolling stock of the required capacity, which can be used for various urban transport

networks, in order to ensure the most complete and high-quality satisfaction of the population's needs for transportation and the necessary level of service quality, in conjunction with economic feasibility. The model assumes that passengers choose from all possible routes connecting any two nodes in the public transport network. The proposed method of selecting the optimal type of rolling stock allows you to reduce operating costs.
Keywords: transport system, mathematical model, variables, choice of rolling stock, route.

Бул статьяда транспорттун керектүү кыймылдагы бөлүмүн тандоонун оптималдык эки деңгелдүү математикалык модели каралган. Ал экономикалык масаттулук менен шаардык тургундарды транспорттук ташууну, тейлөөнү сапататтуу жана толугураак камсыз кылуу максатында, ар кандай шаардык транспорттук торду анлиздөөгө колдонушу мүмкүн. Моделде, пассажирлер транспорттук тордогу эки түйүндү туташтырган баардык мүмкүн болгон матруттардан керектүүсүн тандап алат деп снушталат. Сунушталган кыймылдагы составдын оптималдуу түрүн тандап алуу методикасы экспулатациялоо чыгымдарды азайтууга мүмкүнчүлүк берет.
Урунттуу сөздөр: транспорттук система, математикалык модель, өзгөрмө, кыймылдуу составты тандоо, маршрут.

В статье рассмотрена оптимизационная двухуровневая математическая модель выбора подвижного состава необходимой вместимости, которая может быть использована для анализа работы различных городских транспортных сетей, с целью обеспечения наиболее полного и качественного удовлетворения потребности населения в перевозках и необходимого уровня качества обслуживания, во взаимосвязи с экономической целесообразностью. Модель предполагает, что пассажиры выбирают из всех возможных маршрутов, соединяющих любые два узла в сети общественного транспорта. Предлагаемая методика выбора оптимального вида подвижного состава позволяет снизить эксплуатационные затраты.
Ключевые слова: транспортная система, математическая модель, переменные, выбор подвижного состава, маршрут.

The solution to this problem of transport accessibility is associated with the development and optimization of the urban transport network, reducing travel time, improving the environmental situation, and improving the quality of life in cities. All these factors directly or indirectly affect the solution of most of the problems of modern cities.

The optimal combination of different types of transport, as well as rolling stock of various capacities, will ensure the most complete and high-quality satisfaction of the population's needs for transportation. The purpose of the urban passenger transport system (UPTS) is to meet the demand for the movement of the population and provide the necessary level of quality of service, in conjunction with economic feasibility.

The operation of urban public passenger transport depends on various factors: network structure, distances between stops, routes covering the network in question; pricing policies, tolls, traffic intervals, and type of rolling stock [1].

Therefore, in this article we will consider models with respect to two variables - the interval of movement and the type of rolling stock. We assume that other factors affecting the transport system are fixed.

Let's consider an optimization two-level mathematical model that offers the choice of the optimal size of the rolling stock. At the top level, a function for social well-being that represents the costs of both the user and the management company, taking into account technological constraints and meeting demand. The lower level includes the ride distribution model.

Variables are the travel intervals of each route, where *n* is the number of routes in the network that are a fixed variable (0,1), assuming that the value "1" is assigned if the bus type *k* is used on route 1, and "0" in other cases.

The cost structure used in this model is passenger costs (UC) and operating costs (OS). Passenger costs are obtained by modeling and depend on variables, as shown in the following formula:

$$ZP = t_o TT + t_w TO + t_d TD + t_p TP \quad (1)$$

where

TT – is the total approach time to the stop point;

TO – total waiting time;

TD – total travel time;

TP – total time spent on the transfer;

t_o – value of the approach time to the stop point;

t_w – the timeout value;

t_d – value of time of movement;

t_p – value of the time spent on the transfer.

The total cost is equal to (km):

$$CK = \sum_i \sum_k L_i IN_i CK_k \delta_{k,i} \qquad (2)$$

where

L_i – length of route i;

IN_i – interval of the traffic on the route i;

CK_k – unit cost of a kilometer on a k -type bus;

$\delta_{k,i}$ – the variable is assigned the value "1" if the bus type is assigned to the route, and the value "0" in other cases.

The cost of waiting of rolling stock on the stopping points:

$$CR = TG_k \sum_i \sum_k CR_k \delta_{k,i} Y_i \qquad (3)$$

where:

TG_k – average time of embarkation and disembarkation of passengers, for the bus of the form k;

CR_k – cost per passenger per hour when idle rolling stock on the k-type of bus;

Y_i – demand on trip (derived from the modelling of the transport network).

$$TG_k = \beta_0 + max\{\beta_s NS_j + \beta_B NB_j\} \qquad (4)$$

where

$NS_j(NB_j)$ – number of passengers who entered/exited using the j-door at the stop;

$\beta_0, \beta_s, \beta_B -$ are parameters.

If the stopping point is overloaded, then the movement of urban passenger transport becomes disorganized and the time increases (β_0), and the time limit before the arrival of the next rolling stock (β_s) may increase. Similarly, the time limit for getting off the bus (β_B) increases if the bus is full and it takes longer for passengers to move around the cabin. Staff costs are taken as:

$$CP = C_p \sum_i f_i, \qquad (5)$$

where $C_p -$ cost of passenger transportation per hour.

Fixed costs for buses are calculated according to the following formula:

$$CF = \sum_i \sum_k f_i CF_k \delta_{k,i} \qquad (6)$$

On the basis of the above presented cost structure, the optimization problem for the upper level as follows:

$$minZP = t_o TT + t_w TO + t_d TD + t_p TP + \sum_i \sum_k L_i IN_i CK_k \delta_{k,i} + TG_k \sum_i \sum_k CR_k \delta_{k,i} Y_i + C_p \sum_i f_i + \sum_i \sum_k f_i CF_k \delta_{k,i} \qquad (7)$$

When

$$\delta_{k,i} \in (0,1);$$

$$\sum_k \delta_{k,i} = 1 \text{ for any } i;$$

$$\sum_i f_i = \sum_i \sum_k \frac{Y_i \delta_{k,i}}{K_k O_k}.$$

The first limitation is relative to the characteristics of the binary variables $\delta_{k,i}$.

The second restriction is that each route can only be assigned one type of bus.

The third constraint is the demand satisfaction depending on the capacity of different types of buses, where $K_k -$ is the capacity of the k bus type, and O_k is the bus load factor, which varies depending on the filling and takes a value from 0 to 1.

The lower level is optimized by applying a public transport destination model.

To solve the equilibrium model in a public transport network, you need to represent a complex network as a graph

$$G' = (\bar{N}, S), \tag{8}$$

where S is the sum of arcs in the network that form sections of the route. A route section is a section of a route between two consecutive nodes connected by a route group. Then the optimization problem will look like this:

$$min \sum_{s \in S} \int_0^{V_s} c_s\,(x)dx\,; \tag{9}$$

When

$$\sum_{r \in R_w} h_r = T_w \text{, for any } w \in W;$$

$$h_r \geq 0, \text{for any } r \in R.$$

where

$c_s -$ is the fare for public transport passengers on the stage with *c*;

$V_s -$ the passenger traffic on the route *s*;

s – section of the transport route indicating the starting point;

S – group of crossings on routes accessible to public transport passengers;

W – source and end points in the matrix;

w – elements of group *W*, in which *w=(i, j)*;

$h_r -$ passenger traffic on route *r*;

T_w – total number of passenger trips in the matrix;

R - group of routes available for public transport passengers;

r is the route number.

The model assumes that passengers choose from all possible routes that connect any two nodes in the public transport network. The chosen path is reduced to minimizing the total travel time (cost). The total travel time is made up of: transport costs, travel time, waiting time, and approach time to the stop point.

The model assumes that between each pair of nodes in the public transport network there is a group of "shared routes" that are equally attractive to passengers.

Meeting the population's demand for transportation and providing quality services will increase the income of urban passenger transport companies by reducing operating costs, by increasing the volume of traffic. With an increase in traffic volumes, the cost price decreases, which, taking into account the growth of revenues, leads to an increase in profits and profitability of enterprises.

The method of choosing the optimal type of rolling stock presented in the article allows to reduce operating costs, but for the effectiveness of implementation, a well-coordinated work of all participants in the transportation process is necessary. The effect of implementing the proposed method is to minimize the costs and expenses of passengers.

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MSC 49M37

DETERMINATION OF THE RANGE OF AGRICULTURAL PRODUCTS PRODUCED WITH A LIMITED AMOUNT OF FINANCIAL CAPACITY OF THE HOUSEHOLD

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The paper develops a mathematical model for determining the optimal size of the sown area for each type of agricultural crop and the amount of mineral (organic) fertilizers used with a limited amount of financial capacity of the household.

Key words: mathematical model, costs, income, yield, acreage, economy, price, production.

Бул жумушта чарбанын чектелген финансалык мүмкүкчүлүк учурунда эгилүүчү ар бир айыл чарба өсүмдүктөрүнүн айдоо аянтарындагы оптимальдуу

ченемин жана аларга колдонулуучу минеральдык (органикалык) жер семирткичтердин көлөмүн табуу маселесине математикалык модель иштелип чыккан.
Урунттуу сөздөр: математикалык модель, чыгым, киреше, түшүмдүүлүк, айдоо аянты, чарба, баа, өндүрүш.

В работе разработана математическая модель задачи определения оптимального размера посевной площади под каждый вид сельхоз культуры и объемы используемых минеральных (органических) удобрений при ограниченном размере финансовой возможности хозяйства.
Ключевые слова: математическая модель, расход, доход, урожайность, посевная площадь, хозяйство, цена, производство.

Problem statement and mathematical model. Let the household has the financial assets in the amount of D soms and has k categories of acreage $s_k, k \in K$ (irrigated, rain-fed, etc.) where was planned to grow n types of agricultural crops. For growing agricultural crops used r types of mineral (organic) fertilizers (nitrogen, saltpeter, etc.)

It is assumed to determine the yield of each type of crop in each category of sown area and the corresponding costs, as well as the price of sales of a unit of agricultural production.

In addition, the purchase price of a unit volume of mineral fertilizers and the rate of consumption of each type of mineral (organic) fertilizers per unit size of each category of acreage are known and etc.

We have to determine the optimal size of acreage for each type of agricultural crop and the amount of mineral (organic) fertilizers used so that the net income received by the household is maximum.

For the mathematical formulation of the problem, we introduce the following notation:

$k -$ index of farm acreage categories, $k \in K = \{1, 2, \ \dots, m\}$;

$j -$ index of the type of agricultural crop, $j \in J = \{1, 2, \ \dots, n\}$;

$r -$ index of the type of mineral (organic) fertilizers used by the farm in the production of agricultural products, $r \in R = \{1, 2, \ldots, p\}$;

<u>Known parameters:</u>

$a_{jk} - is$ yield of $j -$ th type of culture on $k - th$ category of cultivated area, $k \in K$;

d_j – is the selling price of $j - th$ type ofagricultural products, $j \in J$;

$S_k -$ is the size of к –th category of household's acreage, $k \in K$;

c^r –the market price of a unit of volume of $r -$ th type of mineral (organic) fertilizer, $j \in J, r \in R$;

$\mathcal{E}_{jk}^r$ –is the consumption rate of $r -$ th type of mineral (organic) fertilizers per unit of sown area size of $k - th$ category for $j - th$ type culture, $j \in J, r \in R, k \in K$;

$D -$ is the size of financial ability of the household;

$c_{jk} - is$ the cost of farming of $j - th$ type of culture, in the unite of size of $k - th$ category of acreage, $j \in J, k \in K$;

<u>Sought variables:</u>

$x_j -$ volume of agricultural products sold of $j - th$ type, $j \in J$;

$Z_{jr} -$ volume of mineral (organic) fertilizers used agriculture in the cultivation of agricultural products, $j \in J, r \in R$;

x_{jk} –size of $k - th$ acreage for $j - th$ type of culture, $j \in J, k \in K$;

$Z -$ the size of financial resources used by the household.

According to the accepted notation, the mathematical model of the problem has the following form.

$$\sum_{j \in J} d_j x_j - Z \to \max \qquad (1)$$

having conditions

$$\sum_{k\in K} a_{jk}x_{j_k} = x_j,\ \ j\in J, \tag{2}$$

$$\sum_{j\in J}\sum_{k\in K} \mathcal{E}_{jk}^{r}\, x_{jk} = z_r,\ \ r\in R, \tag{3}$$

$$\sum_{j\in J} x_{jk} \le S_k,\ \ k\in K, \tag{4}$$

$$\sum_{r\in R} c^r z_r + \sum_{j\in J}\sum_{k\in K} c_{jk}x_{jk} = Z \le D, \tag{5}$$

$$x_j \ge 0,\ x_{jk}\ge 0,\ \ j\in J, k\in K, \tag{6}$$

$$z_r \ge 0,\ \ r\in R, \tag{7}$$

$$Z \ge 0 \tag{8}$$

Using the relations (2), (3) excluding from (1) and (5) variables Z_r. Then the mathematical model of the problem takes the following form:

To find maximum

$$L(x) = \sum_{j\in J}\sum_{k\in K} d_j a_{jk}x_{jk} - Z \tag{9}$$

having conditions

$$\sum_{j\in J} x_{jk} \le S_k,\ \ k\in K, \tag{10}$$

$$\sum_{j\in J}\sum_{k\in K}(c_{jk} + \sum_{r\in R} c^r \mathcal{E}_{jk}^{r})x_{jk} = Z \le D, \tag{11}$$

$$x_{jk} \ge 0, \qquad j\in J,\ \ k\in K, \tag{12}$$

where $x=\{x_{jk}:\ j\in J,\ \ k\in K\ \}$.

Further, mathematical model (9)-(12) we will write in form of Table 1 and solve the problem with method in [1], where the notation is entered for brevity:

$$q_{jk} = d_j a_{jk},\ \ j\in J,\ \ k\in K;$$

$$b_{jk} = \sum_{r \in R} c^r \mathcal{E}_{jk}^r + c_{jk}\,, \;\; j \in J, \;\; k \in K$$

Table 1

$\boldsymbol{x_{11}}$	$\boldsymbol{x_{12}}$	**…**	$\boldsymbol{x_{1k}}$	$\boldsymbol{x_{21}}$	$\boldsymbol{x_{22}}$	**…**	$\boldsymbol{x_{2k}}$	**…**	$\boldsymbol{x_{n1}}$	$\boldsymbol{x_{n2}}$	**…**	$\boldsymbol{x_{nk}}$	**Z**		
1				**1**					**1**					≤	S_1
	1				1					1				≤	S_2
		…						…			…				…
			1				1					1		≤	S_k
$\boldsymbol{b_{11}}$	b_{12}	…	b_{1k}	b_{21}	b_{22}	…	b_{2k}	…	b_{n1}	b_{n2}	…	b_{nk}	-1	=	0
													1	≤	D
$\boldsymbol{q_{11}}$	q_{12}	…	q_{1k}	q_{21}	$\boldsymbol{q_{22}}$	…	q_{2k}	…	q_{n1}	q_{n2}	…	q_{nk}	-1	→	*max*

Solution algorithm. Calculations begin with determining the values

$$q_{jk} = d_j a_{jk}, j \in J, \;\; k \in K, \qquad (13)$$

and

$$b_{jk} = \sum_{r \in R} c^r \mathcal{E}_{jk}^r + c_{jk}\,, \;\; j \in J, \;\; k \in K. \qquad (14)$$

Next, using known data S_k, $k \in K$, a numerical model of the problem of the form is formulated (9)-(12) and solved.

From the solution of the problem, the size of the sown area allocated for each type of agricultural crop in each category of sown area of the farm is determined $k \in K$, and the amount of financial resources used $Z \leq D$, bringing the maximum net income to the household.

Let's check the working capacity of the mathematical model and algorithm for solving the problem using the following example.

Example. Let the household have a sown area of Let the farm have a sown area of $s = 400$ h, of these irrigated area is $s_1 = 300$ га, and rainfed area is $s_2 = 100$ h and has financial resources in the amount of 2 million soms for growing crops.

The household plans to grow the following types of agricultural products this year: potatoes, onions, wheat and barley, since according to long-term data, the wholesale price of these crops is almost stable and in high demand, price of potato is 20 som per kg, price of onions is - 15 som per kg, price of wheat is 14 som per kg and price of barley is 12 som per kg.

For farming farming uses mineral (organic) fertilizers: gerberit, nitrate, water irrigation etc.

It is necessary to determine the optimal plan for the sown area of irrigated and rainfed fields for each type of crop, as well as the volume of mineral (organic) fertilizers used, taking into account the financial means of the farm, so that the net income of the farm from the sale of grown agricultural products is maximum.

Known: - yield (kg) of each type of crop $|a_{j,k}|_{j \in J, k \in K}$ on irrigated and rain-fed acreage:

$$|a_{jk}|_{4,2} = \begin{bmatrix} 12000.0 & 0.0 \\ 15000.0 & 0.0 \\ 3200.0 & 2000.0 \\ 2600.0 & 1600.0 \end{bmatrix};$$

- vector of realized prices per unit volume of agricultural products of d, (som per kg)

$$d = (d_1, d_2, d_3, d_4) = (20.0, 15.0, 14.0, 12.0);$$

- vector of the size of the sown areas s (hectar)

$$s = (s_1, s_2) = (300.0, 100.0);$$

- market price per unit volume of mineral fertilizers: herbicide - 55 som per kg, saltpeter - 27 som per kg, irrigation water - 0,40 som per m^3;

- the rate of consumption of mineral fertilizers per hectare, depending on the category of sown area for each type of crop, $|\varepsilon_{jk}^{r}|_{|j|,|R|}$ $k \in K$, т.е.

a) on the irrigated acreage (*k=1*)

$$|\varepsilon_{j1}^{r}|_{4,3} = \begin{bmatrix} - & 50.0 & 2000.0 \\ - & 50.0 & 2000.0 \\ 10.0 & 50.0 & 1000.0 \\ 10.0 & 50.0 & 1000.0 \end{bmatrix};$$

b) on rainfed cultivated area (*k=2*)

$$|\varepsilon_{j2}^{r}|_{4,3} = \begin{bmatrix} - & 50.0 & - \\ - & 50.0 & - \\ 10.0 & 50.0 & - \\ 10.0 & 50.0 & - \end{bmatrix};$$

- matrix of expenses for growing each type of crop per unit of sown area size by category, $|c_{jk}|_{|j|,|k|}$, (som per hectare), so

$$|c_{jk}|_{4,2} = \begin{bmatrix} 41000.0 & 40000.0 \\ 43000.0 & 42000.0 \\ 10000.0 & 9000.0 \\ 9500.0 & 8500.0 \end{bmatrix};$$

- size of the financial means of the household *D*=2000000.0 soms.

Let's form a numerical model of the problem (9)-(13). According to the algorithm for solving the problem, we determine the values q_{jk} и $b_{jk}, j = \{1, 2, 3, 4\}, k \in K = \{1, 2\}$ by formulas (13) and (14).

we have $\left|q_{jk}\right|_{4,2} = \begin{bmatrix} 240000.0 & 0.0 \\ 22500.0 & 0.0 \\ 44800.0 & 28000.0 \\ 31200.0 & 21600.0 \end{bmatrix}$; and $\left|b_{jk}\right|_{4,2} = \begin{bmatrix} 240000.0 & 41350.0 \\ 45150.0 & 43350.0 \\ 12300.0 & 10900.0 \\ 11800.0 & 10400.0 \end{bmatrix}$.

The numerical model of the problem looks like this.

Finding the maximum

$$L(x) = 240000x_{11} + 0x_{12} + 225000x_{21} + 0x_{22} + 44800x_{31} + \\ +28000x_{32} + 31200x_{41} + 21600x_{42} - z \quad (15)$$

under conditions

$$\sum_{j \in J} x_{j1} = 300, \sum_{j \in J} x_{j2} = 100, \quad (16)$$

$$43150x_{11} + 41350x_{12} + 45150x_{21} + 43350x_{22} + 12300x_{31} + \\ +10900x_{32} + 11800x_{41} + 10400x_{42} = z \leq 20000000, \quad (17)$$

$$x_{jk} \geq 0, \quad j = \{1, 2, 3, 4\}, \quad k \in K = \{1, 2\}. \quad (18)$$

problems (15) –(18) it can be presented as a Table 2.

Table 2

x_{11}	x_{12}	x_{21}	x_{22}	x_{31}	x_{32}	x_{41}	x_{42}	z		
1		1		1		1			=	300
	1		1		1		1		=	100
43150	41350	45150	43350	12300	10900	11800	10400	-1	=	0
								1	<=	20000000
240000	0	225000	0	44800	28000	31200	21600	-1	→	max

Then after solving the problem (15)–18) with method as in [1], we get optimal plan as

$$x^* = \{ x_{11} = 300, \quad x_{42} = 100\}$$

of acreage for each type of crop that provides the maximum net income of the household is $L(x) = 60175000$ som.

The volume of potatoes grown is 3600 t. , of barley -180 t., and the amount of mineral (organic) fertilizers used is determined from (3), that is used for growing potatoes 15 t. of saltpeter, 600 thousands m^3 of irrigation water, and for barley is – 1 t. For herbicidea, 5 t. of saltpeter.

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THE PROBLEM OF DETERMINING THE SIZE OF THE LOAN AND THE LEASED ACREAGE IN THE PRODUCTION OF PRODUCTS

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The paper develops a mathematical model of the problem of determining the optimal size of the sown area for each type of crop on their own and leased plots and the necessary financial credit to the household according to the criterion of maximum net income. A method for solving the problem for the case with two-way restrictions on production volumes is developed.

Key words: mathematical model, acreage, crop, credit, economy, income, costs, production.

Бул жумушта чарбанын таза кирешеси максималдуу болгондой критериясы менен эгилүүчү ар бир айыл чарба өсүмдүктөрүн өзүнүн жана ижарага алынуучу айдоо аянтарындагы оптималдуу ченемин жана керектелуучу финансалык насыяны табуу маселесине математикалык модель иштелип чыккан. Өндүрүү көлөмү эки тарабынан тен чектелген учуру үчүн чыгаруу ыкмасы көрсөтүлгөн.

Урунттуу сөздөр: математикалык модель, айдоо аянты, айыл чарба өсүмдүктөрү, насыя, чарба, киреше, чыгым, өндүрүш.

В работе разработана математическая модель задачи определения оптимального размера посевной площади под каждый вид культуры на своих и арендуемых участках и необходимого финансового кредита хозяйству по критерию максимума чистого дохода. Разработан метод решения задачи для случая с двусторонними ограничениями на объемы производства.

Ключевые слова: математическая модель, посевная площадь, сельскохозяйственная культура, кредит, хозяйство, доход, расход, производство.

Problem statement. A household with a sown area of s and does not have a financial assets, planned to get a loan for cultivation n type of agricultural crop, $j \in J = \{1, 2, \ldots, n\}$ and drew up a contract with the processing company to sell them at the agreed price c_j. Production volume of $x_j,\ j \in J$ is limited by upper and lower limitations due to agreement, so $d_j' \leq x_j \leq d_j'', j \in J$.

For the household, the yield of each type of agricultural crop is known, the rate of consumption of each type of fertilizer used (nitrogen, saltpeter, herbicide, irrigation water, etc.) per unit area for each type of crop and the wholesale purchase price, as well as the interest rate of the financial loan.

It is necessary to determine the optimal size of the sown area for each type of crop on their own and leased plots, as well as the size of the financial loan so that the planned volume of agricultural products of each type is fulfilled under the contract and the household's profit is maximized.

Let's formulate a mathematical model of the problem.

We shall introduce the following notation:

j – is crop type index, $j \in J = \{1, 2, \ldots, n\}$;

J – is the set of crop indices;

$r -$ is index of the type of mineral fertilizers used by the household for growing agricultural crops, $r \in R$;

$R -$ is set of indexes of the type of mineral fertilizers purchased by the household;

Known parameters.

$c_j -$ is contractual sales price per unit of volume of j- th type of agricultural products by the household, $j \in J$;

$\overline{c_j}$ – is the cost of growing a unit of the size of the acreage under j- th type of culture, $j \in J$;

$c_r -$ is wholesale price per unit volume of mineral fertilizers used of r -th type on the wholesale market;

D – is maximum amount of financial credit received by the household at interest;

$S -$ is the size of the acreage on the household;

$a_{jr} -$ is consumption rate per unit of sown area of $r - th$

types of mineral fertilizers under $j - th$ type of culture, $j \in J, r \in R$;

$b_j -$ is yield of $j - th$ type of agricultural crop, $j \in J$;

α – is loan interest rate, λ - loan term, in paper it is assumed that the term of the loan is one year, $\lambda = 1$;

β – is the amount of payment by the household for rent per unit of sowing size area;

Sought quantities:

$x_j -$ agricultural production of $j - th$ type produced by household according to agreement with customer, $j \in J$;

$Z_r -$volume of $r - th$ type of mineral fertilizers purchased by the household

According the agreement, $r \in R$

y_j −sown area held by $j-th$ type of culture in its household area, $j \in J$;

$V-$ the amount of the financial loan received by the household at interest;

ω_j- the size of the cultivated area of the household held by $j-th$ type of culture in rented area, $j \in J$;

$\omega-$ the size of the leased sown area by the household.

According to the adopted designations, the mathematical model of the problem of determining the optimal size of the sown area for each type of crop and the amount of financial resources received at interest, as well as the size of the leased area for agricultural crops due to agreement of production can be represented as.

To find the maximum

$$L(x)=\sum_{j\in J} c_j x_j-(1+\alpha\lambda)V \rightarrow \max \qquad (1)$$

under conditions

$$\sum_{j\in J} y_j=S, \qquad (2)$$

$$\sum_{j\in J} \omega_j=\omega, \qquad (3)$$

$$\sum_{j\in J} \alpha_{jr}\left(y_j+\omega_j\right)=Z_r,\ \ r\in R, \qquad (4)$$

$$b_j(y_j+\omega_j)=x_j,\ j\in J, \qquad (5)$$

$$d_j'\le x_j\le d_j'',\ j\in J, \qquad (6)$$

$$\sum_{j\in J} c_j\left(y_j+\omega_j\right)+\beta\omega+\sum_{r\in R} c_r z_r=V\le D, \qquad (7)$$

$$x_j\ge 0,\ y_j\ge 0,\ \omega_j\ge 0,\ \ j\in J, \qquad (8)$$

$$Z_r \geq 0, \quad r\epsilon R, \tag{9}$$

$$\omega \geq 0, \;\; V \geq 0, \tag{10}$$

where $\quad x = \{x_j : \; j \in J\}$.

Let's add substation of variables

$$x_j = x_j^1 + x_j^2, \quad y_j = y_j^1 + y_j^2, \; \omega_j = \omega_j^1 + \omega_j^2, \;\; j \in J.$$

Let’s modify the problem (1)-(10).

Then the mathematical model of the problem of determining the maximum net income and the amount of financial credit at interest, as well as the size of the leased acreage to meet the contractual volume of agricultural production, has the form.

To find maximum

$$l(\bar{y}, \bar{\omega}) = \sum_{j \in J} c_j b_j((y_j^1 + y_j^2) + (\omega_j^1 + \omega_j^2)) - (1 + \alpha\lambda)V \tag{11}$$

under conditions

$$\sum_{j \in J} (y_j^1 + y_j^2) = S, \tag{12}$$

$$\sum_{j \in J} (\omega_j^1 + \omega_j^2) = \omega, \tag{13}$$

$$b_j(y_j^1 + \omega_j^1) = d_j', \;\; j \in J, \tag{14}$$

$$b_j(y_j^2 + \omega_j^2) \leq d_j'' - d_j', \;\; j \in J, \tag{15}$$

$$\sum_{j \in j} [(\bar{c_J} + \sum_{r \in R} c_r \alpha_{jr})\,(y_j^1 + y_j^2) + (\bar{c_J} + \beta +$$

$$+ \sum_{r \epsilon R} c_r \alpha_{jr})(\omega_j^1 + \omega_j^2)] = V \leq D, \tag{16}$$

$$y_j^1 \geq 0, \quad y_j^2 \geq 0, \ \ \omega_j^1 \geq 0, \ \ \omega_j^2 \geq 0, \ j \in J, \qquad (17)$$

where

$$\bar{y} = \{ \, y_j^1, y_j^2 \colon \ j \in J \, \}, \ \ \bar{\omega} = \{\omega_j^1, \omega_j^2 \colon \ j \in J\} \, .$$

Mathematical model of a problem of the form (11)-(17) let's represent it as in Table 1, where the notation is entered for brevity: $W_j = c_j b_j, \ j \in J, \quad q_{j=}\overline{c_j} + \sum_{r \epsilon R} c_r \alpha_{jr\,,} \quad \overline{q_J} = q_j + \beta, \ j \in J,$ and we will solve the proposed method in the work [1].

Table 1

y_1^1	y_1^2	y_2^1	y_2^2	…	y_n^1	y_n^2	ω_1^1	ω_1^2	ω_2^1	ω_2^2	…	ω_n^1	ω_n^2	V	ω		
1	1	1	1	…	1	1										=	S
							1	1	1	1	…	1	1		-1	=	0
b_1							b_1									=	d_1'
		b_2							b_2							=	d_2'
				…							…					…	…
					b_n							b_n				=	d_n'
	b_1							b_1								≤	$d_1'' - d_1'$
			b_2							b_2						≤	$d_2'' - d_2'$
				…							…					…	…
						b_n							b_n			≤	$d_n'' - d_n'$
q_1	q_1	q_2	q_2	…	q_n	q_n	$\overline{q_1}$	$\overline{q_1}$	$\overline{q_2}$	$\overline{q_2}$	…	$\overline{q_n}$	$\overline{q_n}$	-1		=	0
														1		≤	D
w_1	w_1	w_2	w_2	…	w_n	w_n	w_1	w_1	w_2	w_2	…	w_n	w_n	$-(1+\alpha\lambda)$	0	→	max

From the problem solution of (11) – (17) is determining the optimal amount of sown area for each type of crop on our own $y_j^*, \ j \in J$ and leased area $\omega_j^*, \quad j \in J,$ the amount of the financial loan V^* received by the household for the production of agricultural products and the size of the leased acreage ω^*. Further, according to the system of equalities (4) and (5) we determine the volume of agricultural product produced and the volume of mineral fertilizers purchased for each type of crop cultivation by the household.

Let's check the working capacity of the mathematical model and algorithm for solving the problem using the following example.

Example. The household signed a contract with a processing enterprise for the production and supply of two types of agricultural products (potatoes, onions) at the price of: 20 som per kg for potatoes, 15 som per kg for onions in volume not less than 1300 tons and not more 2700 tons of potatoes, for onions not less than 1000 tons and not more than 1500 tons.

Known:

- size of the household's acreage s=100 hectare;

- crop yield: of potatoes 12 tons per hectare and onions 15 tons per hectare;

- the cost of growing a unit of acreage: 41 thousand som per hectare and 43 thousand som per hectare;

- the price per unit of the volume of used mineral fertilizers: ammonium nitrate is 55 som per hectare, irrigation water – 0,40 som per m^3;

- rate of consumption of mineral fertilizers per unit size of the sown area: saltpeter – 50 kg per hectare, irrigation water 2000 m^3/hectare;

- the amount of payment by the farm for renting a unit of sown area for crops is 10 thousand som per hectare;

- credit interest is α =6%, the period of issuing for one year, so $\lambda = 1$.

The maximum amount of financial credit received by the household is up to 20 million soms.

It is necessary to determine the optimal size of the sown area for each type of crop, the amount of financial credit and the size of the leased area for agricultural crops so that the planned volume of agricultural products of each type is fulfilled under the contract and the net income of the household has to be maximum.

According to the known data, the numerical model of the problem can be represented as finding the maximum

$$L(x) = 20.0x_1 + 15x_2 - (1 + 0{,}06 * 1) \qquad (18)$$

under conditions

$$\sum_{j=1}^{2} y_j = 100, \quad \sum_{j=1}^{2} \omega_j = \omega, \qquad (19)$$

$$50.0(y_1 + \omega_1) + 50.0(y_2 + \omega_2) = z_1,$$

$$2000.0(y_1 + \omega_1) + 2000.0(y_2 + \omega_2) = z_2, \qquad (20)$$

$$12000.0(y_1 + \omega_1) = x_1, \quad 15000.0(y_2 + \omega_2) = x_2, \qquad (21)$$

$$1300000.0 \leq x_1 \leq 2700000.0,$$

$$1000000.0 \leq x_2 \leq 1500000.0, \qquad (22)$$

$$41000.0(y_1 + \omega_1) + 43000.0(y_2 + \omega_2) + 10000.0\omega + 55z_1 +$$

$$+0{,}40z_2 = v \leq 20000000.0 \qquad (23)$$

$$y_j \geq 0, \quad \omega_j \geq 0, \qquad j \in J = \{1, 2\}, \qquad (24)$$

$$z_r \geq 0, \quad r \in R \qquad (25)$$

$$\omega \geq 0, v \geq 0. \qquad (26)$$

Let's add substation of variables

$$x_j = x_j^1 + x_j^2, y_j = y_j^1 + y_j^2, \ \omega_j = \omega_j^1 + \omega_j^2, j \in J = \{1, 2\}.$$

Converting the problem (18) – (26) to form

Finding maximum

$$L(\bar{y}, \bar{\omega}) = 140000(y_1^1 + y_1^2) + 240000(\omega_1^1 + \omega_1^2) +$$

$$+225000(y_2^1+y_2^2)++225000(\omega_2^1+\omega_2^2)-(1+0.06*1) \quad (27)$$

under conditions

$$\sum_{j=1}^{2}(y_j^1+y_j^2) \quad =100, \quad \sum_{j=1}^{2}(\omega_j^1+\omega_j^2)=\omega, \qquad (28)$$

$$12000(y_1^1+\omega_1^1)=1300000, \quad 15000(y_2^1+\omega_2^1)=1000000,$$

$$12000(y_1^2+\omega_1^2)\le 1400000, \quad 15000(y_2^2+\omega_2^2)\le 500000, \qquad (29)$$

$$43150(y_1^1+y_1^2)+53150(\omega_1^1+\omega_1^2)+45150(y_2^1+y_2^2)+$$

$$+55150(\omega_2^1+\omega_2^2)=v\le 20000000, \qquad (30)$$

$$y_j^1\ge 0,\ y_j^2\ge 0,\ \omega_2^1\ge 0,\ \omega_j^2\ge 0,\ j\in J=\{1,2\}, \qquad (31)$$

where $\bar{y}=\{y_j^1,\ y_j^2:\ \ j=1,2\}$ $\quad \bar{\omega}=\{\omega_j^1,\ \omega_j^2:\ j=1,2\}$.

problem (27) – (31) can be presented in the form of Table 2.

Table 2.

y_1^1	y_1^2	y_2^1	y_2^2	ω_1^1	ω_1^2	ω_2^1	ω_2^2	v	ω		
1	1	1	1							=	400
				1	1	1	1		-1	=	0
12000				12000						=	1300000
	12000				12000					≤	1400000
		15000				15000				=	1000000
			15000				15000			≤	500000
43150	43150	45150	45150	53150	53150	55150	55150	-1		=	0
								1		≤	20000000
240000	240000	225000	225000	240000	240000	225000	225000	-1,06	0	→	max

After solving the problem (27) – (31) using the method in [1], we get the optimal plan of the allocated sown area for each type of crop (potatoes, onions) on our own and leased area:

$$y=\{ \quad y_j^1=100\}, \quad \omega=\{ \omega_1^1=8{,}33, \quad \omega_2^1=66{,}67\}.$$

It follows from this plan that to fulfill the contractual conditions, the farm must use all available acreage of 100 hectares, as well as 8.33 hectares of rental field for potatoes, and a rental field of 66.67 hectares for onions. At the same time, 1,300 tons of potatoes and 1,000 tons of onions were produced.

The household received a financial loan 8434583.0 som under 6% for a year.

The net income of the farm is $L(x)$ =32059342.0 som.

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