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Altay Borubaev was born in village Kara-Oy, Talas region, Kyrgyzstan on December 31, 1950.

In 1967 he graduated from secondary school with medal.

In 1972 he graduated from the Kyrgyz State University with the degree “mathematician”, he was on an internship at M.V. Lomonosov Moscow State University and in 1975 he finished the graduate school at the KSU.

In 1972-1975 he worked as a teacher at the KSU and in 1975-76 he worked as a teacher at the Frunze Polytechnic Institute.

In 1975-1992 he worked as a teacher, as a head of department, as the dean of Mechanical-mathematical faculty, as the Vice-rector for scientific work at the KSU; in 1983-1984 he was on a scientific internship at the Charles University in Prague; in 1988-1990 he was in a doctorate at Lomonosov MSU.

In 1992-1994 he was the First Deputy Minister of Education.

In 1994-1998 he was the rector of the Kyrgyz State Pedagogical University named after I. Arabaev.

In 1998-2000 he was the rector of the Kyrgyz National University named after J. Balasagyn. He was elected a member of Jogorku Kenesh of KR.

In 2000-2005 he was elected the speaker of the Assembly of People's Representatives of Jogorku Kenesh of KR.

In 2005-2013 he was the Chairs of (National) Highest Attestation Commission of KR.

In 2013-2016 he was a Vice-President of NAS of KR.

Since 2016 he works as the director of Institute of Mathematics of NAS of KR.

He published more than 150 scientific works including 10 monographs and 6 text-books in many countries, made reports at conferences and lectured in England, China, Greece, Russia, Italy, Hungary, France, Uzbekistan, Azerbaijan, Czechia, Kazakhstan, India and Swiss. He organized some international conferences. He prepared more than 20 candidates of sciences and 8 doctors of sciences. He is the founder of world-wide known scientific school in topology.

He participated in drafting laws “On education” and ”On science and state scientific-technical policy framework”.

He was awarded the Order “Commonwealth” of Inter-Parliamentary Assembly of the CIS, the Order of Peter the Great, First class (Russia), the Honorable medal of the Charles University in Prague, Russian medal “In memory of the 300th anniversary of St. Petersburg”, the World Intellectual Property Organization Gold Medal, Gold Medal (Socrates International Committee).

By proposal of group of scientists, for outstanding contribution in the theory of spaces the Moscow International Observatory assigned the name “Altay” to one of stars.

President of Turkic World Mathematical Society (since 2014), Honorable President of Turkic World Mathematical Society (since 2017).

MSC 91F20

MATHEMATICAL MODELS OF RELATIONS IN INDEPENDENT COMPUTER PRESENTATION OF NATURAL LANGUAGES

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In this paper we consider mathematical models of objects to be presented as interactive software. Supra, the authors proposed independent computer presentation of natural languages and implemented some notions of Kyrgyz and English. In the paper such mathematical models are described for some relations including family relationship.
Keywords: mathematical model, language, computer model, relation, independent presentation, learning.

Интерактивдик программалык жабдуу түрүндө чагылдырууга тийиштүү нерселердин математикалык моделдери макалада каралат. Мурда авторлор тарабынан табигый тилдерди компьютерде көз карандысыз түрдө чагылдыруу сунушталган жана кээ бир түшүнүктөр кыргыз жана англис тилдеринде ишке ашырылган. Макалада кээ бир байланыштар (алардын ичинде үй-бүлөлүк байланыштар) үчүн тийиштүү математикалык моделдер сүрөттөлгөн.
Урунттуу сөздөр: математикалык модель, тил, компьютердик модель, байланыш, көз карандысыз чагылдыруу, үйрөнүү.

В статье рассматриваются математические модели объектов для их представления в форме интерактивного софта. Ранее авторы предложили независимое компьютерное представление естественных языков и реализовали некоторые понятия кыргызского и английского языков. В статье описаны такие математические модели для некоторых отношений, включая родственные отношения.
Ключевые слова: математическая модель, язык, компьютерная модель, отношение, независимое представление, обучение.

1. Introduction

One of main tasks of present day informatics is developing of interactive computer presentations of all familiar real and virtual objects to offer the user the opportunity to master them safely and effectively before real treating. If such computer presentation does not depend on the user's knowledge and skills on similar objects then we call it *independent*. In our opinion, such presentations are more effective because the user can learn inductively - without referencing other objects in mind. In regards with learning a language, the user begins to thinking in it, without translation in mind.

Earlier, investigating and learning a living language were implemented with the assistance (including bilingual dictionaries and text-books) of persons who had a complete command of it; investigating of a dead language was done by means of remained bilingual texts and texts with additional implicit suggestions and conclusions. Invention of recording sounds gave possibility to fix examples of an oral language objectively. Invention of talking pictures fixed examples of phrases with connection to situations and actions. Computer games gave the user the opportunity to choose actions with corresponding phrases. Existing software to learn languages base on languages native to the user, nevertheless some notions are presented independently. This survey demonstrates that there were not completely independent presentations of natural languages.

Using ideas [1], [2], [3] we [4-11] gave definitions and developed elements of such presentations. We described mathematical models in general [12]. A candidate dissertation had been confirmed [13]. We will base on Kyrgyz language mentioning other languages too. We shall consider also feedback for checking-up knowledge of a language. We use random generation of tasks and situations for independent presentation of notions and objective estimation of knowledge.

2. Definitions for independent presentation

Definition 1. If low energetic outer influences can cause sufficiently various reactions and changing of the inner state of the object (by means of inner energy of the object or of outer energy entering into object besides of commands) at any time then such (permanently unstable) object is an *affectable object*, or a *subject*, and such outer influences are *commands.*

Definition 2. A system of commands such that any subject can achieve desired efficiently various consequences from other one is a *language*.

Hypothesis 1. A human's genuine understanding of a text in a natural language can be clarified by means of observing the human's actions in real life situations corresponding to the text.

Definition 3. Simple mathematical models consist of fixed (F_i) and movable (M_j) sets and temporal sequence of conditions of types $(M_j \subset F_i)$, $(M_j \cap F_i = \varnothing)$, $(M_j \cap F_i \neq \varnothing)$.

Computer interactive presentations are built on the base of mathematical models.

Definition 4. Let any *notion* (word of a language) be given. If an algorithm acting at a computer: generates (randomly) a sufficiently large amount of instances covering all essential aspects of the *notion* to the user, gives a command involving this notion in each situation, perceives the user's actions and performs their results clearly on a display, detects whether a result fits the command, then such algorithm is said to be a *computer interactive presentation* of the *notion.*

Certainly, commands are to contain other words too. But these words must not give any definitions or explanations of the notion.

Definition 5. If all words being used in Definition 4 are unknown to the user nevertheless s/he is be able to fulfil the meant action (because it is the only natural one in this situation) then the notion (word of a language) is said to be *primary*. If the user has to know supplementary words to complete the action then the notion is said to be *secondary*. Thus, there arises a natural hierarchy of notions.

Using this method we can present not only real notions (objects and actions) but also notions which have imaginary concepts.

Hypothesis 2. A person learning a natural language without references to any other ones, hearing a notion in various situations begins to form a kind of *mathematical model* in mind corresponding to this notion by means of trial and error method and attempts to fulfill operations similar to mathematical ones: closing and compactification. After successful completing such operations, the human feels “mastering” this notion.

Hypothesis 3. Any notion has a minimalistic mathematical model (involving minimal number of *entities* in Occam’s sense).

3. Mathematical models for relations

Nouns are remembered better if they are demonstrated with relations. Many of nouns can be included to any relations.

We will use verbs 1) *бер (give), 2) келтир (let it/him/her come to).*

Images of objects being fixed in a model but mobile in reality are marked with a low fence.

1) Mathematical model. Fixed F_i *(human or animal),* G_i*(hands or paws)*$\subset F_i$*;*

movable M_j *(food or thing).*

Temporal sequence: *1.* $(M_j \cap G_j = \varnothing)$*, 2.* $(M_j \cap G_j \neq \varnothing)$*.*

Example 1. Relation food. Fixed

F_1 *= киши (human),* F_2 *= пил(elephant),* F_3 *= тыйын(squirrel);*

these words are known to the user,

movable M_1 *=нан(bread),* M_2 *=коон(melon),*

M_3 *=жаңгак(nut),* M_4 *=таш(stone);* these words are unknown to the user.

Commands. *1. Нанды кишиге бер! (Give bread to human)!*

2. Коонду пилге бер! 3. Жаңгакты тыйынга бер!

2) Mathematical model. Fixed F_i *(human or animal),* G_i*(neighborhood)*$\supset F_i$*;* movable M_j *(human or animal).*

Temporal sequence: *1.* $(M_j \cap G_j = \varnothing)$*, 2.* $(M_j \cap G_j \neq \varnothing)$*.*

Example 2. Family relation. There are *adults* (*fathers* and *mothers*) in solid color clothing (fathers' colors differ from mothers' ones) and *children (boys* and *girls);* color of upper part of child's clothes is of their father's; color of lower part of child's clothes is of their mother's.

No preceding known words.

Scene 2.1. Fixed: fathers. Movable: children appear one by one. Firstly, one child (boy or girl) is present.

Command:

Эркек баланы/кызды атасына келтир! (Let boy/girl come to his/her father)!

Next child (boy or girl) enters into the screen, next command …

Scene 2.2. Fixed: fathers and mothers. Movable: children appear one by one.

Preceding scene repeats with commands

Эркек баланы/кызды апасына келтир! (Let boy/girl come to his/her mother)!

Scene 2.3. Fixed: children. Movable fathers and mothers appear couple by couple. Firstly, one couple is present.

Command:

Атаны/апаны уулуна/кызына келтир! (Let father/mother come to his/her son/daughter)!

5. Conclusion

This paper is a next contribution to our general project of developing mathematical models of various notions for independent presentation of natural languages. We hope that such software would be interesting and useful for people to learn languages.

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ON CONSTANTS RELATED TO EFFECT OF "NUMEROSITY"

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Mathematical constants can be defined as dimensionless numbers not depending on linear transformations of tasks. Supra, by the authors' definition, appearance of phenomena in systems only with large number of components is said to be the effect of numerosity. The least number of components preserving such phenomenon is said to be the constant related to it. Supra, the authors constructed a system of random difference equations to simulate the ancient popular synergetic process and a system of difference

equations to simulate mutual repelling of electrical charges on a topological torus. General notions of “effect” and “phenomenon” are presented and constants related to the mentioned phenomena are estimated in the paper.
Keywords: constant, numerosity, effect, phenomenon, definition, differential equation, difference equation

Маселени сызыктуу өзгөрткөндө өзгөртпөс, өлчөмсүз сан математикалык туруктуу болуп саналат. Мурда, авторлор сунуштаган аныктама боюнча, көп бөлүккө ээ болгон системада кубулуштар пайда болуу «көпчө» эффектиси деп аталган. Мындай кубулушту козгоочу бөлүктөрдүн эң аз мүмкүн болгон саны ал кубулушка байланган туруктуу болуп саналат. Мурда, авторлор белгилүү иргөө кубулушун чагылдыруучу, кокустук менен айырма теңдемелер системасын жана топологиялык шакекте электр заряддардын өз ара түртүлүшүн чагылдыруучу айырма теңдемелер системасын түздү. Макалада «эффект» жана кубулуш түшүнүктөрүнүн жалпы аныктамалары көрсөтүлдү жана айтылган кубулуштарга байланган туруктуулар бааланды.
Урунттуу сөздөр: туруктуу, «көпчө», эффект, кубулуш, аныктама, дифференциалдык теңдеме, айырмалуу теңдеме

Математические константы можно определить, как безразмерные числа, не изменяющиеся при линейных преобразованиях задач. Ранее, по определению авторов, возникновение явлений только для систем с большим количеством компонент названо эффектом «множественности». Самое малое число, вызывающее такое явление, называется постоянной, связанной с этим явлением. Ранее авторы построили систему случайных разностных уравнений, отражающую давно известный синергетический процесс, и систему случайных разностных уравнений, отражающую взаимное отталкивание электрических зарядов на топологическом торе. В статье представлены общие определения понятий «эффекта» и «явления» и оценены константы, связанные с упомянутыми явлениями.
Ключевые слова: константа, множественность, эффект, явление, определение, дифференциальное уравнение, разностное уравнение.

1. Introduction

Mathematical constants can be defined as dimensionless numbers being solutions of some tasks and not depending on linear transformations of tasks.

Discoveries of new "phenomena" and "effects" used to be sufficient steps in developing science but there were not definitions of these notions before our publication [1]. We gave corresponding definitions and examples, proposed methodic to search new phenomena.

Besides well-known effects of infinity, of multi-dimensionality, of singular perturbations we distinguished the effects of analyticity, of self-ordering.

Some of them correspond to real effects, other (the effect of infinity) are specifically mathematical.

The law of large numbers can be considered as some phenomena in statistic.

Supra, by our definition, appearance of phenomena in systems only with large number of components was said to be the effect of numerosity.

We found some phenomena due to this effect not related to statistic.

In this paper we propose a definition of constants related to this effect and estimate two of them.

2. Definitions

Consider a mathematical statement (theorem) in general as an implication of conditions $A \Rightarrow B$, or, more concrete, if there is a general class X of objects x and $A \subset X$, $B \subset X$ then $A \subset B$ or $((x \in A) \Rightarrow (x \in B))$. To search "phenomena" and "effects" more systematically we have proposed

Definition 1 [1]. To prove sufficiency of A for B one is to construct an example without both A and B. An (interesting, nonpresumable, single) way of violating B is said to be a *phenomenon*.

The notion “single” can be defined more exactly. Let X be a set and a measure *mes* can be introduced in it. Then a subset $P \subset X \backslash B$ is a phenomenon if $mes(P)=0$. In other words, if $x \in X$ then $x \in P$ ”almost never”.

Definition 2. If P is a property (or some properties) of elements $x \in X$ having a property E such that a logical proof $(E \wedge C) \Rightarrow P$ (where C is any additional condition) is too complicated and the property P was discovered not by a logical way but by meeting paradoxes, by experiments in physics and chemistry or by computational experiments in mathematics then E is said to be an *effect*.

These definitions yield the following methodic. If some objects $x \in X$ with different but similar unexpected properties have the same property E then this property is considered to be an effect. Putting additional conditions on x new phenomena may be found in the class X.

Definition 3. Appearance of phenomena in systems only with large number of components is said to be the effect of *numerosity*.

Definition 4. If a phenomenon occurs less often for number of components less than N and does more often for number of components greater than N then the number N is said to be the *constant of numerosity* for this phenomenon.

3. Constant related to the phenomenon of irgöö

The common Kyrgyz word *irgöö* means: discrete optimization by means of synergetic, or "random vibration of balls of different sizes of same material in a wide symmetrical vessel yields migration of the biggest one to the center of their surface." This experimental fact is too difficult to be proven by any mathematical model but can be corroborated by numerical experiments with a system of difference equations.

Thus, we stated

Hypothesis 1 [2]. For a large number of balls in a vessel, in a certain class of processes described by random difference equations, the probability of the event "the biggest ball is close to the center of surface of heap of balls" is 1 as time tends to infinity.

The cylinder of radius 1 is taken as a vessel. Let a (large) natural number n and (small) positive radiuses $r_1 > r_2 \geq ... \geq r_n$ be given.

Definition 5. If a set of n points $\{(x_k, y_k, z_k): k=1..n\} \subset R^3$ fulfills the conditions

1) $r_k \leq z_k$, $x_k^2 + y_k^2 \leq (1 - r_k)^2$ for all k in $1..n$ (all balls are in the vessel);

2) $(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2 \geq (r_j + r_k)^2$ for all $k \neq j$ in $1..n$ (the balls do not overlap) then such set is said to be *admissible.*

Definition 6. A (short) vector $\{u, v, w\}$ $(w<0)$ is said to be *admissible* for a given admissible set of points and a number k in $1..n$ if the set obtained by means of changing the k-th point to the point (x_k+u, y_k+v, z_k+w) is admissible too. Such passing from one set of points to a new set of points is said to be an *admissible* shift.

Algorithm 1 (of approximate calculations). For any initial admissible set of points repeat the following steps:

1) shift all points up with a (short) vector;

2) while it is possible, in the obtained admissible set of points execute random admissible shifts.

The adjusted

Hypothesis 2. With the probability 1, there exists such number M that after M steps there will be $x_1^2 + y_1^2 \leq r_1^2$ and there will not be other points over this point.

To verify this hypothesis a program was written in *pascal*. For example, let $n=50$ and $r_k=0.3-0.01k,\ k=1..19;\ r_k=0.1,\ k=20..50.$

Some runs of this program gave the constant of numerosity $N \sim 50$.

4. Constant related to self-ordering of electrical charges

We applied the effect of numerosity to search self-ordering of discrete electrical charges in viscous media [3], [4], [5]. Motion of equal, repelling by the Coulomb law electrical charges from a random initial distribution on a topological torus form a final regular grid was modeled by computer.

Motion of N electrical charges is described by a system of N two-dimensional differential equations. These differential equations are approximated by a system of difference equations.

The following program with graphical demonstration of the initial distribution and of the final one was written in *pascal* (with $N=256$).

```
program sabina; uses crt, graph;
var hxy,vx,vy,dx,dy,dxy,dxy1,hxy1,z,z2,xj,yj,dxy2,dxyd: double;
i,j,nxy,it,nt,np,ihand,n_time,ik: longint;
var drv, mode,f,n: integer; x,y:array[1..300] of double;
xn,yn:array[1..300] of integer;
begin {main} drv:=0; mode:=VgaHi; InitGraph(drv,mode,'c:\tp\bgi');
randomize; SetTextStyle(0,0,2);
OutTextXY(30,20,'Repelling 256 electrical charges on torus');
OutTextXY(100,40,'(Wait a little)'); z:=700.; z2:=z/2.0;
np:=10; {nt:=500*n_time;} hxy:=1.0; hxy1:=hxy; nt:=1000; nxy:=256;
for ik:=1 to nxy do begin  x[ik]:=z*random; y[ik]:=z*random;
xn[ik]:=round(x[ik]); yn[ik]:=round(y[ik]);
Setcolor(green); circle(xn[ik]+80,yn[ik]+70,2); end;
   for it:=0 to nt do begin {it} if it>np then hxy:=2.0*hxy1; if it>2*np then
hxy:=4.0*hxy1;
         for i:=1 to nxy do begin {i=ix} vx:=0.; vy:=0.; for j:=1 to nxy do
begin if j<>i then begin
```

```
xj:=x[j]; if xj>x[i]+z2 then xj:=xj-z; if xj<x[i]-z2 then xj:=xj+z;
yj:=y[j]; if yj>y[i]+z2 then yj:=yj-z; if yj<y[i]-z2 then yj:=yj+z;
dxy2:=sqr(x[i]-xj)+sqr(y[i]-yj)+1.;
dxy1:=z/(dxy2*sqrt(dxy2)); if dxy1<sqr(z)/nxy*0.5 then begin
dx:=(x[i]-xj)*dxy1; dy:=(y[i]-yj)*dxy1; vx:=vx+dx; vy:=vy+dy; end; end; end;
 x[i]:=x[i]+vx*hxy; if x[i]>z then x[i]:=x[i]-z; if x[i]<0. then x[i]:=x[i]+z;
 y[i]:=y[i]+vy*hxy; if y[i]>z then y[i]:=y[i]-z; if y[i]<0. then y[i]:=y[i]+z; end {i=ix};
for ik:=1 to nxy do begin xn[ik]:=round(x[ik]); yn[ik]:=round(y[ik]) end; end {it};
Setcolor(white);
repeat for ik:=1 to nxy do begin circle(xn[ik]+80,yn[ik]+70,8);
circle(xn[ik]+80,yn[ik]+70,6); circle(xn[ik]+80,yn[ik]+70,4);
circle(xn[ik]+80,yn[ik]+70,2) end;
delay(100); until keypressed; end.
```

Runs of this program found the constant of numerosity $N \sim 110$.

Also, when the number of charges is a square of even number then the grid is square in most of experiments; when it is a square of odd number then the grid is triangular in most of experiments.

Conclusion

Search for constants of numerosity yield certain conclusions on the indefinite notion “many”. We hope that such constants would be found for other real and virtual processes.

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MSC 54D05

AXIOMATIZATION OF KINEMATICAL SPACES

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Survey of peculiarities of axiomatization in mathematics based on works by A.A. Borubaev and G.M. Kenenbaeva, axiomatization of kinematical spaces is presented in the paper. A new axiomatization of controlled motion of objects in a space with bounded velocity is proposed in the paper. A new notion of generalized kinematical space is defined.
Key words: axiomatization, topological space, kinematical space, velocity, motion, length.

А.А. Бөрүбаев жана Г.М. Кененбаевалардын иштеринде математикадагы аксиомалаштыруунун өзгөчөлүктөрүн карoo жана кинематикалык мейкиндиктерди аксиомалаштыруу макалада көрсөтүлдү. Ылдамдыгы чектелген, мейкиндикте нерселердин башкарылган кыймылдоосун жаңы аксиомалаштыруу макалада сунушталды. Жалпыланган кинематикалык мейкиндиктин жаңы түшүнүгү аныкталган.
Урунттуу сөздөр: аксиомалаштыруу, топологиялык мейкиндик, кинематикалык мейкиндик, ылдамдык, кыймылдоо, узундук.

В статье представлены обзор особенностей аксиоматизации в математике на основе работ А.А. Борубаева и Г.М. Кененбаевой и аксиоматизация кинематических пространств. В статье предложена новая аксиоматизация управляемого движения объектов в пространстве с ограниченной скоростью. Определено новое понятие обобщенного кинематического пространства.
Ключевые слова: аксиоматизация, топологическое пространство, кинематическое пространство, скорость, движение, длина.

1. Introduction

The goal of this work is developing of a system of axioms to present controlled motion of stretched objects with bounded velocity.

The second section presents a survey of axiomatization as whole. A.A. Borubaev [1] made an extended survey of ideas and axiomatization of topology and uniform topology. On this base, in the series of works [2-4] a general survey of mathematics was done: firstly, some ideas appeared, effects and phenomena had been discovered; further, systems of axioms were developed.

The third section contains a survey of axiomatization of controlled motion of points in a topological space.

The fourth section presents controlled motion of stretched objects with bounded velocity. Topological structures on sets are built by introducing families of subsets meeting some properties. To generalize the notion of a kinematical space we propose to use a family of subsets having “length” (we will call them “passes”) and a family of subsets (we will call them “things”) which are to be moved along “passes”.

2. Survey of axiomatization in mathematics

A.A. Borubaev [1] wrote: Axiomatization of the notion of continuity had led to the notion of a topological space. There were two ways of axiomatization of the notion of uniform continuity: 1) through the proximity relation of two sets A and B (distance(A,B) is zero in a metric space) as development of P.S. Alexandroff’s and K. Kuratowski’s viewpoint on a topological space; 2) through axiomatization of properties of the system of ε-neighborhoods in a metric space as development of Hausdorff’s viewpoint. The first way had led to the construction of proximity spaces (V.A. Efremovich), the analysis of proximity spaces was held by Ju.M. Smirnov, the second way had led to the construction of uniform spaces (A. Weyl).

The first systematic exposition of the theory of uniform spaces in terms of entourages was given in Bourbaki’s book. Another, but equivalent to the previous concept of a uniform space and defined in terms of a family of coverings was introduced and studied by Tukey. Later, a broad and important study of uniform spaces in the terms of coverings was carried by Yu.M. Smirnov. Isbell’s book, in which the theory of uniform spaces got an important development, was also written in terms of the coverings.

One can see that the uniform spaces can also be described in terms of pseudometrics (L. Gilman, M. Jerison); in terms of metrics over semifields (M.J. Antonovskii, V.G. Boltyanskii, T.A. Sarymsakov); in terms of equivalent nets (V.A. Yefremovich, A.S. Schwarz) and small sets (V. Sandberg) and others.

On the base of this, in the series of works [2-4] a general survey of mathematics was done: firstly, some ideas appeared, effects and phenomena had been discovered; further, systems of axioms were developed. As different systems of axioms codify the same idea, they are equivalent.

3. Survey of axiomatization of motion of points with bounded velocity

As a codifying the ancient idea of controlled motion with bounded velocity, the notion of kinematical space was introduced [5].

Definition 3.1. A computer program is said to be a **presentation** of a computer kinematical space if:

P1) there is an (infinite) metrical space X of points and a set X_1 of display-presentable points being sufficiently dense in X;

P2) the user can pass from any point x_1 in X_1 to any other point x_2 by a sequence of adjacent points in X_1 by their will;

P3) the minimal time to reach x_2 from x_1 is (approximately) equal of the minimal time to reach x_2 from x_1.

The space X is said to be a **kinematic space**; the space X_1 is said to be a **computer kinematic space**; this minimal time is said to be the **kinematical distance** ρ_X between x_1 and x_2; a sequence of adjacent points is said to be a **route**. Passing to a limit as X_1 tends to X we obtain the following.

There is a set K of **routes**; each route M, in turn, consists of the positive real number T_M (**time** of route) and the function $m_M : [0, T_M] \to X$ (**trajectory** of route);

(K1) For $x_1 \neq x_2 \in X$ there exists such $M \in K$ that $m_M(0) = x_1$ and $m_M(T_M) = x_2$, and the set of values of such T_M is bounded with a positive number below;

(K2) If $M = \{T_M, m_M(t)\} \in K$ then the pair $\{T_M, m_M(T_M - t)\}$ is also a route of K (the reverse motion with same speed is possible); (cf. P3).

(K3) If $M = \{T_M, m_M(t)\} \in K$ and $T^* \in (0, T_M)$ then the pair: T^* and function $m^*(t) = m_M(t) \quad (0 \le t \le T^*)$ is also a route of K (one can stop at any desired moment);

(K4) concatenation of routes for three distinct points x_1, x_2, x_3.

If there exists a kinematic consistent with the given metric then the metric space is said to be **kinematizable**.

A similar definition also based on the notion of path was proposed in [6].

Denote the set of connected subsets of R as In. A *path* is a continuous map $\gamma : In \rightarrow X$ (a topological space).

Definition 3.2. The following definition is composed of some definitions in [6] reduced to a "a priori" bounded, path-connected space X. A length structure in X consists of a class A of admissible paths together with a function (length) $L: A \rightarrow R_+$.

The class A has to satisfy the following assumptions:

(A1) The class A is closed under restrictions: if $\gamma \in A$, $\gamma : [a, b] \rightarrow X$ and $[u, v] \subset [a, b]$ then the restriction $\gamma|_{[u, v]} \in A$ and the function L is continuous with respect to u,v;

(A2) The class A is closed under concatenations of paths and the function L is additive correspondingly. If a path $\gamma : [a, b] \rightarrow X$ is such that its restrictions γ_1, γ_2 to $[a, c]$ and $[c, b]$ belong to A, then so is γ.

(A3) The class A is closed under linear reparameterizations and the function L is invariant correspondingly: for a path $\gamma \in A$, $\gamma : [a, b] \rightarrow X$ and a homeomorphism $\varphi : [c, d] \rightarrow [a, b]$ of the form $\varphi(t) = \alpha t + \beta$, the composition $\gamma(\varphi(t))$ is also a path.

(A4) (similar to (Kl)).

The metric in X is defined as

$\rho_L(z_0, z_1) := \inf\{L(\gamma) \mid \gamma : [a, b] \rightarrow X; \gamma \in A; \gamma(a) = z_0; \gamma(b) = z_1\}$.

4. Axiomatization of motion of points with bounded velocity

We [7] proposed controlled motion of stretched sets in topological spaces with bounded velocity based on motion of points as [1].

We propose more general definition.

Consider the practical task. Let there be a thing and obstacles. It is necessary to move the thing to another place. Is possible? If yes then in what minimal time it can be done?

Definition 4.1. There is a family P of connected subsets of the set X (we will call them **passes**); each pass has the positive **length (time)** and a family Q of connected subsets of the set X (we will call them **things**).

(It means that a thing moves along a pass).

The space X is said to be a **generalized kinematic space**.

(G1) For each $x\in p\in P$ there exists such $q\in Q$ that $x\in q$ [a thing can be in each place of a pass].

(G2) For each $x_1 \neq x_2\in X$ there exists such pass $p\in P$ that x_1 , x_2 $\in P$ and the set of lengths of such p is bounded with a positive number below; this infinum is said to be the **generalized kinematical distance** ρ_X between points x_1 and x_2.

(G3) For each $q_1 \neq q_2\in Q$ there exists such pass $p\in P$ that q_1 , q_2 $\in P$ and the set of lengths of such p is bounded with a positive number below; this infinum is said to be the **generalized kinematical distance** ρ_X between things p_1 and p_2.

(G4) If x_1 , x_2 $\in p_1$ and x_2 , x_3 $\in p_2$ then there exists such pass $p_3\in P$ that x_1 , x_2, x_3 $\in p_3$ and $length(p_3) \leq length(p_1)+ length(p_2)$.

(G5) If for each $x_1 \neq x_2\in X$ there exists such pass $p_{12}\in P$ that $length(p_{12}) =\rho_X (x_1 , x_2)$ then the generalized kinematical space X is said to be **flat** (with respect to P).

If there exists a generalized kinematic (families P and Q) consistent with the given Hausdoff metric for Q then the metric space is said to be K-kinematizable.

If $Q=X$ then Definition 4.1 generalizes Definition 3.1.

Example. It is known that the space R^2 with the metric

$$\rho_0((x_1,y_1), (x_2,y_2)):=max\{| x_1-x_2|, | y_1-y_2|\}$$

is not kinematizable in the sense of Definition 3.1. Let passes in R^2 be squares

$$S(x_0,y_0 ,h):= \{(x,y)| 0\leq x_0 \leq x\leq x_0+h\leq 1; 0\leq y_0\leq y\leq y_0+h\}$$

with length h.

Then the set

$$\{(x_1,y_1), (x_2,y_2)\}\subset S(min\{x_1,x_2\}, min\{y_1,y_2\}, max\{|x_1-x_2|, |y_1-y_2|\})$$

with the length being equal to $\rho_0((x_1,y_1), (x_2,y_2))$.\

Hence, the space R^2 with the metric ρ_0 is a generalized kinematic space.

5. Conclusion

We hope that the new definitions in this paper would provide effective computer presentations for motion of things in virtual and real spaces.

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MSC 45D05

SPECTRAL PROPERTIES OF LINEAR EQUATIONS WITH INTEGRALS ON UNBOUNDED DOMAINS IN ANALYTIC FUNCTIONS

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Supra the author constructed and implemented the following algorithms on a computer. Given an equation of Volterra type with power coefficients by integral summands, the algorithm presents data to detect existence of solutions and occurrence of arbitrary constants in it; also, given an equation with a coefficient, specific values of the coefficient are found. In this paper such items are considered for equations on unbounded domains.

Keywords: integral equation, unbounded domain, Volterra equation, algorithm, analytical function.

Мурда, автор төмөнкү алгоритмдерди түзүп жана компьютерде жүзөгө ашырган. Даражалуу көбөйтүндүлүү интегралдык кошулуучулары бар теңдеме берилген. Теңдемеде үчүн алгоритм чыгарылышынын жашоосун аныктоо жолуна мүмкүндүгүн жана анда каалагандай турактуу сан бар экендигин аныктоо үчүн малыматты берет. Ошондой эле коэффициент менен теңдеме сунушталган, бул теңдемеде коэффициенттин өзгөчө маанисин табуу, анализдөө маселеси каралат. Бул макалада бул сыяктуу маселелер чектелбеген аймакта теңдемелер үчүн каралат.

Урунттуу сөздөр: интегралдык теңдеме, чектелбеген аймак, Вольтерра тибиндеги теңдеме, алгоритм, аналитикалык функция.

Ранее автор построила и реализовала на компьютере следующие алгоритмы. Дано уравнение со степенными сомножителями при интегральных слагаемых, алгоритм представляет данные для определения существования решения и наличия в нем произвольных постоянных; также дано уравнение с коэффициентом, находятся особые значения коэффициента. В данной статье такие вопросы рассматриваются для уравнений на неограниченных областях.

Ключевые слова: интегральное уравнение, неограниченная область, уравнение типа Вольтерра, алгоритм, аналитическая функция.

Introduction

Before our publications, we did not find investigations on spectra of Volterra equations with a parameter. Supra we constructed and implemented the following algorithms on a computer [1-4]. Given a linear equation with power coefficients by integral summands, the algorithm presents data to detect existence of solutions and occurrence of arbitrary constants in it; also, given an equation with a coefficient, specific values of the coefficient are found. Non-linear equations are considered in [5].

In this paper such items are considered for equations on unbounded domains.

1. Statement of problem

We will use denotations

***R** := (-∞;∞); **R**$_+$:= [0; ∞); **R**$_{++}$:= (0; ∞); **N**$_0$:= {0, 1, 2, 3, ...}; **N** := { 1, 2, 3, ...}.*

We will use the term "Algorithm" as it is usually understood in Analysis: arithmetical operations and comparison over numbers in ***R*** (for rational numbers this definition coincides with the strict one).

We will write discrete arguments in brackets to bring denotations nearer to algorithmic ones and to bypass the common ambiguity of expressions such as a_{2j}.

We will consider equations of type

$$P(t,D)u(t)+\lambda\int_{-\infty}^{t}K(t,s)s^{n}u(s)ds=e^{t}f(t) \qquad (1)$$

where $D=d/dt$, $P(t,D)$ is a polynomial with respect to D with analytic coefficients, $f(t)$ and $K(t,s)$ is an analytic function, $\lambda\in\boldsymbol{R}$ is a coefficient, the integral is supposed to converge.

Particularly, we will consider the equation

$$u(t)+\lambda\int_{-\infty}^{t}s^{n}u(s)ds=e^{t}f(t),\ n\in N_0. \qquad (2)$$

We will consider given and unknown real-valued analytical functions in the form

$$f(t)=f[0]+f[1]t+f[2]t^2+\ldots, \qquad (3)$$

$$u(t)=e^{t}(u[0]+u[1]t+u[2]t^2+\cdots). \qquad (4)$$

Denote $Z_n(t)=e^{-t}\int_{-\infty}^{t}e^{s}s^{n}ds, n\in N_0$. It is known that $Z_0(t)=1$,

$$Z_n(t)=t^n-nZ_{n-1}(t), n\in N.$$

$$Z_1(t)=t-1, Z_2(t)=t^2-2t+2,\ Z_3(t)=t^3-3t^2+6t-6\ \ldots \qquad (5)$$

2. Particular cases of integral equations

Let $n=0$ in (2):

$$u(t)+\lambda\int_{-\infty}^{t}u(s)ds=e^{t}f(t). \qquad (6)$$

Substituting (3) and (4) we obtain

$$e^{t}(u[0]+u[1]t+u[2]t^2+u[3]t^3+\cdots)+$$

$$+\lambda\int_{-\infty}^{t}e^{s}(u[0]+u[1]s+u[2]s^2+u[3]s^3+\cdots)ds=$$

$$= e^t(f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots),$$

$$u[0] + u[1]t + u[2]t^2 + u[3]t^3 + \cdots +$$

$$+\lambda(Z_0(t)u[0] + Z_1(t)u[1] + Z_2(t)u[2] + Z_3(t)u[3] + \cdots) =$$

$$= f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots,$$

$$u[0] + u[1]t + u[2]t^2 + u[3]t^3 + \cdots +$$

$$+\lambda(u[0] + (t-1)u[1] + (t^2 - 2t + 2)u[2] + (t^3 - 3t^2 + 6t - 6)u[3] + \cdots)$$

$$= f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots.$$

We obtain the following system of linear algebraic equations

$$u[0] + \lambda(u[0] - u[1] + 2u[2] - 6u[3] + \cdots) = f[0];$$

$$u[1] + \lambda(u[1] - 2u[2] + 6u[3] + \cdots) = f[1];$$

$$u[2] + \lambda(u[2] - 3u[3] + \cdots) = f[2];$$

$$u[3] + \lambda(u[3] + \cdots) = f[3]$$

. . .

Theorem 1. If *f(t)* is a polynomial then the equation (6) has the unique polynomial solution for $\lambda \neq -1$; If $\lambda = -1$ then the equation (6) either has no solution or has endless number of solutions; the spectrum is $\lambda \in \{-1\}$.

Let $n=1$ in (2):

$$u(t) + \lambda \int_{-\infty}^{t} su(s)ds = e^t f(t). \tag{7}$$

Substituting (3) and (4) we obtain

$$e^t(u[0] + u[1]t + u[2]t^2 + u[3]t^3 + \cdots) +$$

$$+ \lambda \int_{-\infty}^{t} e^s s(u[0] + u[1]s + u[2]s^2 + u[3]s^3 + \cdots)ds =$$

$$= e^t(f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots),$$

$$u[0] + u[1]t + u[2]t^2 + u[3]t^3 + \cdots +$$

$$+\lambda(Z_1(t)u[0] + Z_2(t)u[1] + Z_3(t)u[2] + Z_4(t)u[3] + \cdots) =$$

$$= f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots,$$

$$u[0] + u[1]t + u[2]t^2 + u[3]t^3 + \cdots +$$

$$+\lambda\big((t-1)u[0] + (t^2 - 2t + 2)u[1] + (t^3 - 3t^2 + 6t - 6)u[2] + \cdots\big) =$$

$$= f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots$$

$$u[0] + \lambda(-u[0] + 2u[1] - 6u[2] + \cdots) = f[0];$$
$$u[1] + \lambda(-2u[1] + 6u[2] + \cdots) = f[1];$$
$$u[2] + \lambda(-3u[2] + \cdots) = f[2].$$

Theorem 2. If the function *f(t)* is a polynomial then the equation (7) has the unique polynomial solution for $\lambda \neq 1/k,\ k \in N$ otherwise for some *f(t)* then the equation (6) either has no solution or has endless number of solutions; the spectrum is endless.

Theorem 3. If $n>0$ and the function *f(t)* is a polynomial then the equation (2) has endless spectrum.

3. An integro-differential equation

We will consider the equation

$$u'(t) + \lambda \int_{-\infty}^{t} u(s)ds = e^t f(t). \qquad (8)$$

Substituting (3) and (4) we obtain

$$e^t(u[0] + u[1] + (u[1] + 2u[2])t + (u[2] + 3u[3])t^2 + \cdots) +$$
$$+ \lambda \int_{-\infty}^{t} e^s (u[0] + u[1]s + u[2]s^2 + u[3]s^3 + \cdots)ds =$$
$$= e^t(f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots),$$
$$u[0] + u[1] + (u[1] + 2u[2])t + (u[2] + 3u[3])t^2 + \cdots +$$
$$+\lambda(Z_0(t)u[0] + Z_1(t)u[1] + Z_2(t)u[2] + Z_3(t)u[3] + \cdots) =$$
$$= f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots,$$
$$u[0] + u[1] + (u[1] + 2u[2])t + (u[2] + 3u[3])t^2 + \cdots +$$
$$+\lambda(u[0] + (t-1)u[1] + (t^2 - 2t + 2)u[2] + (t^3 - 3t^2 + 6t - 6)u[3] + \cdots)$$
$$= f[0] + f[1]t + f[2]t^2 + f[3]t^3 + \cdots.$$

We obtain the following system of linear algebraic equations

$$u[0] + u[1] + \lambda(u[0] - u[1] + 2u[2] - 6u[3] + \cdots) = f[0];$$
$$u[1] + 2u[2] + \lambda(u[1] - 2u[2] + 6u[3] + \cdots) = f[1];$$
$$u[2] + 3u[3] + \lambda(u[2] - 3u[3] + \cdots) = f[2];$$
$$u[3] + 4u[4] + \lambda(u[3] + \cdots) = f[3]$$

. . .

Theorem 4. If *f(t)* is a polynomial then the equation (8) has the unique polynomial solution for $\lambda \neq -1$; If $\lambda = -1$ then the equation (8) either has no solution or has endless number of solutions; the spectrum is $\lambda \in \{-1\}$.

5. Conclusion

There can be various phenomena for equations of type (1) including infinite spectrum. Algorithms can be constructed for some subclasses of such equations.

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QUOTIENT SPACES ARISING IN ASYMPTOTICAL BEHAVIOR OF SOLUTIONS OF DELAY-DIFFERENTIAL EQUATIONS

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Supra, the author introduced the following equivalence relation is in the space of solutions of initial value problems for dynamical systems: distance between two solutions tends to zero while time increases. The phenomenon "the dimension of the quotient space is less than one of the initial space" was called "asymptotical reduction of dimension of space of solutions". Also, the author introduced the Hausdorff asymptotical equivalence relation is introduced: distance between two solutions with invertible transformation of argument tends to zero. The corresponding quotient spaces are considered in this paper.

Keywords: quotient space, equivalence relation, asymptotical equivalence, delay-differential equation, initial value problem, Hausdorff metric.

Мурда автор динамикалык системалардын чыгарылыштарынын мейкиндигинде төмөнкүдөй асимптотикалык эквиваленттүүлүктүн түшүнүгүн киргизди: убактыт өскөндө эки чыгарылыштын арасында аралык нөлгө умтулат. Фактор-мейкиндиктин ченеми баштапкы мейкиндиктин ченеминен кичүү болгон кубулуш, «чыгарылыштар мейкиндигинин ченемин асимптотикалык төмөндөтүү» деп айтылган. Дагы, автор хаусдорфтук асимптотикалык эквиваленттүүлүктүн түшүнүгүн киргизди: убактыт өскөндө аргументин кайра калыбына келтирүүчү өзгөртүү менен эки чыгарылыштын арасында аралык нөлгө умтулат. Бул макалада дал келген фактор-мейкиндиктер каралат.

Урунттуу сөздөр: фактор-мейкиндик, эквиваленттик катышы, кечигүүчү аргументтүү дифференциалдык теңдеме, баштапкы маселе, хаусдорфтук метрика.

Ранее автор ввела следующее отношение асимптотической эквивалентности в пространстве решений начальных задач для динамических систем: расстояние между двумя решениями стремится к нулю при увеличении времени. Явление «размерность фактор-пространства меньше, чем размерность исходного пространства» было названо «асимптотическое уменьшение размерности пространства решений». Автор также ввела понятие хаусдорфовой асимптотической эквивалентности: неограниченное сближение решений с обратимым преобразованием аргумента с увеличением времени. Соответствующие фактор-пространства рассматриваются в этой статье.

Ключевые слова: фактор-пространство, отношение эквивалентности, асимптотическая эквивалентность, дифференциальное уравнение с запаздывающим аргументом, начальная задача, хаусдорфова метрика

1. Introduction

Is one of the main in the theory of dynamical systems. Many mathematical methods were developed for investigation of the problem of behavior of solutions of initial value problems as time tends to infinity including the theory of stability [1]-[2], method of characteristic equations for autonomous and periodical dynamical systems, method of special solutions for delay-differential equations [3]-[4]. Various sufficient conditions were obtained to provide some kinds of behavior of solutions. Various definitions and denotations were introduced for each kind.

Supra, the author [5] introduced the following equivalence relation is in the space of solutions of initial value problems for dynamical systems: distance between two solutions tends to zero while time increases. The phenomenon "the dimension of the quotient space is less than one of the initial space" was called "asymptotical reduction of dimension of space of solutions". Also, the author [6] introduced the Hausdorff asymptotical equivalence relation: distance between two solutions with invertible transformation of argument tends to zero.

This paper demonstrates that the corresponding quotient spaces generate new mathematical objects.

Section 2 contains definitions of asymptotical equivalence and λ-exponential asymptotical equivalence and the phenomenon of asymptotical reduction of dimension.

Section 3 proposes definitions of Hausdorff asymptotical equivalence and Hausdorff asymptotical quotient space.

Section 4 contains examples of Hausdorff asymptotical quotient space for various types of differential equations.

Denote $\boldsymbol{R}_{+}:=[0,\infty)$; $\boldsymbol{R}_{++}:=(0,\infty)$.

2. Review of preceding definitions

"Ordinary" equations and systems of equations with delay in more general form can be presented as follows (we are restricted with existence and uniqueness of solution of an initial value problem).

Definition 1. A dynamical system is a tuple of a number $h\geq 0$ [delay], a totally ordered set Λ of real numbers with the least element but without the greatest one [domain of functions]: $\Lambda=\boldsymbol{R_h}:=[-h,\infty)$ or $\Lambda=\boldsymbol{N_0}:=\{0,1,2,3,\ldots\}$, a topological space Z [range of functions]; a set Φ of functions $[-h,0]\to Z$ [initial conditions]; if $h=0$ then $\Phi=Z$; a function $W(t,\varphi)$: $\Lambda\times\Phi\to Z$ such that its restriction on $[-h,0]$ equals φ

[solutions of initial value problems]. If $\Lambda=\boldsymbol{R_h}$ then $W(t,\varphi)$ is supposed to be continuous with respect to t.

We will consider the following classes of spaces with their dimensions:

1-spaces: $Z = \boldsymbol{R}$; dimension $=1$;

d-spaces: $Z =\boldsymbol{R}^d$, $d \in N:=\{1,2,...\}$; dimension = d;

N-spaces: Z is a normed linear space with norm $||\cdot||_Z$; dimension (finite or infinite) is the number of elements in the basis;

M-spaces: Z is a metric space with metric ρ_Z; the inductive Ind-dimension is used;

U-spaces: Z is a uniform space with set of entourages Y_Z; Ind-dimension is used.

Definition 1. (The most general are U-Spaces). The following equivalence is said to be asymptotical equivalence (λ-exponential asymptotical equivalence):

For N-spaces

$$(\varphi_1 \sim \varphi_2) \Leftrightarrow (\lim\{ \, ||W(t,\varphi_1)-W(t,\varphi_2)||_Z: t\to\infty\}= 0);$$

$$((\varphi_1 \sim_\lambda \varphi_2) \Leftrightarrow (\sup\{ \, ||W(t,\varphi_1)-W(t,\varphi_2)||_Z \exp(\lambda t): t\in\Lambda\}< \infty));$$

For M-spaces

$$(\varphi_1 \sim \varphi_2) \Leftrightarrow (\lim\{ \rho_Z(W(t,\varphi_1), W(t,\varphi_2)): t\to\infty\}= 0).$$

$$((\varphi_1 \sim_\lambda \varphi_2) \Leftrightarrow (\sup\{ \rho_Z(W(t,\varphi_1), W(t,\varphi_2)) \exp(\lambda t): t\in\Lambda\}< \infty)).$$

For U-spaces:

$$(\varphi_1 \sim_\lambda \varphi_2) \Leftrightarrow (\forall V\in Y_Z)(\exists\, t_1 \in\Lambda) \, (\forall t >t_1)((W(t,\varphi_1), W(t,\varphi_2)) \in V).$$

Definition 2. The factor-space $\Phi^*:=\Phi /\sim$ of the space Φ by the asymptotical equivalence is said to be an asymptotical quotient space; respectively, the quotient space $\Phi^*_\lambda:=\Phi /\sim_\lambda$ of the space Φ by the λ-exponential asymptotical equivalence is said to be λ-exponential asymptotical quotient space.

Example 1. (The Floquet-Lyapunov theory).

Some types of linear autonomous delay-differential equations have countable sets of characteristic values $\{\mu_1, \mu_2,...\}$ which can be semi-ordered:

$$Re(\mu_1)\geq Re(\mu_2) \geq ...;$$

$\lim\{Re \, (\mu_k): k \to\infty\} = -\infty$ such that functions $\exp(\mu_k \, t)$ (and for multiple values also $\exp(\mu_k \, t+ \nu_k \ln t)$, $\nu_k \in \boldsymbol{N}$), are (components of) particular solutions.

If $W(t, \varphi)$ can be presented as $\Sigma\{c_k(\varphi)\exp(\mu_k t+\nu_k \ln t): k\in \mathbf{N}\}$ where $c_k(\varphi)$ are linear operators then the phenomenon "asymptotical reduction of dimension of space of solutions" takes place, the infinite-dimensional space Φ reduces to the space with basis $\{\exp(\mu_k t+\nu_k \ln t): Re\ \mu_k \geq 0\}$.

3. Definitions of Hausdorff asymptotical equivalence and asymptotical quotient space

Definition 3 ($\Lambda=\mathbf{R}_h$ in this section). Let $s\in\mathbf{R}_+, \vartheta:[s,\infty)\to \mathbf{R}_+$ be of the class Θ of strictly increasing continuous functions, $\lim\{\vartheta(t):t\to\infty\}=\infty$.

The following equivalence is said to be Hausdorff asymptotical equivalence:

For N-spaces $(\varphi_1 \simeq \varphi_2) \Leftrightarrow (\forall\varepsilon\in \mathbf{R}_{++})(\exists s, \vartheta)(\forall t\in[s,\infty))\ (||W(t,\varphi_1)- W(t,\varphi_2)||_Z <\varepsilon)$;

For M-spaces $(\varphi_1 \simeq \varphi_2) \Leftrightarrow (\forall\varepsilon\in \mathbf{R}_{++})(\exists s, \vartheta)\ (\forall t\in[s,\infty))(\rho_Z(W(t,\varphi_1), W(\vartheta(t),\varphi_2))<\varepsilon)$;

For U-spaces $(\varphi_1 \simeq \varphi_2) \Leftrightarrow (\forall\varepsilon\in\Gamma_Z)\ (\forall t\in[s,\infty))(\ (W(t,\varphi_1), W(\vartheta(t),\varphi_2))\ \in\varepsilon)$.

Lemma 1 [6]. The introduced relation is a correct relation of equivalence.

Proof. Reflexivity of the relation $\simeq$ is obvious. Prove the symmetricity. Let $\varphi_1 \simeq \varphi_2$. There exists the inverse function $\zeta(t)\in\Theta$ to the function $\vartheta(t)$.

Substituting $\zeta(t)$ instead of t into (1), we obtain:

$$(\forall\zeta(t)\in[s,\infty))(\ (W(\zeta((t)),\varphi_1), W(\vartheta(\zeta(t)),\varphi_2))\ \in\varepsilon).$$

The condition $\zeta(t)\geq s$ is equivalent to the condition $\vartheta(\zeta(t))\geq \vartheta(s)$. Hence

$$(\forall t\in[\vartheta(s),\infty))(\ W(t),\varphi_2), (W(\zeta(t),\varphi_1))\in\varepsilon);\ \varphi_2 \simeq \varphi_1.$$

Prove the transitivity. For given $\varepsilon\in \Gamma_Z$ find such $\varepsilon_1\in \Gamma_Z$ that $\varepsilon_1 \circ \varepsilon_1 \subset \varepsilon$. There exist such s_{12}, s_{23}, $\vartheta_{12}(t)$, $\vartheta_{23}(t)$, that

$$(\forall t\in[s_{12},\infty))(\ (W(t,\varphi_1), W(\vartheta_{12}(t),\varphi_2))\ \in\varepsilon_1),$$

$$(\forall t\in[s_{23},\infty))(\ (W(t,\varphi_2), W(\vartheta_{23}(t),\varphi_3))\ \in\varepsilon_1).$$

Substituting $\vartheta_{12}(t)$ instead of t, we obtain

$$(\forall\vartheta_{12}(t)\in[s_{23},\infty))(\ (W(\vartheta_{12}(t),\varphi_2), W(\vartheta_{23}(\vartheta_{12}(t)),\varphi_3))\ \in\varepsilon_1).$$

The condition $\vartheta_{12}(t)\geq s_{23}$ is equivalent to the condition $t \geq \zeta_{12}(s_{23})$. Hence the preceding assertion can be written as

$$(\forall t\in[\zeta_{12}(s_{23}),\infty))(\ W(\vartheta_{12}(t),\varphi_2), (W((\vartheta_{23}\vartheta_{12})(t),\varphi_3))\in\varepsilon_1).$$

If we choose $s_{13}=max\{s_{12}, \zeta_{12}(s_{23})\}$ then the above assertions imply

$(\forall t\in[s_{13},\infty))(((W(t,\varphi_1), W(\vartheta_{12}(t),\varphi_2)) \in\varepsilon_1)\wedge (W(\vartheta_{12}(t),\varphi_2), (W((\vartheta_{23}\vartheta_{12})(t),\varphi_3))\in\varepsilon_1))$.

Hence

$$(\forall t\in[s_{13},\infty))((W(t,\varphi_1), W((\vartheta_{23}\vartheta_{12})(t),\varphi_3))\in\varepsilon_1\circ\varepsilon_1).$$

The transitivity is proven. Lemma is proven. A Hausdorff asymptotical quotient space will be denoted as $\Phi^{*=}$.

4. Examples of objects generated by quotient spaces

Solutions of scalar differential equations ($\Lambda=Z=\boldsymbol{R}$).

Example 2 [6]. All continuous and increasing to infinity functions are Hausdorff asymptotically equivalent. All continuous, increasing and tending to any number fun-ctions are Hausdorff asymptotically equivalent.

Example 3 [6]. All solutions of the equation $z'(t)=az(t),\ a>0$ form three classes of Hausdorff quotient space $\Phi^{*=}$.

Solutions of vector differential equations.

Example 4. We consider from out point of view [7]. System of differential equations

$$x''(t) = Z_x'(x(t),y(t)) \,/\left(\left(Z_x'(x(t),y(t))\right)^2 + (Z_x'(x(t),y(t)))^2 + 1\right),$$

$$y''(t) = Z_x'(x(t),y(t)) \,/\left(\left(Z_x'(x(t),y(t))\right)^2 + (Z_x'(x(t),y(t)))^2 + 1\right).$$

Z(x,y) is the surface defined by the formula

$$Z(x,y) = \sum_{j=1}^{3}\left(\left(x-\cos(2\pi j/3)\right)^2 + \left(y-\sin(2\pi j/3)\right)^2 + 0.01\right)^{-1} + x^2 + y^2.$$

If the initial conditions is $(-a;0),\ 0< a<1$ then the point moves along the li-ne $(-a<x<0;\ y=0)$, further does along the line $(0<x<\varepsilon;\ y=0),\ \varepsilon<a$, and further motion is unpredicable.

Conclusion

We hope that consecutive revealing of functions being Hausdorff asymptotically equivalent for various types of differential equations would yield new mathematical objects and it would be interesting for investigation of equations.

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MSC 34A26, 35A16

ENLARGING OF DOMAINS OF SOLUTIONS BY MEANS OF FUNCTIONAL RELATIONS

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Solutions of some differential equations have values connected by functional relations. For examples, even, odd and periodical solutions, Vallée-Poussin’s assertion, Lagrange interpolation polynomial for ordinary differential equations, Asgeirsson’s identity for partial differential equations of hyperbolic type are considered. In the paper functional relations are used for enlarging of domains of solutions. An algorithm for rectangular domains is built.

Keywords: functional relation, ordinary differential equation, partial differential equation, solution, domain, algorithm.

Ар кандай типтеги айрым дифференциалдык теңдемелердин чечимдеринде функционалдык өз ара байланыштарга байланыштуу маанилер бар. Мисалы, жуп, так жана мезгилдүү чыгарылыш, Валле-Пуссен катышы, кадимки дифференциалдык теңдемелер үчүн Лагранждын интерполяциялык полиному, гиперболалык типтеги дифференциалдык теңдемелер үчүн Асгейрссон бирдейлиги каралат. Макалада функционалдык өз ара байланыштар чыгарылыштарды аныктоо чөйрөсүн кеңейтүү үчүн колдонулат. Тик бурчтуу аймактар үчүн алгоритм курулат.
Урунттуу сөздөр: функционалдык өз ара байланыш, чыгарылыш, кадимки дифференциалдык теңдеме, айрым туундулуу дифференциалдык теңдеме, чыгарылыш, аныктоо аймагы, алгоритм.

Решения некоторых дифференциальных уравнений различных типов имеют значения, связанные функциональными соотношениями. Например, рассмотрены четные, нечетные и периодические решения, соотношение Валле-Пуссена, интерполяционный многочлен Лагранжа, тождество Асгейрссона для дифференциальных уравнений гиперболического типа. В статье функциональные соотношения применяются для расширения областей определения решений. Построен алгоритм для прямоугольных областей.
Ключевые слова: функциональное соотношение, решение, обыкновенное дифференциальное уравнение, дифференциальное уравнение в частных производных, решение, область определения, алгоритм.

1. Introduction

It is known that solutions of some types of differential equations have functional relations (in our terminology) connecting their values in different points. By given values of solutions in several points one can find their values in other points. In the paper we use functional relations for enlarging of domains of solutions.

Sometimes known values of function in some points (multi-point value problem) define it within all the domain. Otherwise, we propose

Definition. If the function $f(x):X\to F$ is known on a set $X_0 \subset X$, there is a functional relation (*) on values of $f(x)$ and $f(x)$ can be defined on a set X_1 by means of (*), $X_0 \subset X_1 \subset X$ then X_1 is said to be an (*)-enlarging of X_0.

The second section contains examples of functional relations with one-step enlargings. The third section contains an algorithm for enlarging of rectangular domains for Asgeirsson's identity.

In this paper we will use functional denotations of type *x[n]* instead of x_n.

2. Examples of functional relations with enlarging and value problems

Denote the functional relation number F for every equation as the minimal number of connected points (if it exists). We will give either mention of k-point value problem (k-PVP) or a formula for (*)-enlarging.

2.1. Odd and even functions on R. $F=2$, $X_1=\{x\in R|\ (\exists x_0\in R)(\ |x|=|\ x_0|)$.

2.2. The set of IVP $y'(x)=a$, $y(0)=0$, arbitrary $a\neq 0$: $F=2$:

$$(*)\ y(x[1])x[2]-y(x[2])x[1]=0.\ 1\text{-PVP}.$$

2.3. The set of IVP: $y'(x)=a$, $y(0)=y_0$, arbitrary $a\neq 0, y_0$; $F=3$:

$$(*)\ (y(x[1])-y(x[3]))(x[1]-x[2])-(y(x[1])-y(x[2]))(x[1]-x[3])=0.\quad 2\text{-PVP}.$$

2.4. The linear differential equation of the k-th order $y^{(k)}(x)=0$, or a polynomial of $(k-1)$-th order: $F=k+1$. Let numbers $x[1], x[2],\dots,x[k+1], y[1], y[2],\dots,y[k+1]$ be given. Construct the Lagrange interpolation polynomial of the $(k-1)$-th order by the values $x[1], x[2],\dots,x[k]$ и $y[1], y[2],\dots,y[k]$ then (*) $L(x[k+1])-y[k+1]=0$. k-PVP.

2.5. The first result on functional relations (in our terms) for a linear ordinary differential equation was obtained by C.J. de la Vallée Poussin (for instance see [1]): the k-PVP $y^{(k)}(x)+p_1(x)\,y^{(k-1)}(x)+\dots+p_k(x)\,y(x)=0,\ a\le x\le b,$

$p_k(x)\in C[a,b],\ y(x[i])=c[i],\ i=1,\dots,k$ has a unique solution when

$$||\,p_1||_{[a,b]}(b-a)+||\,p_2||_{[a,b]}(b-a)^2/2!+\dots+||\,p_n||_{[a,b]}(b-a)^n/n! < 1.$$

2.6. A solution of the hyperbolic equation $\frac{\partial^2}{\partial x_1\,\partial x_2}u(x_1,x_2)=0$ meets the Asgeirsson's identity $(F=4)$: (*) $u(w_1,v_1)+u(w_2,v_2)-u(w_1,v_2)-u(w_2,v_1)\equiv 0$.

It is considered in the next section.

2.7. A solution of the wave equation $\frac{\partial^2}{\partial x_1^2}u(x_1,x_2)=\frac{\partial^2}{\partial x_2^2}u(x_1,x_2)$ meets the similar Asgeirsson's identity $(F=4)$: for four vertices of a rectangle obtained by means of rotation of the rectangle (6) on 45^0.

3. Algorithm for Asgeirsson's identity

The definition: $N_0=\{0, 1,2,\dots\}$.

$$X_1=\{(x,y)\in R^2|\ (\exists (x_0,y_0)\in R^2)$$
$$(\ ((x_0,y_0)\in X_0)\ and\ ((x_0,y)\in X_0)\ and\ ((x,y_0)\in X_0)))\}.$$

In other words, only three points defined by four numbers

$$(x_0,y_0),\ (x_0,y_1),\ (x_1,y_0) \qquad (1)$$

generate the fourth point (x_1,y_1).

Example of infinitely many steps: the set

$$X_0 := \{(2-2^{1-n}, 2-2^{-n}) \in R^2 | n \in N\} \cup \{(2-2^{-n}, 2-2^{-n}) \in R^2 | n \in N\} \cup \{(0,0)\}.$$

At the first step, the only triple (1) is $(0,0)$, $(2-2^{1-0}, 2-2^{-0}) \equiv (0,1)$, $(2-2^{-0}, 2-2^{1-0}) \equiv (1,0)$, the fourth point is $(1,1)$.

At the second step, the only triple (1) is $(1,1)$, $(2-2^{1-1}, 2-2^{-1}) \equiv (1,1.5)$, $(2-2^{-1}, 2-2^{1-1}) \equiv (1.5,1)$, the fourth point is $(1.5,1.5)$.

At the third step, the only triple (1) is $(1.5, 1.5)$, $(2-2^{1-2}, 2-2^{-2}) \equiv (1.5,1.75)$, $(2-2^{-2}, 2-2^{1-2}) \equiv (1.75,1.5)$, the fourth point is $(1.75,1.75)$ etc.

Let the set X_0 be a union of several coordinate rectangles.

Firstly, consider the domain being the union of two rectangles

$$X_0 := (U_1 \times V_1) \cup (U_2 \times V_2) \subset R^2.$$

Algorithm 1.

A) If $U_1 \cap U_2 = \varnothing$ *and* $V_1 \cap V_2 = \varnothing$ *then no additional points appear*.

B) If $((U_1 \cap U_2 = \varnothing$ *and* $V_1 \cap V_2 \neq \varnothing)$ *or* $(U_1 \cap U_2 \neq \varnothing$ *and* $V_1 \cap V_2 = \varnothing)$ *or* $(U_1 \cap U_2 \neq \varnothing$ *and* $V_1 \cap V_2 \neq \varnothing))$ *then* $X_1 := (U_1 \cup U_2) \times (V_1 \cup V_2)$.

After enlarging all possible pairs of rectangles consider triples of rectangles (three points (1) are in three different rectangles).

4. Conclusion

There are publications [2], [3], [5], [6], [12], [18], [20], [21] and other ones. Their survey demonstrates that there did not exist a unified classification of multidimensional partial differential equations and some existing classifications were based on formal writings of them. Authors of [23], [24] proposed to classify equations by properties of their solutions. Examples in this paper substitute this point of view. The algorithms mentioned above can be implemented in any algorithmic language.

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ON THE INFLUENCE OF INTEGRAL PERTURBATIONS TO THE BOUNDEDNESS OF SOLUTIONS OF A FOURTH-ORDER DIFFERENTIAL EQUATIONS ON THE HALF-AXIS

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The boundedness of all solutions of Volterra type fourth order weakly nonlinear integro-differential equation is investigated in the cases when the corresponding linear homogeneous, linear inhomogeneous and weakly nonlinear differential equations can have solutions unbounded on the half-axis. Sufficient conditions are established and illustrative examples are constructed.
Keywords: integro-differential equation of the fourth order, weak nonlinearity, boundedness, influence of Volterra type integral terms.

Вольтерра тибиндеги төртүнчү тартиптеги сызыктуу сымал интегро-дифференциалдык теңдемененин бардык чыгарылыштарынын жарым окто чектелгендиги, ага тиешелүү төртүнчү тартиптеги сызыктуу бир тектүү, сызыктуу бир тектүү эмес жана сызыктуу сымал дифференциалдык теңдемелеринин жарым окто чектелбеген чыгарылыштары болушу мүмкүн учурунда, изилденет. Жетиштүү шарттар табылат жана иллюстративдик мисалдар тургузулат.
Урунттуу сөздөр: төртүнчү тартиптеги интегро-дифференциалдык теңдеме, сызыктуу сымалдык, чектелгендик, Вольтерра тибиндеги интегралдык мүчөлөрдүн таасири.

Исследуется ограниченность всех решений слабо нелинейного интегро-дифференциального уравнения четвертого порядка типа Вольтерра в случаях, когда соответствующие линейное однородное, линейное неоднородное и слабо нелинейное дифференциальные уравнения могут иметь неограниченные на полуоси решения. Устанавливаются достаточные условия и строятся иллюстративные примеры.
Ключевые слова: интегро-дифференциальное уравнение четвертого порядка, слабая нелинейность, ограниченность, влияние интегральных членов типа Вольтерра.

All functions involved in the study and their derivatives are continuous and the relations hold for $t \geq t_0$, $t \geq \tau \geq t_0$, $|u_j| < \infty$ $(j = 0, 1, 2, 3, 4)$; $J = [t_0, \infty)$; IDE - integro-differential equation; DE - differential equation.

Problem. Establish sufficient conditions for the boundedness on the half-interval J of all solutions to Volterra type fourth-order IDE of the form:

$$x^{(4)}(t) + \int_{t_0}^{t} [Q_0(t,\tau)x(\tau) + Q_1(t,\tau)x'(\tau) + Q_2(t,\tau)x''(\tau) + Q_3(t,\tau)x'''(\tau)]d\tau =$$

$$= f(t) + F\left(t, x(t), x'(t), x''(t), x'''(t), \int_{t_0}^{t} H\left(t, \tau, x(\tau), x'(\tau), x''(\tau), x'''(\tau)\right)d\tau\right),$$

$$t \geq t_0 \qquad (1)$$

with weak nonlinearities $F(t, u_0, u_1, u_2, u_3, u_4)$, $H(t, \tau, u_0, u_1, u_2, u_3)$:

$$\begin{cases} |F(t, u_0, u_1, u_2, u_3, u_4)| \leq F_0(t) + \sum_{k=0}^{4} g_k(t)|u_k| , \\ |H(t, \tau, u_0, u_1, u_2, u_3)| \leq \sum_{\nu=0}^{3} h_\nu(t,\tau)|u_k| \end{cases} \qquad (F, H)$$

with non-negative $F_0(t)$, $g_k(t)$, $h_\nu(t,\tau)$ $(k = 0, 1, 2, 3, 4,\ \nu = 0, 1, 2, 3)$ and in cases when the following equations:

$$x^{(4)}(t) = 0, \quad t \geq t_0, \qquad (1_0)$$

$$x^{(4)}(t) = f(t), \quad t \geq t_0, \qquad (1_1)$$

$$x^{(4)}(t) = f(t) + F(t, x(t), x'(t), x''(t), x'''(t), 0), \quad t \geq t_0 \qquad (1_2)$$

can have solutions unbounded on J.

We are talking about solutions $x(t) \in C^4(J, R)$ of IDE (1) with any initial data $x^{(k)}(t_0)$ $(k = 0, 1, 2, 3)$. Each such solution exists by virtue of conditions (F, H).

Note that such a problem was investigated earlier in [1] for fourth-order linear IDE of the form:

$$x^{(4)}(t) + \sum_{k=0}^{3} \left[a_k(t)x^{(k)}(t) + \int_{t_0}^{t} Q_k(t,\tau)x^{(k)}(\tau)\, d\tau\right] = f(t), \ \ t \geq t_0 \quad (l)$$

in case, when corresponding fourth-order linear DEs:

$$L(t; x) \equiv x^{(k)}(t) + \sum_{k=0}^{3} a_k(t)x^{(k)}(t) = 0, \ \ t \geq t_0, \qquad (l_0)$$

$$L(t; x) \equiv f(t), \ \ t \geq t_0 \qquad (l_1)$$

can have unbounded solutions on J.

To solve the above problem, we will follow the scheme of the method from [1], namely, a non-standard method for reducing to a system [2,3], method of cutting functions [4, p. 41], method of integral inequalities [5] are developed and lemma 3.3 [4, p. 110-111] or lemma [6] on an integral inequality of the first kind are applied.

Let us present the main results of this work below.

Following [2, 3, 1], in IDE (1) we make a nonstandard substitution:

$$x''(t) + \lambda^2 x(t) = W(t)y(t), \tag{2}$$

where $0 \neq \lambda$ – some auxiliary parameter, $\lambda \in R; 0 < W(t)$ – some weight function, $y(t)$ – a new unknown function. Then, similarly to [3, 1], IDE (1) turns into the following equivalent system:

$$\begin{cases} x''(t) + \lambda^2 x(t) = W(t)y(t), \\ y''(t) + 2W'(t)\big(W(t)\big)^{-1} y'(t) + \Big[W''(t)\big(W(t)\big)^{-1} - \lambda^2\Big] y(t) + \lambda^4 \big(W(t)\big)^{-1} \times \\ \times x(t) + \displaystyle\int_{t_0}^{t} [P_0(t,\tau)x(\tau) + P_1(t,\tau)x'(\tau) + P_2(t,\tau)y(\tau) + K(t,\tau)y'(\tau)]d\tau = \\ = \big(W(t)\big)^{-1} f(t) + \big(W(t)\big)^{-1} F\Big(t, x(t), x'(t), -\lambda^2 x(t) + W(t)y(t), \\ -\lambda^2 x'(t) + W(t)y'(t) + W'(t)y(t), \displaystyle\int_{t_0}^{t} H(t,\tau,x(\tau),x'(\tau),-\lambda^2 x(\tau) + W(\tau)y(\tau), \\ \quad -\lambda^2 x'(\tau) + W(\tau)y'(\tau) + W'(\tau)y(\tau)\Big) d\tau\Big), \qquad t \geq t_0, \end{cases} \tag{3}$$

where

$$P_0(t,\tau) \equiv \big(W(t)\big)^{-1}\big[Q_0(t,\tau) - \lambda^2 Q_2(t,\tau)\big],$$

$$P_1(t,\tau) \equiv \big(W(t)\big)^{-1}\big[Q_1(t,\tau) - \lambda^2 Q_3(t,\tau)\big],$$

$$P_2(t,\tau) \equiv \big(W(t)\big)^{-1}[Q_2(t,\tau)W(\tau) + Q_3(t,\tau)W'(\tau)],$$

$$K(t,\tau) \equiv \big(W(t)\big)^{-1} Q_3(t,\tau)W(\tau).$$

Then we proceed in the same way as in [4, 1]. Let

$$K(t,\tau) = \textstyle\sum_{i=0}^{n} K_i(t,\tau), \tag{K}$$

$$(W(t))^{-1} f(t) = \textstyle\sum_{i=0}^{n} f_i(t), \tag{f}$$

$\psi_i(t)$ $(i = 1..n)$ – some cutting functions,

$$R_i(t,\tau) \equiv K_i(t,\tau)\left(\psi_i(t)\psi_i(\tau)\right)^{-1}, E_i(t) \equiv f_i(t)\left(\psi_i(t)\right)^{-1},$$

$$R_i(t,t_0) = A_i(t) + B_i(t)\ (i = 1..n), \qquad (R)$$

$c_i(t)$ $(i = 1..n)$ – some functions.

For an arbitrary fixed solution $(x(t), y(t))$ of system (3) we multiply its first equation by $x'(t)$, the second equation – by $y'(t)$ [7, p. 194-217] and then we add them. Then integrate between the limits from t_0 to t, including by parts. For this, similarly to [4, 1], we introduce the conditions (*K*), (*f*), functions $\psi_i(t)$, $R_i(t,\tau)$, $E_i(t)$, condition (*R*), functions $c_i(t)$ $(i = 1..n)$ and apply lemmas 1.4, 1.5 [8]. Next, after some transformations (for example, we represent $W''(t)\big(W(t)\big)^{-1} - \lambda^2 \equiv \gamma^2 + W''(t)\big(W(t)\big)^{-1} - \lambda^2 - \gamma^2, \gamma = const \neq 0$) we obtain the following identity:

$$\big(x'(t)\big)^2 + \lambda^2\big(x(t)\big)^2 + \big(y'(t)\big)^2 + \gamma^2\big(y(t)\big)^2 + \sum_{i=1}^{n}\{A_i(t)\big(Y_i(t,t_0)\big)^2 +$$

$$+B_i(t)\big(Y_i(t,t_0)\big)^2 - 2E_i(t)Y_i(t,t_0) + c_i(t) - \int_{t_0}^{t}[B_i'(s)\big(Y_i(s,t_0)\big)^2 -$$

$$-2E_i'(s)Y_i(s,t_0) + c_i'(s)]ds + \int_{t_0}^{t} R_{i\tau}'(t,\tau)\big(Y_i(t,\tau)\big)^2 d\tau\} \equiv c_* +$$

$$+2\int_{t_0}^{t} W(s)y(s)x'(s)ds + \frac{1}{2}\sum_{i=1}^{n}\int_{t_0}^{t}\Big[A_i'(s)\big(Y_i(s,t_0)\big)^2 +$$

$$+\int_{t_0}^{s} R_{is\tau}''(s,\tau)\big(Y_i(s,\tau)\big)^2 d\tau\Big]ds + 2\int_{t_0}^{t} y'(s)\{f_0(s) - 2W'(s)\big(W(s)\big)^{-1}y'(s) +$$

$$+\Big[\lambda^2 + \gamma^2 - W''(s)\big(W(s)\big)^{-1}\Big]y(s) - \lambda^4\big(W(s)\big)^{-1}x(s) -$$

$$-\int_{t_0}^{s}[P_0(s,\tau)x(\tau) + P_1(s,\tau)x'(\tau) + P_2(s,\tau)y(\tau) + K_0(s,\tau)y'(\tau)]d\tau\}ds +$$

$$+2\int_{t_0}^{t}\big(W(s)\big)^{-1}y'(s)F(s, x(s), x'(s), -\lambda^2 x(s) + W(s)y(s), -\lambda^2 x'(s) +$$

$$+W(s)y'(s)+W'(s)y(s),\int_{t_0}^{s}H(s,\tau,x(\tau),x'(\tau),-\lambda^2x(\tau)+W(\tau)y(\tau),$$

$$-\lambda^2x'(\tau)+W(\tau)y'(\tau)+W'(\tau)y(\tau))d\tau)ds, \qquad (4)$$

where

$$Y_i(t,\tau)\equiv\int_{\tau}^{t}\psi_i(\eta)y'(\eta)d\eta \quad (i=1..n),$$

$$c_*=\left(x'(t_0)\right)^2+\lambda^2\left(x(t_0)\right)^2+\left(y'(t_0)\right)^2+\gamma^2\left(y(t_0)\right)^2+\sum_{i=1}^{n}c_i(t_0).$$

Turning to the integral inequality taking into account condition (*F*,*H*), similarly to theorems 1.1, 2.1 from [4] and the theorem from [1], we prove the following theorem.

Theorem. Suppose that 1) $\lambda>0$, $W(t)>0$, $\gamma>0$; conditions (*K*), (*f*), (*R*) holds; 2) $A_i(t)\geq 0$, $B_i(t)\geq 0$, $B_i'(t)\leq 0$, $R_{i\tau}'(t,\tau)\geq 0$, exist functions $A_i^*(t)\geq 0$, $c_i(t)$, $R_i^*(t)\geq 0$ such that $A_i'(t)\leq A_i^*(t)A_i(t)$, $\left(E_i^{(k)}(t)\right)^2\leq B_i^{(k)}(t)c_i^{(k)}(t)$, $R_{it\tau}''(t,\tau)\leq R_i^*(t)R_{i\tau}'(t,\tau)$ $(i=1..n;k=0,1)$. Then, for any solution $(x(t),y(t))$ of system (3) the following energy estimate is valid:

$$E(t,c_*)\equiv\left(x'(t)\right)^2+\lambda^2\left(x(t)\right)^2+\left(y'(t)\right)^2+\gamma^2\left(y(t)\right)^2+$$

$$+\sum_{i=1}^{n}\left[A_i(t)\left(Y_i(t,t_0)\right)^2+\int_{t_0}^{t}R_{i\tau}'(t,\tau)\left(Y_i(t,\tau)\right)^2d\tau\right]\leq M(t,c_*), \qquad (5)$$

where

$$M(t,c_*)\equiv\left\{\sqrt{c_*}+\int_{t_0}^{t}\left[|f_0(s)|+\left(W(s)\right)^{-1}F_0(s)\right]exp\left(-\frac{1}{2}\int_{t_0}^{s}V(\eta)d\eta\right)ds\right\}^2\times$$

$$\times\, exp\left(\int_{t_0}^{t}V(s)ds\right),$$

$$V(t)\equiv\sum_{i=1}^{n}[A_i^*(t)+R_i^*(t)]+2\left\{\gamma^{-1}W(t)+2|W'(t)|\left(W(t)\right)^{-1}+\right.$$

$$+\gamma^{-1}\left|\lambda^2+\gamma^2-W''(t)\big(W(t)\big)^{-1}\right|+\lambda^3\big(W(t)\big)^{-1}+\int_{t_0}^{t}[\lambda^{-1}|P_0(t,\tau)|+$$

$$+|P_1(t,\tau)|+\gamma^{-1}|P_2(t,\tau)|+|K_0(t,\tau)|]d\tau\}+2\big(W(t)\big)^{-1}\{\lambda^{-1}g_0(t)+$$

$$+g_1(t)+g_2(t)\big(\lambda+\gamma^{-1}W(t)\big)+g_3(t)\big(\lambda^2+W(t)+\gamma^{-1}|W'(t)|\big)+$$

$$+g_4(t)\int_{t_0}^{t}[\lambda^{-1}h_0(t,\tau)+h_1(t,\tau)+h_2(t,\tau)\big(\lambda+\gamma^{-1}W(\tau)\big)+$$

$$+h_3(t,\tau)\big(\lambda^2+W(\tau)+\gamma^{-1}|W'(\tau)|\big)]d\tau\}.$$

Let, in addition, 3) $A_j(t)>0$, $\psi_j(t)>0$, $\psi'_j(t)\ge 0$ $(1\le j\le n)$, $q_j(t,c_*)\ge 0$, $q'_j(t,c_*)\ge 0$, $q'_j(t,c_*)\left(\psi_j(t)\right)^{-1}\in L^1(J,R_+)$,

where

$$q_j(t,c_*)\equiv\big(A_j(t)\big)^{-\frac{1}{2}}\big(M_j(t,c_*)\big)^{\frac{1}{2}}.$$

Then $y(t)=O(1)$.

From this theorem we have

Corollary. If all conditions of theorem are hold and $W(t)\in L^1(J,R_+\backslash\{0\})$, then any solution of fourth-order IDE (1) $x(t)=O(1)$, i.e. is bounded on the half-interval J.

The validity of this corollary is obtained from the following Cauchy's formula [9, c. 393 - 394]:

$$x(t)=x(t_0)\cos\lambda(t-t_0)+\frac{1}{\lambda}x'(t_0)\sin\lambda(t-t_0)+\frac{1}{\lambda}\int_{t_0}^{t}[\sin\lambda(t-s)]W(s)y(s)ds$$

produced from replacement (2), i.e. from the first equation of system (3), for any initial values $x(t_0)$, $x'(t_0)$.

Let us give the simplest examples.

Example 1. For the weakly nonlinear fourth-order IDE:

$$L(t,x)\equiv x^{(4)}(t)+\int_{0}^{t}\left\{\left[\frac{\sqrt[3]{cos\tau}}{2t+3\tau+4}+4Q_2(t,\tau)\right]x(\tau)+\right.$$

$$+\left[-\frac{\sqrt[5]{sint+cos\tau}}{t+\tau+5}+4Q_3(t,\tau)\right]x'(\tau)+Q_2(t,\tau)x''(\tau)+Q_3(t,\tau)x'''(\tau)\Bigg\}d\tau=$$

$$=\int_0^t \frac{|x(\tau)|\sin x\prime(\tau)}{t+2\tau+9}d\tau,\quad t\geq 0, \qquad (1_*)$$

where

$$Q_2(t,\tau)\equiv 2Q_3(t,\tau)(\tau+1)^{-1}+\sqrt{t^2+\tau^2+1}\cos(t\tau),$$

$$Q_3(t,\tau)\equiv (t+1)^{-2}(\tau+1)^2\left\{\frac{1}{t-\tau+4}+\left[exp\left(\frac{\cos t}{t+3}\right)+\tau\right]^{\frac{1}{2}}\right\}exp[t^7+\tau^7+$$

$$+e^t+e^\tau+exp(e^{2t})+exp(e^{2\tau})]-\frac{|sin\tau|}{t+\tau+1},$$

all conditions of the theorem and corollary hold for $\lambda=2$, $W(t)\equiv(t+1)^{-2}$, $\gamma=4$, here $t_0=0$,

$$P_0(t,\tau)\equiv\frac{(t+1)^2\sqrt[3]{cos\tau}}{2t+3\tau+4},\ P_1(t,\tau)\equiv-\frac{(t+1)^2\sqrt[5]{sint+cos\tau}}{t+\tau+5},$$

$$P_2(t,\tau)\equiv(t+1)^2(\tau+1)^{-2}\sqrt{t^2+\tau^2+1}\cos(t\tau),$$

$$K(t,\tau)\equiv\left\{\frac{1}{t-\tau+4}+\left[exp\left(\frac{\cos t}{t+3}\right)+\tau\right]^{\frac{1}{2}}\right\}exp[t^7+\tau^7+e^t+e^\tau+exp(e^{2t})+$$

$$exp(e^{2\tau})]-\frac{(t+1)^2(\tau+1)^{-2}|sin\tau|}{t+\tau+1},\ g_4(t)\equiv 1,\ h_0(t,\tau)\equiv\frac{1}{t+2\tau+9},\ g_k(t)\equiv 0\ (k=$$

$$0,1,2,3),\ h_\nu(t,\tau)\equiv 0\ (\nu=1,2,3);n=1,\ \ \psi_1(t)\equiv exp[t^7+e^t+exp(e^{2t})],$$

$$R_1(t,\tau)\equiv\frac{1}{t-\tau+4}+\left[exp\left(\frac{\cos t}{t+3}\right)+\tau\right]^{\frac{1}{2}},\ A_1(t)\equiv exp\frac{\cos t}{2(t+3)},\ B_1(t)\equiv\frac{1}{t+4},\ A_1^*(t)\equiv$$

$$\frac{t+4}{(t+3)^2},\ R_1^*(t)\equiv\frac{t+4}{(t+3)^2},\ K_0(t,\tau)\equiv\frac{(t+1)^2(\tau+1)^{-2}|sin\tau|}{t+\tau+1},\ f_0(t)\equiv f_i(t)\equiv 0\ (i=1..n).$$

Therefore, any solution of the IDE (1_*) is bounded on the half-axis $R_+=[0,\infty)$. However, all solutions of the corresponding homogeneous DE: $x^{(4)}(t)=0, t\geq 0$ under the condition $x(0)\neq 0$ are unbounded on the R_+, which follows from the general solution of this simplest DE:

$x(t)=c_0+c_1t+c_2t^2+c_3t^3$ (c_k – arbitrary constants ($k=0,1,2,3$)).

Example 2. Weakly nonlinear fourth-order IDE:

$$L(t;x)=-\frac{(t+1)^{-2}exp[t^7+e^t+exp(e^{2t})]}{t+5}-\frac{|\cos t|}{\sqrt{t+2}}+t+$$

$$+\int_0^t \frac{|x(\tau)|\sin x'(\tau)}{t+2\tau+9}d\tau,\ t\geq 0,$$

where $L(t;x)$ – the same operator as in the IDE (1_*) of Example 1, satisfies all conditions of the theorem and the corollary, for the same λ, $W(t)$, γ. Here

$$f_1(t)\equiv -\frac{(t+1)^{-2}exp[t^7+e^t+exp(e^{2t})]}{t+5},\ E_1(t)\equiv -\frac{1}{t+5},\ c_1(t)\equiv \frac{1}{t+5};$$

$$f_0(t)=-\frac{(t+1)^2|\cos t|}{\sqrt{t+2}}+t(t+1)^2.$$

This means that all solutions of the given IDE are bounded on the half-axis R_+. Wherein, it can be shown that for the corresponding DE:

$$x^{(4)}(t)=-\frac{(t+1)^{-2}exp[t^7+e^t+exp(e^{2t})]}{t+5}-\frac{|\cos t|}{\sqrt{t+2}}+t,\qquad t\geq 0$$

all it solutions are not bounded on the R_+.

Example 3. For the weakly nonlinear fourth-order IDE:

$$L(t;x)=e^t\cos[x(t)-e^t]+\int_0^t \frac{|x(\tau)|\sin x'(\tau)}{t+2\tau+9}d\tau,\quad t\geq 0,$$

where $L(t;x)$ – the same operator as in the Example 1 for IDE (1_*), all conditions of the theorem and the corollary are satisfied, here $F_0(t)\equiv e^t$. Therefore, all solutions of this IDE are bounded on the half-axis R_+. However, the unbounded on the R_+ function $x(t)=e^t$ is a solution of the corresponding weakly nonlinear DE:

$$x^{(4)}(t)=e^t\cos[x(t)-e^t],$$

which is easy to check.

Thus, we have found a class of weakly nonlinear fourth-order IDEs of the form (1), for which the problem under consideration is solvable.

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SUFFICIENT CONDITIONS FOR THE STABILITY OF SOLUTIONS OF FOURTH ORDER LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION

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Sufficient conditions for the stability of solutions, i.e. the boundedness on the semiaxis of all solutions and their derivatives of the first, second, third orders of the linear integro-differential equation of the fourth order of the Volterra type, are established. For this, a non-standard method of reducing equations to a system is being developed with introduction of three certain positive weighting functions. An illustrative example is constructed.
Keywords: linear integro-differential equation of the fourth order, a non-standard method of reducing equations to a system, weighting functions, boundedness, stability.

Сызыктуу Вольтерра тибиндеги төртүнчү тартиптеги интегро-дифференциалдык теңдеменин турумдуулугунун, б. а. бардык чыгарылыштарынын жана алардын биринчи, экинчи, үчүнчү туундуларынын жарым окто чектелгендигинин, жетиштүү шарттары табылат. Бул үчүн теңдемелерди системага стандарттык эмес келтирүү методу үч оң салмактык функцияларды кийирүү аркылуу өнүктүрүлөт. Иллюстративдик мисал тургузулат.
Урунттуу сөздөр: төртүнчү тартиптеги сызыктуу интегро-дифференциалдык теңдеме, теңдемени системага стандарттык эмес келтирүү методу, салмактык функциялар, чектелгендик, турумдуулук.

Устанавливаются достаточные условия устойчивости решений, т. е. ограниченности на полуоси всех решений и их первых, вторых, третьих производных, линейного интегро-дифференциального уравнения четвертого порядка типа Вольтерра. Для этого развивается нестандартный метод сведения уравнений к системе с введением трех некоторых положительных весовых функций. Строится иллюстративный пример.
Ключевые слова: линейное интегро-дифференциальное уравнение четвертого порядка, нестандартный метод сведения уравнений к системе, весовые функции, ограниченность, устойчивость.

All appearing functions and their derivatives are continuous and relations are true when $t \geq t_0$, $t \geq \tau \geq t_0$, $J = [t_0, \infty)$; IDE - integro-differential equation; DE - differential equation. The stability of solutions to a linear IDE of the fourth order is understood as the boundedness on the half-interval J of all its solutions and their derivatives up to the third order inclusive.

The main purpose of this work is to solve the following problem.

Problem. To establish sufficient conditions for the stability of solutions to linear IDEs of the fourth order Volterra type of the form

$$x^{(4)}(t) + a_3(t)x'''(t) + a_2(t)x''(t) + a_1(t)x'(t) + a_0(t)x(t) + \int_{t_0}^{t} [Q_0(t,\tau)x(\tau) +$$

$$+Q_1(t,\tau)x'(\tau) + Q_2(t,\tau)x''(\tau) + Q_3(t,\tau)x'''(\tau)]d\tau = f(t), \;\; t \geq t_0. \qquad (1)$$

Note, that such a problem for other classes of IDEs of the form (1) was studied in [1, 2] by the method of comparison [3, pp. 15-17] with solutions of the corresponding differential equations; in [4] - by a modified method of weighting and cutting functions [5]; in [6] - by the method of squaring equations [3, p.28].

We are considering solutions of the IDE (1) $x(t) \in C^4(J, R)$ with any initial data $x^{(4)}(t_0) \quad (k = 0, 1, 2, 3)$. Each such solution exists and uniqueness.

To solve the above problem, we are developing a method based on the ideas of the method of reduction to the system [7, 8] and the method of cutting functions [3]. The essence of the method is as follows.

First, we make the changes in the IDE (1):

$$x'(t) = W_1(t)y(t), \qquad (2)$$

$$y'(t) = W_2(t)z(t), \qquad (3)$$

$$z'(t) = W_3(t)u(t), \qquad (4)$$

where $0 < W_k(t)$ $(k = 1,2,3)$ - some auxiliary weighting functions, $y(t), z(t), u(t)$ - new unknown functions.

From (2) - (4) by differentiation we have:

$$x''(t) = W_1'(t)y(t) + W_1(t)y'(t) = W_1'(t)y(t) + W_1(t)W_2(t)z(t), \qquad (5)$$

$$x'''(t) = W_1''(t)y(t) + W_1'(t)y'(t) + \left(W_1(t)W_2(t)\right)'z(t) + W_1(t)W_2(t)z'(t) =$$

$$= W_1''(t)y(t) + \left[W_1'(t)W_2(t) + \left(W_1(t)W_2(t)\right)'\right]z(t) + W_1(t)W_2(t)W_3(t)u(t), \; (6)$$

$$x^{(4)}(t) = W_1'''(t)y(t) + W_1''(t)y'(t) + \left[W_1'(t)W_2(t) + \left(W_1(t)W_2(t)\right)'\right]'z(t) +$$

$$+\left[W_1'(t)W_2(t) + \left(W_1(t)W_2(t)\right)'\right]z'(t) + \left(W_1(t)W_2(t)W_3(t)\right)'u(t) +$$

$$+W_1(t)W_2(t)W_3(t)u'(t) = W_1'''(t)y(t) + W_1''(t)W_2(t)z(t) + [W_1'(t)W_2(t) +$$

$$+\left(W_1(t)W_2(t)\right)'\Big]'z(t) + \left[W_1{}'(t)W_2(t) + \left(W_1(t)W_2(t)\right)'\right]W_3(t)u(t) +$$

$$+\left(W_1(t)W_2(t)W_3(t)\right)'u(t) + W_1(t)W_2(t)W_3(t)u'(t) = W_1'''(t)y(t) +$$

$$+\left\{W_1''(t)W_2(t) + \left[W_1'(t)W_2(t) + \left(W_1(t)W_2(t)\right)'\right]'\right\} z(t) + \{[W_1'(t)W_2(t) +$$

$$+\left(W_1(t)W_2(t)\right)'\Big]W_3(t)+\left(W_1(t)W_2(t)W_3(t)\right)'\Big\}u(t)+W_1(t)W_2(t)W_3(t)u'(t). \quad (7)$$

Taking into account relations (2) - (7), from IDE (1) we obtain:

$$W_1'''(t)y(t)+\left\{W_1''(t)W_2(t)+\left[W_1'(t)W_2(t)+\left(W_1(t)W_2(t)\right)'\right]'\right\} z(t)+$$
$$+\{\left[W_1'(t)W_2(t)+\left(W_1(t)W_2(t)\right)'\right]W_3(t)+\left(W_1(t)W_2(t)W_3(t)\right)'\}u(t)+$$
$$+W_1(t)W_2(t)W_3(t)u'(t)+a_3(t)\{W_1''(t)y(t)+[W_1'(t)W_2(t)+$$
$$+\left(W_1(t)W_2(t)\right)'\big]z(t)+W_1(t)W_2(t)W_3(t)u(t)\big\}+a_2(t)\{W_1'(t)y(t)+$$
$$+W_1(t)W_2(t)z(t)\}+a_1(t)W_1(t)y(t)+a_0(t)x(t)+\int_{t_0}^{t}\{Q_0(t,\tau)x(\tau)+$$
$$+Q_1(t,\tau)W_1(\tau)y(\tau)+Q_2(t,\tau)[W_1'(\tau)y(\tau)+W_1(\tau)W_2(\tau)z(\tau)]+$$
$$+Q_3(t,\tau)W_1''(\tau)y(\tau)+Q_3(t,\tau)\left[W_1'(\tau)W_2(\tau)+\left(W_1(\tau)W_2(\tau)\right)'\right]z(\tau)+$$
$$+Q_3(t,\tau)W_1(\tau)W_2(\tau)W_3(\tau)u(\tau)\}d\tau=f(t), \quad t\ge t_0. \quad (8)$$

We introduce the following notation:

$$W(t)\equiv W_1(t)W_2(t)W_3(t),$$
$$b_3(t)\equiv a_3(t)+\left(W(t)\right)^{-1}\left\{\left[W_1'(t)W_2(t)+\left(W_1(t)W_2(t)\right)'\right]W_3(t)+W'(t)\right\},$$
$$b_2(t)\equiv\left(W(t)\right)^{-1}\left\{W_1''(t)W_2(t)+\left[W_1'(t)W_2(t)+\left(W_1(t)W_2(t)\right)'\right]'+\right.$$
$$\left.+a_3(t)\left[W_1'(t)W_2(t)+\left(W_1(t)W_2(t)\right)'\right]+a_2(t)W_1(t)W_2(t)\right\},$$
$$b_1(t)\equiv\left(W(t)\right)^{-1}\{W_1'''(t)+a_3(t)W_1''(t)+a_2(t)W_1'(t)+a_1(t)W_1(t)\},$$
$$b_0(t)\equiv\left(W(t)\right)^{-1}a_0(t),\, P_0(t,\tau)\equiv\left(W(t)\right)^{-1}Q_0(t,\tau),$$
$$P_1(t,\tau)\equiv\left(W(t)\right)^{-1}\{Q_1(t,\tau)W_1(\tau)+Q_2(t,\tau)W_1'(\tau)+Q_3(t,\tau)W_1''(\tau)\},$$
$$P_2(t,\tau)\equiv\left(W(t)\right)^{-1}\{Q_2(t,\tau)W_1(\tau)W_2(\tau)+$$
$$+Q_3(t,\tau)\left[W_1'(\tau)W_2(\tau)+\left(W_1(\tau)W_2(\tau)\right)'\right]\},$$
$$K(t,\tau)\equiv\left(W(t)\right)^{-1}Q_3(t,\tau)W(\tau), \quad F(t)\equiv\left(W(t)\right)^{-1}f(t).$$

Then dividing both sides of (8) by $W_1(t)W_2(t)W_3(t)$ and using introduced notation, from (2) - (4), (8) we obtain following system of three differential equations (2) - (4) and IDE:

$$\begin{cases} x'(t) = W_1(t)y(t), \\ y'(t) = W_2(t)z(t), \\ z'(t) = W_3(t)u(t), \\ u'(t) + b_3(t)u(t) + b_2(t)z(t) + \\ +b_1(t)y(t) + b_0(t)x(t) + \int_{t_0}^{t}[P_0(t,\tau)x(\tau) + P_1(t,\tau)y(\tau) + \\ +P_2(t,\tau)z(\tau) + K(t,\tau)u(\tau)]d\tau = F(t), \quad t \geq t_0. \end{cases} \tag{9}$$

This system equivalents to original IDE of fourth order (1).

Let [3]:

$$K(t,\tau) = \sum_{i=0}^{n} K_i(t,\tau), \tag{K}$$

$$F(t) = \sum_{i=0}^{n} F_i(t), \tag{F}$$

$\psi_i(t)$ $(i = 1..n)$ - some cutting functions,

$$R_i(t,\tau) \equiv K_i(t,\tau)\left(\psi_i(t)\psi_i(\tau)\right)^{-1}, \quad E_i(t) \equiv F_i(t)\left(\psi_i(t)\right)^{-1} \ (i = 1..n),$$

$$R_i(t,t_0) = A_i(t) + B_i(t) \ (i = 1..n), \tag{R}$$

$c_i(t)$ $(i = 1..n)$ - some functions.

For any solution $\left(x(t), y(t), z(t), u(t)\right)$ of the system (9), its first DE we multiply by $x(t)$, the second DE - by $y(t)$, third - by $z(t)$, fourth IDE - by $u(t)$ [9 pp. 194 -217], add the resulting relations. Then we integrate between t_0 and t, including by parts, at the same time we introduce the conditions (*K*), (*F*), functions $\psi_i(t), R_i(t,\tau), E_i(t)$ $(i = 1..n)$, conditions (*R*), functions $c_i(t)$ $(i = 1..n)$, using lemmas 1,4, 1,5 [10]. As a result from the system (9) we obtain the following identity:

$$\left(x(t)\right)^2 + \left(y(t)\right)^2 + \left(z(t)\right)^2 + \left(u(t)\right)^2 + 2\int_{t_0}^{t} b_3(s)\left(u(s)\right)^2 ds +$$

$$+\sum_{i=1}^{n}\Big\{A_i(t)\left(U_i(t,t_0)\right)^2 + B_i(t)\left(U_i(t,t_0)\right)^2 - 2E_i(t)U_i(t,t_0) + c_i(t) -$$

$$-\int_{t_0}^{t}\left[B_i'(s)\left(U_i(s,t_0)\right)^2 - 2E_i'(s)U_i(s,t_0) + c_i{}'(s)\right]ds +$$

$$+\int_{t_0}^{t} R'_{i\tau}(t,\tau)\big(U_i(t,\tau)\big)^2 d\tau\Bigg\} \equiv c_* + \sum_{i=1}^{n}\int_{t_0}^{t}\Big[A_i{}'(s)\big(U_i(s,t_0)\big)^2 +$$

$$+\int_{t_0}^{s} R''_{is\tau}(s,\tau)\big(U_i(s,\tau)\big)^2 d\tau\Big] ds + 2\int_{t_0}^{t}[W_1(s)y(s)x(s) + W_2(s)z(s)y(s) +$$

$$+W_3(s)u(s)z(s)]ds + 2\int_{t_0}^{t} u(s)\{F_0(s) - b_2(s)z(s) - b_1(s)y(s) - b_0(s)x(s) -$$

$$-\int_{t_0}^{s}[P_0(s,\tau)x(\tau) + P_1(s,\tau)y(\tau) + P_2(s,\tau)z(\tau) + K_0(s,\tau)u(\tau)]d\tau\}ds, \qquad (10)$$

where

$$U_i(t,\tau) \equiv \int_{\tau}^{t} \psi_i(\eta)u(\eta)d\eta \ \ (i = 1..n),$$

$$c_* = \big(x(t_0)\big)^2 + \big(y(t_0)\big)^2 + \big(z(t_0)\big)^2 + \big(u(t_0)\big)^2 + \sum_{i=1}^{n} c_i(t_0).$$

We prove following theorem, passing from identity (10) to the integral inequality, similarly to Theorems 1.1, 2.1 [3], applying Lemma 1 [11].

Theorem. Let 1) $W_k(t) > 0$ $(k = 1,2,3)$; the conditions (*K*), (*F*), (*R*) are satisfied; 2) $b_3(t) \geq 0$; 3) $A_i(t) \geq 0$, $B_i(t) \geq 0$, $B'_i(t) \leq 0$, $R'_{i\tau}(t,\tau) \geq 0$, there are functions $A^*_i(t) \in L^1(J,R_+)$, $c_i(t)$, $R^*_i(t) \in L^1(J,R_+)$ such that $A'_i(t) \leq A^*_i(t)A_i(t)$, $\big(E^{(k)}_i(t)\big)^2 \leq B^{(k)}_i(t)c^{(k)}_i(t)$ $(k = 0,1)$,
$R''_{it\tau}(t,\tau) \leq R^*_i(t)R'_{i\tau}(t,\tau)$ $(i = 1..n;\ k = 0,1)$;
4) $W_j(t) + |b_k(t)| + |F_0(t)| + \int_{t_0}^{t}|P_k(t,\tau)|d\tau + \int_{t_0}^{t}|K_0(t,\tau)|d\tau \in L_1(J,R_+\backslash\{0\})$ $(j = 1,2,3;\ k = 0,1,2)$; 5) functions $W_j(t)$ $(j = 1,2,3)$, $W'_1(t)$, $W''_1(t)$, $\big(W_1(t)W_1(t)\big)'$ are bounded on semiinterval J. Then any solution $x(t)$ of IDE (1) is stability.

Conditions 1) – 4) of the theorem ensure $x(t) = O(1)$, $y(t) = O(1)$, $z(t) = O(1)$. We have $x'(t) = O(1)$ by virtue $W_1(t) = O(1)$ from (2). We obtain that $x''(t) = O(1)$ based on $W_1{}'(t) = O(1)$, $W_2(t) = O(1)$ from relation (5). Since

$W_j(t) = O(1)$ $(j = 1,2,3)$, $W_1''(t) = O(1)$, $\left(W_1(t)W_2(t)\right)' = O(1)$, then it follows from relation (6) that $x'''(t) = O(1)$. (Here $x(t)$ is any solution of IDE (1), $O(1)$ is the E. Landau symbol, which means boundedness). Therefore, for any solution $x(t)$ of a fourth-order IDE (1), $x^{(k)}(t) = O(1)$ $(k = 0,1,2,3)$ are true, which is equivalent to the stability of $x(t)$.

Let's give the simplest illustrative example.

Example. For the fourth order IDE

$$x^{(4)}(t) + \left[6 + e^{\sqrt{t}}\right]x'''(t) + \left[11 + 3e^{\sqrt{t}} - e^{-2t}\right]x''(t) +$$

$$+\left[6 + 2e^{\sqrt{t}} - e^{-2t} + \frac{e^{-2t}}{(t+1)^2}\right]x'(t) - \frac{e^{-3t}}{t^2 + \sqrt{t} + 1}x(t) +$$

$$+\int_0^t \left\{-\frac{e^{-3t}(sint)^{\frac{1}{7}}}{(t+\tau+4)^5}x(\tau) + \left[2Q_3(t,\tau) - \frac{e^{-3t}}{(t+\tau+1)^2} + \frac{e^{-3t}}{(t+2\tau+3)^4}\right]x'(\tau) +\right.$$

$$\left. + \left[3Q_3(t,\tau) - \frac{e^{-3t}}{(t+\tau+1)^2}\right]x''(\tau) + Q_3(t,\tau)x'''(\tau)\right\}d\tau =$$

$$= \frac{e^{t^2-3t}(sin3t)^{\frac{1}{5}}}{t-\tau+8} - \frac{e^{-3t}}{(t+2)^2}, \quad t \geq 0,$$

where

$$Q_3(t,\tau) \equiv e^{-3t+3\tau}\left\{\frac{t+\tau+6}{t+\tau+7} + \frac{1}{t-\tau+5}\right\}e^{t^2+\tau^2}(sin3tsin3\tau)^{\frac{1}{5}} - \frac{e^{-3t}(cost)^{\frac{1}{3}}}{t^2+\tau+1},$$

all conditions of the theorem are satisfied at $W_j(t) \equiv e^{-t}$ $(j = 1,2,3)$.

Here $t_0 = 0$, $b_3(t) \equiv e^{\sqrt{t}}$, $b_2(t) \equiv -e^{-t}$, $b_1(t) \equiv \frac{1}{(t+1)^2}$, $b_0(t) \equiv -\frac{1}{t^2+\sqrt{t}+1}$, $P_0(t,\tau) \equiv -\frac{(sint)^{\frac{1}{7}}}{(t+\tau+4)^5}$, $P_1(t,\tau) \equiv \frac{e^{-\tau}}{(t+2\tau+3)^4}$, $P_2(t,\tau) \equiv -\frac{e^{-2\tau}}{(t+\tau+1)^2}$, $n = 1$, $\psi_i(t) \equiv e^{t^2}(sin3t)^{\frac{1}{5}}$, $R_1(t,\tau) \equiv \frac{t+\tau+6}{t+\tau+7} + \frac{1}{t-\tau+5}$, $A_1(t) \equiv \frac{t+6}{t+7}$, $B_1(t) \equiv \frac{1}{t+5}$, $A_1^*(t) \equiv \frac{1}{(t+6)(t+7)}$, $R_1^*(t) \equiv 0$, $E_1(t) \equiv \frac{1}{t+8}$, $c_1(t) \equiv \frac{1}{t+5}$, $K_0(t,\tau) \equiv -\frac{e^{-3\tau}(cost)^{\frac{1}{3}}}{t^2+\tau+1}$, $F_0(t) \equiv -\frac{1}{(t+2)^2}$.

Consequently, any solution of given IDE is stability for $t \in R_+$.

Note that, as an illustrative example is shown, the coefficients and kernels of a fourth-order IDE of the form (1) can be nondifferentiable at some points of the semiinterval J.

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ESTIMATES FOR SOLUTIONS AND THEIR FIRST DERIVATIVES OF A WEAKLY NONLINEAR INTEGRO-DIFFERENTIAL SECOND-ORDER EQUATION OF THE VOLTERRA TYPE ON A SEMI-AXIS

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Sufficient conditions are established for estimating on the semiaxis all solutions and their first derivatives of a weakly nonlinear integro-differential equation of the second order of Volterra type. For this, according to the idea of Professor N.V. Azbelev, a certain function with previously known asymptotic properties is introduced into the given equation and the method of partial cutting is developed. An illustrative example is constructed.

Keywords: second-order integro-differential equation, weak nonlinearity, function with known asymptotic properties, estimation of solutions, estimation of the first derivatives of solutions, partial cut-off method.

Экинчи тартиптеги сызыктуу сымал Вольтерра тибиндеги интегро-дифференциалдык теңдеменин бардык чыгарылыштарын жана алардын биринчи туундуларын жарым окто баалоо үчүн жетиштүү шарттары табылат. Бул үчүн берилген теңдемеге профессор Н.В.Азбелевдин идеясы боюнча асимптотикалык касиеттери белгилүү функция кийирилет жана жекече кесүү методу өнүктүрүлөт. Иллюстративдүү мисал тургузулат.
Урунтуу сөздөр: экинчи тартиптеги интегро-дифференциалдык теңдеме, сызыктуу сымалдык, асимптотикалык касиеттери белгилүү функция, чыгарылыштарды баалоо, чыгарылыштардын биринчи туундуларын баалоо, жекече кесүү методу.

Устанавливаются достаточные условия для оценки на полуоси всех решений и их первых производных слабо нелинейного интегро-дифференциального уравнения второго порядка типа Вольтерра. Для этого по идее профессора Н.В.Азбелева в заданное уравнение вводится некоторая функция с заранее известными асимптотическими свойствами и развивается метод частичного срезывания. Строится иллюстративный пример.
Ключевые слова: интегро-дифференциальное уравнение второго порядка, слабая нелинейность, функция с известными асимптотическими свойствами, оценка решений, оценка первых производных решений, метод частичного срезывания.

All appearing functions are continuous and the relations take place at $t \geq t_0$, $t \geq \tau \geq t_0$; $J=[t_0, \infty)$; IDE – integro-differential equation.

Problem. Establish sufficient conditions for the estimate on the half-interval J of all solutions and their first derivatives of a second order IDE of Volterra type of the form

$$x''(t) + a_1(t)x'(t) + a_0(t)x(t) + \int_{t_0}^{t} [K_0(t,\tau)x(\tau) + K_1(t,\tau)x'(\tau)]d\tau =$$

$$= f(t) + F\left(t, x(t), \int_{t_0}^{t} H\big(t,\tau,x(\tau)\big)d\tau\right), t \geq t_0 \qquad (1)$$

with weakly nonlinearity:

$$|F(t,x,u)| \leq F_0(t) + g_0(t)|u| + g_1(t)|u|, |H(t,\tau,x)| \leq g_2(t,\tau)|x| \qquad (N)$$

with non-negative functions $F_0(t)$, $g_0(t)$, $g_1(t)$, $g_2(t,\tau)$.

It's about solutions of $x(t) \epsilon C^2(J,R)$ IDE (1) with any initial data $x^{(k)}(t_0)$ $(k = 0,1)$. By condition (N), each such solution exists.

Note that earlier this problem was solved in the article [1] using the idea of the W-method of N.V. Azbelev [2, pp. 89-99] and the development of the method of weight and cut-off functions [3]. In [1], one can find a list of many works on solving a similar problem for various classes of second-order linear IDEs. In this paper, we develop a method of partial cutting [4], which makes it possible to study the above problem for a different class of IDEs (1) than in [1].

Below we begin to present the main content of this work.

First, using the ideas of N.V. Azbelev's W-method [2, p. 89-99], similarly, as in [1], in IDE (1) we make the substitution

$$x(t) = W(t)y(t), \qquad (2)$$

where $0 < W(t)$ - a function with known asymptotic properties, $y(t)$ is a new unknown function. Then the following second-order IDE is obtained for $y(t)$ [1]:

$$y''(t) + p(t)y'(t) + q(t)y(t) + \int_{t_0}^{t} [P(t,\tau)y(\tau) + Q(t,\tau)y'(\tau)]d\tau =$$

$$= (W(t))^{-1}[f(t) + F(t, W(t)y(t), \int_{t_0}^{t} H\big(t,\tau,W(\tau)y(\tau)\big)d\tau, \ t \geq t_0, \qquad (3)$$

где $p(t) \equiv a_1(t) + 2W'(t)(W(t))^{-1}, q(t) \equiv a_0(t) + a_1(t)W'(t)(W(t))^{-1} +$ $+W''(t)(W(t))^{-1}$, $P(t,\tau) \equiv \big(W(t)\big)^{-1}[K_0(t,\tau)W(\tau) + K_1(t,\tau)W'(\tau)]$,

$Q(t,\tau) \equiv \big(W(t)\big)^{-1}K_1(t,\tau)W(\tau)$.

Let [1,3,4]:

$$Q(t,\tau) = \sum_{i=0}^{n} Q_i(t,\tau), \qquad (Q)$$

$$\big(W(t)\big)^{-1}f(t) = \sum_{i=0}^{n} f_i(t) \qquad (f)$$

$\psi_i(t)$ $(i = 1..n)$ – some cutting functions, $D_i(t) \equiv Q_i(t,t)(\psi_i(t))^{-2}$;

kernels $M_i(t,\tau) \equiv Q_i(t,\tau)(\psi_i(\tau))^{-1}$ $(i=1.. n)$ – partially cut off;

$$E_i(t) \equiv f_i(t)(\psi_i(t))^{-1} \ (i = 1..n);$$

$$D_i(t) = A_i(t) + B_i(t) \quad (i = 1..n), \qquad (D)$$

$c_i(t)$ $(i = 1..n)$ – some functions.

For an arbitrarily fixed solution $y(t)$ to IDE (3), following [5, pp. 194-217], we multiply both sides of the IDE (3) by $y'(t)$, integrate within the limits from t_0 to t, including by parts , in this case, similarly to [1,3,4], we introduce conditions *(Q)*, *(f)*, functions $\psi_i(t), D_i(t), M_i(t,\tau), E_i(t$ condition *(D)*, functions $c_i(t)$ $(i = 1..n)$, apply lemma [4]. Then we get the following identity:

$$\left(y'(t)\right)^2 + 2\int_{t^0}^{t} p(s)\left(y'(s)\right)^2 ds + q(t)\left(y(t)\right)^2 + \sum_{i=1}^{n}\{A_i(t)\left(Y_i(t,t_0)\right)^2 +$$

$$+B_i(t)(Y_i(t,t_0))^2 - 2E_i(t)Y_i(t,t_0) + c_i(t) -$$

$$-\int_{t^0}^{t}[B'_i(s)(Y_i(s,t_0))^2 - 2E'_i(s)Y_i(s,t_0) + c_i'(s)]ds \equiv c_* + \int_{t_0}^{t}\{q'(s)\left(y(s)\right)^2 +$$

$$+\sum_{i=1}^{n}[A'_i(s)(Y_i(s,t_0))^2 + 2\int_{t_0}^{s} M'_{i\tau}(s,\tau)Y_i(\tau,t_0)y'(s)d\tau]\}ds -$$

$$-2\int_{t_0}^{t} y'(s)\{\int_{t_0}^{s}[P(s,\tau)y(\tau) + Q_0(s,\tau)y'(\tau)]d\tau - \left(W(s)\right)^{-1}[f_0(s) +$$

$$+F(s, W(s)y(s), \int_{t_0}^{s} H\left(s,\tau,W(\tau)y(\tau)\right)d\tau)]\}ds, \qquad (4)$$

where

$$Y_i(t,t_0) \equiv \int_{t_0}^{t}\psi_i(\eta)y'(\eta)d\eta \qquad (i = 1..n),$$

$$c_* = (y'(t_0))^2 + q(t_0)\left(y(t_0)\right)^2 + \sum_{i=1}^{n} c_i(t_0).$$

Proceeding from identity (4), similarly to Theorems 1.1, 2.1 [3] and Theorem [4], the following is proved

Theorem. Let 1) *W(t)* > 0, conditions (N), *(Q)*, *(f)*, *(D)* are satisfied; 2) $p(t) \geq 0$; 3) $q(t) \geq q_0 > 0$, there is a function $q^*(t) \in L^1(J, R_+)$ such that $q'(t) \leq q^*(t)q(t)$; 4) $A_i(t) > 0, B_i(t) \geq 0, B_i'(t) \leq 0$, $A_i^*(t) \leq A_i^*(t)A_i(t)$ functions are exist, $(E_i^{(k)}(t))^2 \leq B_i^{(k)}(t)c_i^{(k)}(t) \quad (i = 1..n; k = 0,1)$;
5)$\int_{t_0}^{t}[|P(t,\tau)||Q_0(t,\tau)|]d\tau + g_0(t) + \left(W(t)\right)^{-1}[|f_0(t)| + g_1(t)\int_{t_0}^{t} g_2(t,\tau)d\tau] \in L^1(J, R_+)$.

Then any solution $y(t)$ and its first derivative $y'(t)$ IDE (3) are bounded on the half-interval J and for any solution $x(t)$ and its derivative IDE (1) following estimates are valid:

$$x(t) = W(t)\,O(1), \quad x'(t) = [W(t) + |W'(t)|]\,O(1). \qquad (5)$$

Note that the first estimate from (5) follows from substitution (2), and the second from the relation $x'(t) = W(t)y'(t) + W'(t)y(t)$.

Example. IDE (1) with $a_1(t) \equiv 4 + e^t|\sin t|,\ \ a_0(t) \equiv 4 + 2e^t|\sin t| + \frac{t+2}{t+3}$,

$$f(t) \equiv -e^{-2t}\sin t + \frac{e^{-2t}\cos t}{t^2+g}, \quad K_0(t,\tau) \equiv K_1(t,\tau) + \frac{e^{-2t}\cos\tau}{(t+8\tau+3)^2},$$

$$K_1(t,\tau) \equiv e^{-2t+2\tau}\sqrt{9+(t-\tau)e^{-10t}}\sin t\sin\tau - \frac{e^{-2t+2\tau}}{e^{3t}+\tau+11},$$

$$F(t,x,u) = e^{-2t}\left[\frac{\cos u}{(|x|+1)(t+3)^2} + \frac{|x|\sin u}{e^t+5} - \frac{u^2}{|u|+4}\right], H(t,\tau,x) \equiv \frac{x^3\sin x}{(t+\tau+19)^3(x^2+8)},$$

t_0=0 satisfies all conditions of the theorem for $W(t) = e^{-2t}$, here

$$p(t) \equiv e^t|\sin t|, \qquad q(t) \equiv \frac{t+2}{t+3}, \qquad q_0 = \frac{2}{3}, \qquad q^*(t) \equiv \frac{1}{(t+2)(t+3)},$$

$$P(t,\tau) \equiv \frac{e^{-2\tau}\cos\tau}{(t+8\tau+3)^2},$$

$$Q(t,\tau) \equiv \sqrt{9+(t-\tau)e^{-10t}}\sin t\sin\tau - \frac{1}{e^{3t}+\tau+11},$$

$$n = 1, \psi_1(t) \equiv \sin t, D(t) \equiv 3, A_1(t) \equiv 1, B_1(t) \equiv 2, Q_0(t,\tau) \equiv -\frac{1}{e^{3t}+\tau+1},$$

$$E_1(t) \equiv -1, c_1(t) \equiv 1, f_0(t) \equiv \frac{\cos t}{t^2+9}, F_0(t) \equiv \frac{e^{-2t}}{(t+3)^2}, g_0(t) \equiv \frac{e^{-2t}}{e^t+5},$$

$$g_1(t) \equiv e^{-2t}, \qquad g_2(t,\tau) \equiv \frac{1}{(t+\tau+19)^3}.$$

This means, that for any solution $x(t)$ such IDE the following estimates are valid:

$$x^{(k)}(t) = e^{-2t}O(1) \qquad (k = 0,1).$$

Thus, we have found a class of IDEs of the form (1), for which the above problem can be solved. We believe that the results of this work are an essential complement to the results from article [1].

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SUFFICIENT CONDITIONS FOR THE ASYMPTOTIC STABILITY OF SOLUTIONS OF THE LINEAR VOLTERRA INTEGRO - DIFFERENTIAL EQUATION OF THE FIFTH ORDER WITH INCOMPLETE KERNELS

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Sufficient conditions for the asymptotic stability of solutions of a fifth-order linear integro-differential equation with incomplete kernels of Volterra type on the semi axis are established. The method of auxiliary kernels, non-standard method of reduction to the system, the method of squaring equations, the method of cutting functions and other known methods are developed. The Sylvester criterion and the Lyusternik-Sobolev lemma are applied. An illustrative example is constructed.
Keywords: linear Volterra integral-differential equation of the fifth order, asymptotic stability of solutions, auxiliary kernel method, non-standard method of reduction to the system, the method of squaring equations, the Sylvester criterion, the Lusternik-Sobolev lemma.

Вольтерра тибиндеги толук эмес ядролуу сызыктуу жарым октогу бешинчи тартиптеги интегро-дифференциалдык теңдеменин чыгарылыштарынын асимптотикалык турумдуулугунун жетиштүү шарттары табылат. Кошумча ядролор методу, теңдемени системага стандарттык эмес келтирүү методу, теңдемелерди квадратка көтөрүү методу, кесүүчү функциялар методу жана башка белгилүү методдор өнүктүрүлөт. Сильвестрдин критерийи жана Люстерник-Соболевдин леммасы колдонулат. Иллюстративдик мисал тургузулат.
Урунттуу сөздөр: Вольтерра тибиндеги сызыктуу бешинчи тартиптеги интегро-дифференциалдык теңдеме, чыгарылыштардын асимптотикалык турумдуулугу, кошумча

ядролор методу, теңдемени системага стандарттык эмес келтирүү методу, теңдемелерди квадратка көтөрүү методу, Сильвестрдин критерийи, Люстерник-Соболевдин леммасы.

Устанавливаются достаточные условия асимптотической устойчивости решений линейного интегро-дифференциального уравнения пятого порядка типа Вольтерра с неполными ядрами на полуоси. Развиваются метод вспомогательных ядер, нестандартный метод сведения к системе, метод возведения уравнений в квадрат, метод срезывающих функций и другие известные методы. Применяются критерий Сильвестра и лемма Люстерника-Соболева. Строится иллюстративный пример.

Ключевые слова: линейное вольтеррова интегро-дифференциальное уравнение пятого порядка, асимптотическая устойчивость решений, метод вспомогательных ядер, нестандартный метод сведения к системе, метод возведения уравнений в квадрат, критерий Сильвестра, лемма Люстерника-Соболева.

All the functions and their derivatives appearing in the work are continuous and the relations take place at $t \ge t_0,\ t \ge \tau \ge t_0;\ \mathrm{I} = [t_0, \infty);$ IDE-integro-differential equation; DE-differential equation; the asymptotic stability of solutions of a fifth-order linear IDE is understood as the tendency to zero $t \to \infty$ all solutions and their derivatives up to the fourth order, inclusive.

The problem. To establish sufficient conditions for the asymptotic stability of fifth-order IDE solutions of the Volterra type of the form:

$$x^{(5)}(t) + a_4(t)x^{(4)}(t) + a_3(t)x'''(t) + a_2(t)x''(t) + a_1(t)x'(t) + a_0(t)x(t) +$$

$$+\int_{t_0}^{t} \left[Q_0(t,\tau)x(\tau) + Q_1(t,\tau)x'(\tau) + Q_2(t,\tau)x''(\tau) + Q_3(t,\tau)x'''(\tau)\right] d\tau = f(t), \quad t \ge t_0 \qquad (1)$$

if conditions are met

$$\int_{t_0}^{\infty}\int_{t_0}^{t} |Q_k(t,\tau)|\, d\tau dt = \infty \qquad (k = 0, 1, 2, 3). \qquad (2)$$

Note that, the IDE (1) does not have a kernel with $x^{(4)}(\tau)$, therefore, such an IDE will be called an IDE with incomplete kernels by analogy with the articles of the authors [1, 2].

The problem presented by us is studied for the first time, and for its solution the method of auxiliary kernels is developed [1, 2], by introducing into the IDE (1) a certain kernel $H_4(t,\tau)$ with $x^{(4)}(\tau)$ by the "weight" rule [3, c.114].

Let's start getting the main result.

In the IDE (1), we enter the kernel $H_4(t,\tau)$ с $x^{(4)}(\tau)$ in the following way:

$$\sum_{k=0}^{3} Q_k(t,\tau)x^{(k)}(\tau) = \sum_{k=0}^{3} Q_k(t,\tau)x^{(k)}(\tau) + H_4(t,\tau)x^{(4)}(\tau) - H_4(t,\tau)x^{(4)}(\tau) \quad (3)$$

and we perform integration in parts:

$$-\int_{t_0}^{t} H_4(t,\tau)x^{(4)}(\tau)d\tau = -H_4(t,t)x'''(t) + H_4(t,t_0)x'''(t_0) + \int_{t_0}^{t} H'_{4\tau}(t,\tau)x'''(\tau)d\tau. \quad (4)$$

Then c regard to (3), (4) IDE (1) proceeds to the next loaded with IDE:

$$x^{(5)}(t) + a_4(t)x^{(4)}(t) + \left[a_3(t) - H_4(t,t)\right]x'''(t) + a_2(t)x''(t) + a_1(t)x'(t) + a_0(t)x(t) +$$

$$+\int_{t_0}^{t} \{Q_0(t,\tau)x(\tau) + Q_1(t,\tau)x'(\tau) + Q_2(t,\tau)x''(\tau) + \left[Q_3(t,\tau) + H'_{4\tau}(t,\tau)\right]x'''(\tau) +$$

$$+H_4(t,\tau)x^{(4)}(\tau)\}d\tau = f(t) - H_4(t,t_0)x'''(t_0). \quad (5)$$

To IDE (5), we apply a non-standard method of reduction to the system from [4], namely, in IDE (5), we make the following non-standard replacement [4]:

$$x'''(t) + px''(t) + qx'(t) + rx(t) = W(t)y(t), \quad (6)$$

where p, q, r - are some positive auxiliary parameters;

$0 < W(t)$-some weight function, $y(t)$ - a new unknown function.

From (6) by differentiating, we have

$$x^{(4)}(t) = -px'''(t) - qx''(t) - rx'(t) + W(t)y'(t) + W'(t)y(t) =$$

$$= -p[-px''(t) - qx'(t) - rx(t) + W(t)y(t)] - qx''(t) - rx'(t) + W(t)y'(t) + W'(t)y(t) =$$

$$= (p^2 - q)x''(t) + (pq - r)x'(t) + prx(t) + W_*(t)y(t) + W(t)y'(t), \quad (7)$$

where $W_*(t) \equiv W'(t) - pW(t)$; from (7), (6) we get

$$x^{(5)}(t) = (p^2 - q)x'''(t) + (pq - r)x''(t) + prx'(t) + W_*(t)y'(t) + W_*'(t)y(t) +$$
$$+W(t)y''(t) + W'(t)y'(t) = (p^2 - q)[-px''(t) - qx'(t) - rx(t) + W(t)y(t)] +$$

$$+(pq - r)x''(t) + prx'(t) + [W_*(t) + W'(t)]y'(t) + W_*'(t)y(t) + W(t)y''(t) =$$

$$= [2pq - r - p^3]x''(t) + [pr + q^2 - p^2q]x'(t) + +[qr - p^2r]x(t) +$$

$$+[W_*'(t) + (p^2 - q)W(t)]y(t) + [W_*(t) + W'(t)]y'(t) + W(t)y''(t). \quad (8)$$

Substituting relations (6) - (8) into IDE (5), we will have

$$[2pq - r - p^3]x''(t) + [pr + q^2 - p^2q]x'(t) + [qr - p^2r]x(t) +$$

$$+[W_*'(t) + (p^2 - q)W(t)]y(t) + [W_*(t) + W'(t)]y'(t) + W(t)y''(t) +$$

$$+a_4(t)[(p^2 - q)x''(t) + (pq - r)x'(t) + prx(t) + W_*(t)y(t) + W(t)y'(t)] +$$

$$+[a_3(t)-H_4(t,t)][-px''(t)-qx'(t)-rx(t)+W(t)y(t)]+a_2(t)x''(t)+$$

$$+a_1(t)x'(t)+a_0(t)x(t)+\int_{t_0}^{t}\{Q_0(t,\tau)x(\tau)+Q_1(t,\tau)x'(\tau)+Q_2(t,\tau)x''(\tau)+$$

$$+[Q_3(t,\tau)+H'_{4\tau}(t,\tau)][-px''(\tau)-qx'(\tau)-rx(\tau)+W(\tau)y(\tau)]+$$

$$+H_4(t,\tau)[(p^2-q)x''(\tau)+(pq-r)x'(\tau)+prx(\tau)+W_*(\tau)y(\tau)+W(\tau)y'(\tau)]\}d\tau=$$

$$=f(t)-H_4(t,t_0)x'''(t_0). \quad (9)$$

After the simplest transformations and dividing both sides of (9) by the function $W(t)$, we receive a second-order IDE of Volterra type for $y(t)$. Combining this IDE and the replacement (6), we come to the following system:

$$\begin{cases} x'''(t)+px''(t)+qx'(t)+rx(t)=W(t)y(t), \\ y''(t)+b_4(t)y'(t)+b_3(t)y(t)+b_2(t)x''(t)+b_1(t)x'(t)+b_0(t)x(t)+ \\ +\int_{t_0}^{t}\left[T_0(t,\tau)x(\tau)+T_1(t,\tau)x'(\tau)+T_2(t,\tau)x''(\tau)+T_3(t,\tau)y(\tau)+K(t,\tau)y'(\tau)\right]d\tau= \\ =(W(t))^{-1}f(t)-(W(t))^{-1}H_4(t,t_0)x'''(t_0), \end{cases} \quad (10)$$

equivalent to IDE (5), where

$$b_4(t)\equiv a_4(t)+[W_*(t)+W'(t)](W(t))^{-1},$$
$$b_3(t)\equiv a_3(t)-H_4(t,t)+a_4(t)W_*(t)(W(t))^{-1}+p^2-q+W_*'(t)(W(t))^{-1},$$
$$b_2(t)\equiv\{a_2(t)-p[a_3(t)-H_4(t,t)]+(p^2-q)a_4(t)+2pq-r-p^3\}(W(t))^{-1},$$
$$b_1(t)\equiv\{a_1(t)-q[a_3(t)-H_4(t,t)]+(pq-r)a_4(t)+pr+q^2-p^2q\}(W(t))^{-1},$$
$$b_0(t)\equiv\{a_0(t)-r[a_3(t)-H_4(t,t)]+pr\,a_4(t)+qr-p^2r\}(W(t))^{-1},$$
$$T_0(t,\tau)\equiv(W(t))^{-1}\{Q_0(t,\tau)-r[Q_3(t,\tau)+H'_{4\tau}(t,\tau)]+prH_4(t,\tau)\},$$
$$T_1(t,\tau)\equiv(W(t))^{-1}\{Q_1(t,\tau)-q[Q_3(t,\tau)+H'_{4\tau}(t,\tau)]+(pq-r)H_4(t,\tau)\},$$
$$T_2(t,\tau)\equiv(W(t))^{-1}\{Q_2(t,\tau)-p[Q_3(t,\tau)+H'_{4\tau}(t,\tau)]+(p^2-q)H_4(t,\tau)\},$$
$$T_3(t,\tau)\equiv(W(t))^{-1}\{[Q_3(t,\tau)+H'_{4\tau}(t,\tau)]W(\tau)+H_4(t,\tau)W_*(\tau)\},$$
$$K(t,\tau)\equiv(W(t))^{-1}H_4(t,\tau)W(\tau).$$

For further investigation of the system (10), we also proceed in the same way as in [4], i.e. first, we will carry out separate transformations for each equation of system (10), then we add them and we will obtain the final energy identity.

To the first equation of the system (10), we apply the method of squaring the equations [3, p. 28]. For any solution $(x(t),y(t))$ system (10) we square both parts of its first equation-the third-order DE for $x(t)$, we integrate in the range from t_0 to t, including by parts, and we have the following identity [4]:

$$V_1(t)\equiv\int_{t_0}^{t}[(x'''(s))^2+(p^2-2q)(x''(s))^2+(q^2-2pr)(x'(s))^2+r^2(x(s))^2]ds+$$

$$+qr(x(t))^2+(pq-r)(x'(t))^2+p(x''(t))^2+2prx(t)x'(t)+2rx(t)x''(t)+$$

$$+2qx'(t)x''(t)\equiv c_*+\int_{t_0}^{t}(W(s))^2(y(s))^2ds, \qquad (11)$$

where $c_*=V_1(t_0)$.

For the second equation, a second-order IDE for $y(t)$ we apply the method of cutting functions [3, p. 41]. Suppose that,

$$K(t,\tau)=\sum_{i=0}^{n}K_i(t,\ \tau), \qquad (K)$$

$$(W(t))^{-1}f(t)=\sum_{i=0}^{n}f_i(t), \qquad (f)$$

$\psi_i(t)\quad (i=1..n)$ - some cutting functions,

$$R_i(t,\tau)\equiv K_i(t,\tau)\left(\psi_i(t)\psi_i(\tau)\right)^{-1}, \qquad E_i(t)\equiv f_i(t)\left(\psi_i(t)\right)^{-1}\quad (i=1..n),$$

$$R_i(t,t_0)=A_i(t)+B_i(t)\ (i=1..n), \qquad (R)$$

$c_i(t)\ \ (i=1..n)$ - some functions.

For an arbitrarily fixed solution $(x(t),y(t))$ to system (10) its second equation is multiplied by $y'(t)$ [5, c. 194-217], we integrate in the range from t_0 to t, including by parts, similarly to [3], we introduce conditions *(K)*, *(f)*, the functions $\psi_i(t), R_i(t,\tau), E_i(t), c_i(t)\ \ (i=1..n),$ condition (R); in this case, we apply lemmas 1.4, 1.5 [6]. The result is the following identity:

$$V_2(t)\equiv(y'(t))^2+2\int_{t_0}^{t}b_4(s)(y'(s))^2ds+b_3(t)(y(t))^2+\sum_{i=1}^{n}\{A_i(t)(Y_i(t,t_0))^2+B_i(t)(Y_i(t,t_0))^2-$$

$$-2E_i(t)Y_i(t,t_0)+c_i(t)-\int_{t_0}^{t}\left[B_i'(s)(Y_i(s,t_0))^2-2E_i'(s)Y_i(s,t_0)+c_i'(s)\right]ds+\int_{t_0}^{t}R_{i\tau}'(t,\tau)(Y_i(t,\tau))^2d\tau\}\equiv$$

$$\equiv c_{***}+\int_{t_0}^{t}\{b_3'(s)(y(s))^2+\sum_{i=1}^{n}[A_i'(s)(Y_i(s,t_0))^2+\int_{t_0}^{s}R_{is\tau}''(s,\tau)(Y_i(s,\tau))^2d\tau]\}ds+$$

$$+2\int_{t_0}^{t}y'(s)[f_0(s)-(W(s))^{-1}H_4(s,t_0)x'''(t_0)]ds-2\int_{t_0}^{t}y'(s)\{b_2(s)x''(s)+b_1(s)x'(s)+$$

$$+b_0(s)x(s)+\int_{t_0}^{s}[T_0(s,\tau)x(\tau)+T_1(s,\tau)x'(\tau)+T_2(s,\tau)x''(\tau)+T_3(s,\tau)y(\tau)+$$

$$+K_0(s,\tau)y'(\tau)]d\tau\}ds, \qquad (12)$$

where

$$Y_i(t,\tau)\equiv\int_{\tau}^{t}\psi_i(\eta)y'(\eta)d\eta \quad (i=1..n), \quad c_{**}=V_2(t_0)=(y'(t_0))^2+b_3(t_0)(y(t_0))^2+\sum_{i=1}^{n}c_i(t_0).$$

Add up the identities (11), (12) and we will have the following final identity for any solution $(x(t),y(t))$ of the system (10):

$$V(t)\equiv\int_{t_0}^{t}[(x'''(s))^2+(p^2-2q)(x''(s))^2+(q^2-2pr)(x'(s))^2+r^2(x(s))^2]ds+$$

$$+qr(x(t))^2+(pq-r)(x'(t))^2+p(x''(t))^2+2prx(t)x'(t)+2rx(t)x''(t)+2qx'(t)x''(t)+(y'(t))^2+$$

$$+2\int_{t_0}^{t}b_4(s)(y'(s))^2ds+b_3(t)(y(t))^2+\sum_{i=1}^{n}\{A_i(t)(Y_i(t,t_0))^2+B_i(t)(Y_i(t,t_0))^2-2E_i(t)Y_i(t,t_0)+$$

$$+c_i(t)-\int_{t_0}^{t}\left[B_i'(s)(Y_i(s,t_0))^2-2E_i'(s)Y_i(s,t_0)+c_i'(s)\right]ds+\int_{t_0}^{t}R_{i\tau}'(t,\tau)(Y_i(t,\tau))^2d\tau\}\equiv V(t_0)+$$

$$+\int_{t_0}^{t}\{(W(s))^2(y(s))^2+2y'(s)[f_0(s)-(W(s))^{-1}H_4(s,t_0)x'''(t_0)]\}ds+$$

$$+\int_{t_0}^{t}\{b_3'(s)(y(s))^2+\sum_{i=1}^{n}[A_i'(s)(Y_i(s,t_0))^2+\int_{t_0}^{s}R_{is\tau}''(s,\tau)(Y_i(s,\tau))^2d\tau]ds-2\int_{t_0}^{t}y'(s)\{b_2(s)x''(s)+$$

$$+b_1(s)x'(s)+b_0(s)x(s)+\int_{t_0}^{s}[T_0(s,\tau)x(\tau)+T_1(s,\tau)x'(\tau)+T_2(s,\tau)x''(\tau)+T_3(s,\tau)y(\tau)]+$$

$$+K_0(s,\tau)y'(\tau)]d\tau\}ds, \qquad (13)$$

where $V(t_0)=V_1(t_0)+V_2(t_0)$.

Passing to the integral inequality from identity (13), using the generalized Sylvester criterion [7, p. 137], the Cauchy-Bunyakovsky inequality and applying lemma 1 [8], similar to the theorem from [4], we prove

Theorem. Let be 1) $p>0,\ q>0,\ r>0,\ W(t)>0,$ the conditions are met *(K), (f), (R);* 2) $p^2-2q>0;\ q^2-2pr>0;$

3) all major minors of the matrix A positive, where $A=\begin{pmatrix} qr & pr & r \\ pr & pq-r & q \\ r & q & p \end{pmatrix}$, i. e.

$$\Delta_1 = qr > 0,\ \Delta_2 = \begin{vmatrix} qr & pr \\ pr & pq-r \end{vmatrix} = qr(pq-r) - p^2r^2 > 0,\quad \Delta_3 = \det A = [p^2q^2 + r^2 - p^3r - q^3]r > 0;$$

4) $b_4(t) \ge 0$; 5) $b_3(t) \ge b_{30} > 0$, exists a function $b_3^*(t) \in L^1(I, R_+)$, such that $b_3'(t) \le b_3^*(t) b_3(t)$; 6) $A_i(t) \ge 0,\ B_i(t) \ge 0,\ B_i'(t) \le 0,\ R_{i\tau}'(t,\tau) \ge 0$, exists functions $A_i^*(t) \in L^1(I, R_+),\ c_i(t),\ R_i^*(t) \in L^1(I, R_+)$, such that

$$A_i'(t) \le A_i^*(t) A_i(t), \qquad (E_i^{(k)}(t))^2 \le B_i^{(k)}(t) c_i^{(k)}(t), \qquad R_{it\tau}''(t,\tau) \le R_i^*(t) R_{i\tau}'(t,\tau)\ \ (i = 1..n;\ \ k = 0,1);$$

7) $$(W(t))^2 + \left(b_k(t)\right)^2 + |f_0(t)| + (W(t))^{-1} |H_4(t,t_0)| + \left[\int_{t_0}^{t} (T_k(t,\ \tau))^2 d\tau\right]^{\frac{1}{2}} +$$

$$+ \int_{t_0}^{t} \left[|T_3(t,\tau)| + |K_0(t,\tau)|\right] d\tau \in L^1(I, R_+ \setminus \{0\}) \quad (k = 0,1,2).$$

Then for any solution, $(x(t), y(t))$ systems (10) are valid statements:

$$x^{(k)}(t) \in L^2(I, R) \qquad (k = 0,1,2,3), \tag{14}$$

$$y^{(\nu)}(t) = O(1), \qquad (\nu = 0,1), \tag{15}$$

$$b_4(t)(y'(t))^2 \in L^1(I, R_+), \qquad A_i(t)(Y_i(t,t_0))^2 = O(1)\ \ (i = 1..n).$$

Note that in the proof of this theorem, following the article [4], the following facts are used:

1°) using the generalized Sylvester criterion [7, p. 137], it is shown that the condition 3) of the theorem ensures the non-negativity of the quadratic form:

$$qr(x(t))^2 + (pq - r)(x'(t))^2 + p(x''(t))^2 + 2prx(t)x'(t) + 2rx(t)x''(t) + 2qx'(t)x''(t) \ge 0\,,$$

which is on the left side of identity (13);

2°) for any numbers $\varepsilon_k \in (0,1)$, i. e. $0 < \varepsilon_k < 1\ \ (k = 0,1,2)$, applying inequalities

$2|uv| = 2\frac{1}{\sqrt{\varepsilon_k}}\sqrt{\varepsilon_k}\,|uv| \le \frac{1}{\varepsilon_k}u^2 + \varepsilon_k v^2\ \ (k = 0,1,2)$ $(u, v$ - real functions) we get:

$$2\int_{t_0}^{t} |y'(s) b_0(s) x(s)| ds \le \frac{1}{\varepsilon_0 r^2} \int_{t_0}^{t} (b_0(s))^2 (y'(s))^2 ds + \varepsilon_0 r^2 \int_{t_0}^{t} (x(s))^2 ds,$$

$$2\int_{t_0}^{t} |y'(s) b_1(s) x'(s)| ds \le \frac{1}{\varepsilon_1 (q^2 - 2pr)} \int_{t_0}^{t} (b_1(s))^2 (y'(s))^2 ds + \varepsilon_1 (q^2 - 2pr) \int_{t_0}^{t} (x'(s))^2 ds,$$

$$2\int_{t_0}^{t} |y'(s) b_2(s) x''(s)| ds \le \frac{1}{\varepsilon_2 (p^2 - 2q)} \int_{t_0}^{t} (b_2(s))^2 (y'(s))^2 ds + \varepsilon_2 (p^2 - 2q) \int_{t_0}^{t} (x''(s))^2 ds;$$

3°) application of the Cauchy-Bunyakovsky inequality gives:

$$2\int_{t_0}^{t}\left[\int_{t_0}^{s}\left|T_k(s,\tau)x^{(k)}(\tau)\right|d\tau\right]\left|y'(s)\right|ds \le 2\int_{t_0}^{t}\left[\int_{t_0}^{s}(T_k(s,\tau))^2 d\tau\right]^{\frac{1}{2}}\left[\int_{t_0}^{s}(x^{(k)}(\tau))^2 d\tau\right]^{\frac{1}{2}}|y'(s)|ds\ (k=0,1,2).$$

Note that the inequalities 1°), 2°) are essentially used in the transition from the identity (13) to the integral inequality. Also, by virtue of the conditions 1)-6) of the theorem, we get that $V(t)\ge 0$ and the proof of our theorem can be carried out similarly to the proofs of theorem 1.1 [3, p. 46-49], theorem 1.3 [3, p. 58-60], and theorem 2.1 [3, p. 83-85].

From theorem is analogous to consequence [4] it turns out

Corollary. If all the conditions of the theorem are satisfied and $W^{(\nu)}(t)\to 0\ (\nu=0,1)$ at $t\to\infty,$ then all solutions and their derivatives up to and including the fourth-order IDE of the fifth order (1) tend to zero at $t\to\infty$, that is, any solution of the given IDE (1) is asymptotically stable.

Really, by virtue of the conditions of the theorem, we have statements (14), from which, by applying the Lyusternik-Sobolev lemma [9, p. 393-394; 4], we obtain that for any solution $x(t)$ IDE (1) the statements are true: $x^{(k)}(t)\to 0\ (k=0,1,2)$ at $t\to\infty$. Из replacements (6) taking into account $y(t)=O(1)$ approvals (15) and $W(t)\to 0,\ t\to\infty$ it follows that $x'''(t)\to 0$ at $t\to\infty$. Finally, from the relation (7) based on the conditions $W^{(\nu)}(t)\to 0\ \ (\nu=0,1),\ t\to\infty$ and $y'(t)=O(1)$ statement implies that $x^{(4)}(t)\to 0$ at $t\to\infty$. Thus, we have shown that $x^{(j)}(t)\to 0\ \ (j=0,1,2,3,4)$ at $t\to\infty$ for any solution $x(t)$ IDE (1), which is equivalent to proving the assertion of the corollary.

Example. Fifth-order IDE (1) with

$$a_4(t)\equiv 10+e^t\sqrt{t},\ a_3(t)\equiv\frac{t+4}{t+5}+H_4(t,t)+35+7e^t\sqrt{t},$$

$$a_2(t)\equiv\frac{e^{-2t}(\sin t)^{\frac{1}{5}}}{t+1}+5\left(\frac{t+4}{t+5}\right)+49+14e^t\sqrt{t},$$

$$a_1(t)\equiv e^{-2t}\left(\frac{\sin t}{t}\right)+4\left(\frac{t+4}{t+5}\right)+29+9e^t\sqrt{t},\ a_0(t)\equiv-\frac{21e^{-2t}}{t+2}+\frac{t+4}{t+5}+6+2e^t\sqrt{t},$$

$$Q_0(t,\tau)\equiv 2H_4(t,\tau)+\frac{e^{-2t+2\tau}\sin\tau}{(5t+4\tau+8)^3}-\frac{e^{-2t}\cos\tau}{(t+6\tau+2)^4},$$

$$Q_1(t,\tau)\equiv 9H_4(t,\tau)+\frac{4e^{-2t+2\tau}\sin\tau}{(5t+4\tau+8)^3}+\frac{e^{-2t}}{e^t+e^\tau+3},$$

$$Q_2(t,\tau)\equiv 14H_4(t,\tau)+\frac{5e^{-2t+2\tau}\sin\tau}{(5t+4\tau+8)^3}-\frac{e^{-2t}\sin e^{-t}}{t+\tau+1},\ Q_3(t,\tau)\equiv -H'_{4\tau}(t,\tau)+7H_4(t,\tau)+\frac{e^{-2t+2\tau}\sin\tau}{(5t+4\tau+8)^3},$$

$f(t)\equiv -\dfrac{e^{-2t}e^{t^3}t\sqrt[3]{\sin t}}{t+10}+\dfrac{e^{-2t}\sin 6t}{t^2+4}$, $t_0=0$ satisfies all the conditions of the theorem and corollary for

$$H_4(t,\tau)\equiv e^{-2t+2\tau}\left\{\left[\exp\left(\frac{\cos t}{(t+5)^2}\right)+\tau\right]^{\frac{1}{2}}+\frac{1}{t-\tau+9}\right\}e^{t^3+\tau^3}t\tau\sqrt[3]{\sin t\sin\tau}-\frac{e^{-2t+2\tau}}{(2t+3\tau+4)^5},$$

$$p=5,\ q=4,\ r=1,\ W(t)\equiv e^{-2t},\ \text{here}$$

$$\Delta_2=51,\ \Delta_3=212,\ b_4(t)\equiv 1+e^t\sqrt{t},\ b_3(t)\equiv\frac{t+4}{t+5},\ b_{30}=\frac{4}{5},\ b_3^*(t)\equiv\frac{1}{(t+4)(t+5)},$$

$$b_2(t)\equiv\frac{(\sin t)^{\frac{1}{5}}}{t+1},\ b_1(t)\equiv\frac{\sin t}{t},\ b_0(t)\equiv -\frac{21}{t+2},\ T_0(t,\tau)\equiv -\frac{\cos\tau}{(t+6\tau+2)^4},$$

$$T_1(t,\tau)\equiv\frac{1}{e^t+e^\tau+3},\ T_2(t,\tau)\equiv -\frac{\sin e^{-t}}{t+\tau+1},\ T_3(t,\tau)\equiv\frac{\sin\tau}{\left(5t+4\tau+8\right)^3},$$

$$K(t,\tau)\equiv\left\{\left[\exp\left(\frac{\cos t}{(t+5)^2}\right)+\tau\right]^{\frac{1}{2}}+\frac{1}{t-\tau+9}\right\}e^{t^3+\tau^3}t\tau\sqrt[3]{\sin t\sin\tau}-\frac{1}{(2t+3\tau+4)^5},$$

$$n=1,\ \psi_1(t)\equiv e^{t^3}t\sqrt[3]{\sin t},\ R_1(t,\tau)\equiv\left[\exp\left(\frac{\cos t}{(t+5)^2}\right)+\tau\right]^{\frac{1}{2}}+\frac{1}{t-\tau+9},$$

$$A_1(t)\equiv\exp\left(\frac{\cos t}{2(t+5)^2}\right),\quad B_1(t)\equiv\frac{1}{t+9},\quad A_1^*(t)\equiv\frac{t+7}{(t+5)^3},\ R_1^*(t)\equiv\frac{t+7}{(t+5)^3},$$

$$K_0(t,\tau)\equiv -\frac{1}{(2t+3\tau+4)^5},\ E_1(t)\equiv -\frac{1}{t+10},\ c_1(t)\equiv\frac{1}{t+9},\ f_0(t)\equiv\frac{\sin 6t}{t^2+4}.$$

In conclusion, we note that we have found a new class of fifth-order IDE of the form (1), for which the above problem is solvable. Note that our study differs significantly from the studies from [10, 11].

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SUFFICIENT CONDITIONS OF THE CAUCHY PROBLEM FOR NONLINEAR DIFFERENTIAL EQUATIONS IN PRIVATE DERIVATIVES

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Сызыктуу эмес жекече туундулуу дифференциалдык теңдемелерге коюулган Коши маселесинин чыгарымдуулугу жана чыгарылыштарынын түзүлүшү изилденди. ерге жана өздүк функцияларына ээ экендиги аныкталды. Жекече туундулуу дифференциалдык теңдемелерге коюулган баштапкы маселесинин чыгарылышынын жашашынын жетишээрлик шарттары табылды.
Негизги сөздөр: Жекече туундулуу дифференциалдык теңдемелер. II типтеги Вольтерра интегралдык теңдемеси, кысып чагылтуу принциби.

Исследованы разрешимость задачи Коши и структура решений для нелинейных дифференциальных уравнений в частных производных. Найдены достаточные условия существования решений начальной задачи для дифференциальных уравнений в частных производных.
Ключевые слова: дифференциальные уравнения в частных производных, интегральное уравнение Вольтерра II рода, принцип сжатых отображений.

The solvability of the Cauchy problem and the structure of solutions for nonlinear partial differential equations are investigated. Sufficient conditions are found for the existence of solutions to the initial value problem for partial differential equations.
Ключевые слова: дифференциальные уравнения в частных производных, интегральное уравнение Вольтерра II рода, принцип сжатых отображений.

In this paper, we investigate the solvability of the Cauchy problem and the structure of solutions for nonlinear partial differential equations.

I. Consider an equation of the form

$$u_{tx} + \alpha u_t = f(t, x, u(t, x)) \quad (1)$$

with the initial condition

$$u = \varphi(x) \tag{2}$$

where, $\alpha - const$, $f(t, x, u(t, x)) \in \overline{C}(D \times R) \cap Lip(L|_u)$.

The solution to the Cauchy problem (1)-(2) will be sought in the form

$$u(t, x) = c(t, x) + \int_0^t \int_{-\infty}^x e^{-\alpha(x-\rho)} \cos(t-s) Q(s, \rho) d\rho ds \tag{3}$$

where $c(t, x)$ - is a known continuous function, while

$$c(o, x) = \varphi(x),$$

$Q(t, x)$ - is a new unknown function to be determined.

To define the function, it is necessary to substitute (3) in (1). For this purpose, taking into account (3), we sequentially calculate the following relations. We get:

$$u_x(t, x) = c_x + \int_0^t \cos(t-s) Q(s, x) ds - \alpha(u - c); \tag{4}$$

$$u_x + \alpha(u - c) = c_x + \int_0^t \cos(t-s) Q(s, x) ds; \tag{5}$$

$$u_{xt} + \alpha(u_t - c_t) = c_{xt} + Q(t, x) - \int_0^t \sin(t-s) Q(s, x) ds. \tag{6}$$

From relation (6) we have

$$u_{xt} + \alpha u_t = c_{xt} + \alpha c_t + Q(t, x) - \int_0^t \sin(t-s) Q(s, x) ds. \tag{7}$$

Then, instead of the original equation (1), we have a nonlinear integral equation for

$$Q(t, x) = f\left(t, x, c + \int_0^t \int_{-\infty}^x e^{-\alpha(x-\rho)} \cos(x-\rho) Q(s, \rho) d\rho ds\right) + \int_0^t \sin(t-s) Q(s, x) ds + H(t, c), \tag{8}$$

where $H(t, c) = c_{xt} + \alpha c_t$.

We will solve the nonlinear integral equation (8) by the method of squeezed mappings [1].

Let be

$$H(t, c) \in \overline{C}(D) \tag{9}$$

Those $\|H(t, c)\| \le M_0 = const$.

Let us consider the right-hand side of (8) as an operator acting on a function. We have

$$\left\|A[Q]\right\| \le \left\|(f(t,x,u) + \int_0^t \sin(t-s)Q(s,x)ds) + H(t,c)\right\| \le M$$

where $M = M_0 + M_1 = const$.

We will estimate the difference

$$\left\|A[Q_1(t,x)] - A[Q_2(t,x)]\right\| \le \| \quad f\left(t,x,c+\int_0^t \int_{-\infty}^x e^{-\alpha(x-\rho)}\cos(t-s)Q_1(t,s)d\rho ds\right) + \int_0^t \sin(t-s)Q_1(s,x)ds -$$

$$-f\left(t,x,c+\int_0^t \int_{-\infty}^x e^{-\alpha(x-\rho)}\cos(t-s)Q_2(t,s)d\rho ds\right) - \int_0^t \sin(t-s)Q(s,x)ds \quad \| \le$$

$$\le L\left\{\int_0^t \int_{-\infty}^x e^{-\alpha(x-\rho)}\cos(t-s)\left\|Q_1(\rho,s) - Q_2(\rho,s)\right\| d\rho ds\right\} + T\left\|Q_1(\rho,s) - Q_2(\rho,s)\right\| \le$$

$$\le (L\frac{T}{\alpha} + T)\left\|Q_1(t,x) - Q_2(t,x)\right\| \le \frac{1}{2}\left\|Q_1(t,x) - Q_2(t,x)\right\|,$$

where α, T - are chosen such that

$$\frac{LT}{\alpha} + T < \frac{1}{2}. \qquad (10)$$

By the principle of squeezed mappings, it follows that the nonlinear integral equation (8) has a unique solution.

Next, we prove the boundedness of solutions to the Cauchy problem (1), (2). From (3), for all $(t,x) \in D$, we have the inequality

$$\left\|u(t,x)\right\| \le \left\|c(t,x)\right\| + \int_0^t \int_{-\infty}^x e^{-\alpha(x-\rho)}\cos(t-s)\left\|Q(s,\rho)\right\| d\rho ds \le c_0 + \frac{M}{\alpha} \equiv M_{00} = const.$$

So it is proven

Theorem 1. Let conditions (2), (9), and (10) be satisfied. Then the nonlinear second-order partial differential equation (1) with initial data (2) has a unique solution.

II. Now consider an equation of the form

$$u_{txx} + \alpha u_{xx} + u_t + \alpha u = f(t,x,u(t,x)) \qquad (11)$$

with the initial condition

$$u = \varphi(x) \qquad (12)$$

where $\alpha - const$,

$$f(t,x,u(t,x)) \in \overline{C}(D \times R) \cap Lip(L|_u).$$

The solution to the Cauchy problem (11) - (12) will be sought in the form

$$u(t,x) = c(t,x) + \int_0^t \int_{-\infty}^x e^{-\alpha(t-s)} \sin(x-\rho) Q(s,\rho) d\rho ds \quad (13)$$

where $c(t,x)$ - is a known continuous function, while

$$c(o,x) = \varphi(x),$$

$Q(t,x)$ - is a new unknown function to be determined.

To define the function, it is necessary to substitute (13) in (11). For this purpose, taking into account (13), we successively calculate the following relations. We get:

$$u_t(t,x) = c_t - \alpha(u-c) + \int_{-\infty}^x \sin(x-\rho) Q(t,\rho) d\rho; \quad (14)$$

$$u_{tx} + \alpha(u_x - c_x) = c_{tx} + \int_{-\infty}^x cos(x-\rho) Q(t,\rho) d\rho; \quad (15)$$

$$u_{txx} + \alpha(u_{xx} - c_{xx}) = c_{txx} + Q(t,x) - \int_{-\infty}^x sin(x-\rho) Q(t,\rho) d\rho. \quad (16)$$

From relations (14) and (16) we obtain

$$u_{txx} + \alpha u_{xx} + u_t + \alpha u = c_{txx} + \alpha c_{xx} + c_t + \alpha c + Q(t,x). \quad (17)$$

Then, instead of the original equation (11), we have a nonlinear integral equation for $Q(t,x)$:

$$Q(t,x) = f\left(t, x, c + \int_0^t \int_{-\infty}^x e^{-\alpha(x-\rho)} \cos(x-\rho) Q(s,\rho) d\rho ds\right) + H(t,c) \quad (18)$$

where $H(t,c) = -(c_{txx} + \alpha c_{xx} + c_t + \alpha c)$.

We will solve the nonlinear integral equation (18) by the method of squeezed mappings [1].

Let be

$$H(t,c) \in \overline{C}(D) \quad (19)$$

those

$$\|H(t,c)\| \le \overline{M}_1 = const.$$

Let us consider the right-hand side of (18) as an operator $A[Q]$ acting on a function $Q(t,x)$. We have

$$\|A[Q]\| \le \| f(t,x,u) + H(t,c)\| \le M$$

where $M = \bar{M}_0 + \bar{M}_1 = const$.

We will estimate the difference

$$\|A[Q_1(t,x)] - A[Q_2(t,x)]\| \le f\left(t,x,c+\int_0^t\int_{-\infty}^x e^{-\alpha(x-\rho)}\cos(x-\rho)Q(s,\rho)d\rho ds\right) -$$

$$-f\left(t,x,c+\int_0^t\int_{-\infty}^x e^{-\alpha(x-\rho)}\cos(x-\rho)Q(s,\rho)d\rho ds\right) \le$$

$$\le L\left\{\int_0^t\int_{-\infty}^x e^{-\alpha(x-\rho)}\cos(x-\rho)\|Q_1(\rho,s)-Q_2(\rho,s)\|d\rho ds\right\} \le$$

$$\le L\frac{T}{\alpha}\|Q_1(t,x)-Q_2(t,x)\| \le \frac{1}{2}\|Q_1(t,x)-Q_2(t,x)\|,$$

where α - are chosen such that

$$\frac{LT}{\alpha} < \frac{1}{2}. \tag{20}$$

By the principle of squeezed mappings, it follows that the nonlinear integral equation (18) has a unique solution.

Next, we prove the boundedness of solutions to the Cauchy problem (11), (12). From (13), for all, we have the inequality

$$\|u(t,x)\| \le \|c(t,x)\| + \int_0^t\int_{-\infty}^x e^{-\alpha(x-\rho)}\cos(t-s)\|Q(s,\rho)\|d\rho ds \le c_0 + \frac{\bar{M}}{\alpha} \equiv \bar{M}_{00} = const.$$

So it is proven

Theorem 2. Let conditions (12), (19), and (20) be satisfied. Then the nonlinear third-order partial differential equation (11) with initial data (12) has a unique solution

$$u(t,x) \in \bar{C}^{(1,2)}(R_+ \times R).$$

Note that this work uses the approach developed in the works of the authors [2,3].

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ON THE SOLVABILITY OF THE CAUCHY PROBLEM FOR HOMOGENEOUS SUM-DIFFERENCE EQUATIONS IN THE STRUCTURE OF THE CHARACTERISTIC POLYNOMIAL

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This paper deals with the problem of solvability of the Cauchy problem for the homogeneous summary-difference equations with constant coefficients. The method of calculating operating a criterion for the solvability of the initial problem.
Keywords: Cauchy problem, differential equation, sum-difference equations, operational calculus, the entire function.

Бул эмгек турактуу коэффициенттүү бир тектүү суммалуу-айырмалуу теңдемелердин Коши маселесинин чечүү маселесине арналган. Баштапкы маселенин чечилүүчүлүгүнүн критерийи операторду эсептөө ыкмасы менен алынат.
Урунттуу сөздөр: Коши маселеси, айырмалуу теңдеме, суммалуу-айырмалуу теңдеме, оператордук эсептөө, бүтүн функция.

Данная работа посвящена проблеме разрешимости задачи Коши для однородных суммарно-разностных уравнений с постоянными коэффициентами. Методом операторного исчисления получен критерий разрешимости начальной задачи.
Ключевые слова: задача Коши, разностное уравнение, суммарно-разностное уравнение, операторное исчисление, целая функция.

Consider a homogeneous sum-difference equation with constant coefficients of the form

$$\sum_{i=0}^{m} a_i u(n+i) = \sum_{i=0}^{l}\sum_{j=0}^{n-1} K_i(n-1-j)u(j+i), \quad n \geq 0\,, \tag{1}$$

where $u(n)$ is the required function; a_i - constant coefficients; $a_m = 1,\ K_i(n)$ is a given function and has the form

$$K_i(n) = \sum_{v=1}^{m_i} Q_{iv}\lambda_{iv}^n\,, \quad Q_{iv},\ \lambda_{iv} = const\,, \tag{2}$$

$$u(i) = u_i, \quad i = \overline{0,\ \mu-1}; \quad \mu = \max\{m,\ l\} \tag{3}$$

initial condition of problem (1).

To study problem (1), (3), we use the operator calculus of the form [4]: $D\{f(n)\} = \sum_{n=0}^{\infty} s^{-(n+1)} f(n)$, where s is a complex parameter. For the D-transformation, the following formulas are valid [5]:

$$D\{f(n+t)\} = s^t D\{f(n)\} - \sum_{i=0}^{t-1} s^{t-i-1} f(i)\,; \quad D\left\{(n^{(m)}/m!)\lambda^{n-m}\right\} = 1/(s-\lambda)^{m+1}\,; \tag{4}$$

where $n^{(m)} = n(n-1)\cdot\ldots\cdot(n-m+1), n^{(0)} = 1$ is a generalized degree of order m, number n. Applying the operator D to both sides of (1), we obtain

$$\varphi_1(s)U(s) = \psi_1(s)\,, \tag{5}$$

where $U(s) \equiv D\{u(n)\}$; $\quad \psi_1(s) = \sum_{j=0}^{m-1}\sum_{i=j+1}^{m} a_i s^{i-j-1} u(j) - \sum_{j=0}^{l-1}\sum_{i=j+1}^{l} s^{i-1-j}\bar{K}_i(s)u(j);$

$$\varphi_1(s) = \sum_{i=0}^{m} a_i s^i - \sum_{i=0}^{l} s^i \bar{K}_i(s); \quad \bar{K}_i(s) \equiv D\{K_i(n)\} = \sum_{v=1}^{m_i} \frac{Q_{iv}}{s-\lambda_{iv}}. \tag{6}$$

We denote by $h(s)$ the least common denominator of the fractions $1/(s-\lambda_{iv});\ i = \overline{0,\ l},\ v = \overline{1,\ m_i}$. In particular, if there are no coinciding among λ_{iv}, then

$$h(s) = \prod(s-\lambda_{iv}); \quad i = \overline{0,\ l}, \quad v = \overline{1,\ m_i}\ . \tag{7}$$

Next, we introduce the notation

$$\begin{aligned} &b(s) = h(s)\sum_{i=0}^{m} a_i s^i; \quad B(s) = h(s)\sum_{i=0}^{l}\bar{K}_i(s)s^i = \sum_{i=0}^{l}\sum_{v=1}^{m_i} Q_{iv}h_{iv}(s)s^i, \quad h_{iv}(s) = \frac{h(s)}{s-\lambda_{iv}}; \\ &b_j(s) = h(s)\sum_{i=j+1}^{m} a_i s^{i-1-j}, \quad j = \overline{0,\ m-1}; \quad B_j(s) = h(s)\sum_{i=j+1}^{l} s^{i-1-j}\bar{K}_i(s), \quad j = \overline{0,\ l-1}. \end{aligned} \tag{8}$$

According to [1], multiplying both sides of (5) by the polynomial $h(s)$, we obtain

$$\varphi(s)U(s) = \psi(s), \tag{9}$$

where
$$\varphi(s) \equiv h(s)\varphi_1(s) = b(s) - B(s); \tag{10}$$

$$\psi(s) \equiv h(s)\psi_1(s) = \sum_{j=0}^{m-1} b_j(s)u(j) - \sum_{j=0}^{l-1} B_j(s)u(j). \tag{11}$$

From (9) we find the image of the desired solution

$$U(s) = \psi(s)/\varphi(s), \tag{12}$$

as the ratio of two polynomials. If in (12) $cm.\,\psi(s) < cm.\,\varphi(s)$, (13)

then you can go directly to the original. Everywhere we will assume that $m < l$. In this case, as can be seen from (11), (13)

$$0 \le cm.\,\varphi(s) \le M + l - 1. \tag{14}$$

For definiteness, put
$$cm.\,\varphi(s) = M + l - k. \tag{15}$$

By virtue of (14) $1 \le k \le M + l$.

The following three cases are distinguished: **1.** $r \equiv l - m > k$; **2.** $r \le k \le l$; **3.** $l < k$, where k is some natural number under which (15) holds. The first case ($r > k$) was considered in [7]. In this paper, we will consider the second case.

To derive the conditions under which (15) is satisfied, we need to transform $B(s)$. From formula (4) we have

$$s^t D\{K_i(n)\} = D\{K_i(n+t)\} + \sum_{j=1}^{t} s^{j-1} K_i(t-j). \tag{16}$$

Multiplying both sides of (16) by $h^{(t)}(0)/t!$ and summing over t from 0 to M, we obtain

$$\sum_{t=0}^{M} s^t \frac{h^{(t)}(0)}{t!} D\{K_i(n)\} = \sum_{t=0}^{M} \frac{h^{(t)}(0)}{t!} D\{K_i(n+t)\} + \sum_{j=1}^{M} s^{j-1} K_i(t-j). \tag{17}$$

Taking into account the linearity of the operator D and the form (2) of the kernel $K_i(n)$, we have $s_1 = \sum_{v=1}^{m_i} Q_{iv} D\{\lambda_{iv}^n\} \sum_{t=0}^{M} \frac{h^{(t)}(0)}{t!} \lambda_{iv}^t = \sum_{v=1}^{m_i} Q_{iv} h_{iv}(\lambda_{iv}) D\{\lambda_{iv}^n\} = 0$, because λ_{iv} is by virtue of (7) the roots of the polynomial $h(s)$. Then (17) can be written as

$$h(s)\bar{K}_i(s) = \sum_{t=0}^{M} \frac{h^{(t)}(0)}{t!} \sum_{j=1}^{t} s^{j-1} K_i(t-j).$$

Introducing the substitution $j_1 = j-1,\ j_1 \to j,$ and changing the order of summation, we have

$$h(s)\bar{K}_i(s) = \sum_{j=0}^{M-1}\sum_{t=j+1}^{M}\frac{h^{(t)}(0)}{t!}s^j K_i(t-1-j)\,. \tag{18}$$

By virtue of (18), the polynomial $B(s)$ takes the form

$$B(s) = \sum_{i=0}^{l}\sum_{j=0}^{M-1} b_{ij}s^{i+j}\ , \tag{19}$$

where
$$b_{ij}(s) \equiv \sum_{t=j+1}^{M}\frac{h^{(t)}(0)}{t!}K_i(t-1-j)\,. \tag{20}$$

From (8) we have
$$s^{j+1}b_j(s) = b(s) - \sum_{i=0}^{j} a_i\, s^i h(s)\,, \tag{21}$$

$$s^{j+1}B_j(s) = B(s) - \sum_{i=0}^{j}\sum_{v=1}^{m_i} Q_{iv}\, s^i h_{iv}(s)\,. \tag{22}$$

For $0 \le j \le m-1$ we have

$$s^{j+1}[b_j(s) - B_j(s)] = \varphi(s) - \sum_{i=0}^{j} a_i\, s^i h(s) + \sum_{i=0}^{j}\sum_{v=1}^{m_i} Q_{iv}\, s^i h_{iv}(s)\,. \tag{23}$$

From (23), taking into account (15), by virtue of $r > k$, we obtain

$$\max_{0\le j\le m-1} cm.\,[b_j(s) - B_j(s)] \le M + l - k - 1\,. \tag{24}$$

Now let us estimate the magnitude of the degree $B_j(s)$ for $m \le j \le l-1$. Since, in view of (15), from (22), taking into account (24), we find $cm.B_j(s) + j + 1 \le \max\{M + l - k,\ M - 1 + j\}$, whence, in view of $l - k \ge 0$

$$\max_{m\le j\le l-1} cm.\ B_j(s) \le M + l - k - m - 1 \le M + l - k - 1\,. \tag{25}$$

From (24) and (25), taking into account the structure of the polynomial $\psi(s)$, we conclude that $cm.\,\psi(s) \le M + l - k - 1$, i.e. the validity of inequality (13) for $r > k$. By virtue of (13) in (12), we can go directly to the original.

CASE II.
$$r \le k \le l\,. \tag{26}$$

Let, for definiteness, $M < k$ and $r > k - M$. We represent the characteristic polynomial $\varphi(s)$ in the form

$$\varphi(s) = \sum_{i=0}^{m}\sum_{j=0}^{M} a_{ij}\, s^{i+j} - \sum_{i=0}^{l}\sum_{j=0}^{M-1} b_{ij}\, s^{i+j}\ , \tag{27}$$

where $$a_{ij} = \frac{h^{(j)}(0)}{j!} a_i \,. \qquad (27')$$

We draw the domain of definition of the coefficients of the polynomial $\varphi(s)$ on the plane $i0j$. By virtue of (15), (26), we have

$$\sum_{i=l+2-r}^{l} \sum_{j=M+l-r-i+1}^{M-1} b_{ij} s^{i+j} = 0; \qquad (28)$$

$$\sum_{i=l-M}^{l-r} \sum_{j=l-i}^{M} a_{ij} s^{i+j} - \sum_{i=l+1-M}^{l-r} \sum_{j=l-i}^{M-1} b_{ij} s^{i+j} - \sum_{i=l+1-r}^{l} \sum_{j=l-i}^{M+l-r-i} b_{ij} s^{i+j} = 0; \qquad (29)$$

$$\sum_{i=l+1-k+y}^{l-r} a_{i,\,M+l-k+1-i+y} - \sum_{i=l+2-k+y}^{M+l-k+1+y} b_{i,\,M+l-k+1-i+y} = 0; \quad y = \overline{0,\ k-2-M}; \qquad (30)$$

$$\sum_{i=l-k}^{l-r} a_{i,\,M+l-k-i} - \sum_{i=l-k}^{l-k+M} b_{i,\,M+l-k-i} \neq 0. \qquad (31)$$

In (28) and (29), making substitutions $i = l-\delta,\ i+j = M+l-x$ and changing the order of summation, we obtain

$$\sum_{x=1}^{r-1} s^{M+l-x} \sum_{\delta=0}^{x-1} b_{l-\delta,\,M-x+\delta} = 0; \qquad (28_1)$$

$$\sum_{x=r}^{M} s^{M+l-x} \sum_{\delta=r}^{x} a_{l-\delta,\,M-x+\delta} - \sum_{x=r+1}^{M} s^{M+l-x} \sum_{\delta=r}^{x-1} b_{l-\delta,\,M-x+\delta} - \sum_{x=r}^{M} s^{M+l-x} \sum_{\delta=0}^{r-1} b_{l-\delta,\,M-x+\delta} = 0. \qquad (29_1)$$

From (28_1), by virtue of the assumption $r < M$, we have

$$\sum_{\delta=0}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta) = 0, \quad \sigma = \overline{1,\ r-1}. \qquad (32)$$

In (29_1), equating to zero, the coefficients at the same powers of s, we find

$$\sum_{\delta=r}^{x} a_{l-\delta,\,M-x+\delta} - \sum_{\delta=0}^{x-1} b_{l-\delta,\,M-x+\delta} = 0, \ \ x = \overline{r,\ M}. \qquad (29_2)$$

By virtue of (20) and (27 '), expression (29_2) takes the form

$$\sum_{\sigma=r}^{x} \frac{h^{(M-x+\sigma)}(0)}{(M-x+\sigma)!} a_{l-\delta} - \sum_{\sigma=1}^{x} \frac{h^{(M-x+\sigma)}(0)}{(M-x+\sigma)!} \sum_{\delta=0}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta) = 0, \quad x = \overline{r,\ M}, \qquad (29_3)$$

where the order of summation is rearranged in the second sum. Substituting $t - M + x = \sigma$ from the latter, we have

$$\sum_{\sigma=r}^{x} c_{x\sigma} y_\sigma = 0, \quad x = \overline{r,\ M}, \ \ c_{x\sigma} \equiv \frac{h^{(M-x+\sigma)}(0)}{(M-x+\sigma)!}, \ \ y_\sigma \equiv a_{l-\delta} - \sum_{\delta=0}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta). \qquad (33)$$

Since the matrix of system (33) is triangular, and its diagonal elements are nonzero: $c_{rr} = c_{r+1,r+1} = ... = c_{MM} = 1 \neq 0$, this system has only a zero solution. So,

$$a_{l-\delta} - \sum_{\delta=0}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta) = 0, \quad \sigma = \overline{r,\ M}. \tag{34}$$

Now we transform (30) and (31). Further, replacing $i = l - \delta$ from (30) and (31), we have

$$\sum_{\delta=r}^{k-1-y} a_{l-\delta,\, M-k+1+y+\delta} - \sum_{\delta=k-1-M-y}^{k-2-y} b_{l-\delta,\, M-k+1+y+\delta} = 0, \quad y = \overline{0,\ k-2-M}; \tag{30$_1$}$$

$$\sum_{\delta=r}^{k} a_{l-\delta,\, M-k+\delta} - \sum_{\delta=k-M}^{k} b_{l-\delta,\, M+\delta-k} \neq 0. \tag{31$_1$}$$

By virtue of (20) and (27 '), relations (30_1) and (31_1) take the form

$$\sum_{\sigma=r}^{k-1-y} \frac{h^{(M-k+1+y+\sigma)}(0)}{(M-k+1+y+\sigma)!} a_{l-\delta} - \sum_{t=1}^{M} \frac{h^{(t)}(0)}{t!} \sum_{\delta=k-y-M-1}^{t-M+k-y-2} K_{l-\delta}(t-M+k-y-2-\delta) = 0, \quad y = \overline{0,\ k-2-M}; \tag{30$_2$}$$

$$\sum_{\sigma=r}^{k} \frac{h^{(M-k+\sigma)}(0)}{(M-k+\sigma)!} a_{l-\delta} - \sum_{t=1}^{M} \frac{h^{(t)}(0)}{t!} \sum_{\delta=k-M}^{t-M+k-1} K_{l-\delta}(t-M+k-1-\delta) \neq 0, \tag{31$_2$}$$

where the orders of summation are replaced in the second sum. In (30_2) and (31_2), respectively, making another change of $t - M + k - 1 - y = \sigma$ and $t - M + k = \sigma$, we obtain

$$\sum_{\sigma=r}^{k-1-y} \frac{h^{(M-k+1+y+\sigma)}(0)}{(M-k+1+y+\sigma)!} a_{l-\delta} - \sum_{\sigma=k-M-y}^{k-1-y} \frac{h^{(M-k+1+y+\sigma)}(0)}{(M-k+1+y+\sigma)!} \sum_{\delta=k-1-M-y}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta) = 0, \quad y = \overline{0,\ k-2-M}; \tag{30$_3$}$$

$$\sum_{\sigma=r}^{k} \frac{h^{(M-k+\sigma)}(0)}{(M-k+\sigma)!} a_{l-\delta} - \sum_{\sigma=k-M+1}^{k} \frac{h^{(M-k+\sigma)}(0)}{(M-k+\sigma)!} \sum_{\delta=k-M}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta) \neq 0. \tag{31$_3$}$$

We transform relations (30_3) and (31_3) in the following form

$$\sum_{\sigma=r}^{k-1-y} \frac{h^{(M-k+1+y+\sigma)}(0)}{(M-k+1+y+\sigma)!} a_{l-\delta} - \sum_{\sigma=k-1-M-y}^{k-1-y} \frac{h^{(M-k+1+y+\sigma)}(0)}{(M-k+1+y+\sigma)!} \sum_{\delta=0}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta) = 0, \quad y = \overline{0,\ k-2-M}; \tag{30$_4$}$$

$$\sum_{\sigma=r}^{k} \frac{h^{(M-k+\sigma)}(0)}{(M-k+\sigma)!} a_{l-\delta} - \sum_{\sigma=k-M}^{k} \frac{h^{(M-k+\sigma)}(0)}{(M-k+\sigma)!} \sum_{\delta=0}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta) \neq 0. \tag{31$_4$}$$

Since by assumption $k - M \leq r - 1$, by virtue of (32) from (30_4) and (31_4), we have

$$\sum_{\sigma=r}^{k-1-y} \frac{h^{(M-k+1+y+\sigma)}(0)}{(M-k+1+y+\sigma)!} \left[a_{l-\delta} - \sum_{\delta=0}^{\sigma-1} K_{l-\delta}(\sigma-1-\delta) \right] = 0, \quad y = \overline{0,\ k-2-M}; \tag{30$_5$}$$

$$\sum_{\sigma=r}^{k}\frac{h^{(M-k+\sigma)}(0)}{(M-k+\sigma)!}\left[a_{l-\delta}-\sum_{\delta=0}^{\sigma-1}K_{l-\delta}(\sigma-1-\delta)\right]\neq 0. \tag{31$_5$}$$

From (30_5), taking into account (33), (34), we obtain

$$a_{l-\delta}-\sum_{\delta=0}^{\sigma-1}K_{l-\delta}(\sigma-1-\delta)=0,\quad \sigma=\overline{M+1,\ k-1}. \tag{35}$$

Combining equalities (34) and (35) into one and replacing $\sigma=r+x$, we find

$$a_{m-x}-\sum_{\delta=0}^{r-1+x}K_{l-\delta}(r-1+x-\delta)=0,\quad x=\overline{0,\ k-1-r}. \tag{36}$$

From (32_5), by virtue of (35), (31_5), and (36), we obtain

$$a_{l-k}-\sum_{\delta=0}^{k-1}K_{l-\delta}(k-1-\delta)\neq 0. \tag{37}$$

The previous reasoning was carried out under the assumption $M<k$. A similar method is used to study the question for $M\geq k$. In both cases, conditions (32), (36), and (37) have the same form, but the calculations are different.

So, it is fair

Theorem 1. If $r\leq k\leq l$, then for the degree of the polynomial $\varphi(s)$ to be equal to $M+l-k$, it is necessary and sufficient that *k* conditions (32), (36), and (37) hold.

Further, it is true

Theorem 2. If $k\leq l$, then $cm.\varphi(s)>cm.\psi(s)$ whatever $u_i, i=\overline{0,\ l-1}$.

Proof. By virtue of $l-k\geq 0$, (24) holds for $j=\overline{0,\ m-1}$. Now let us estimate the value of $B_j(s)$ at $m\leq j\leq l-1$. Since $l>m, r\leq k$, then $cm.\ B(s)=M+m=M+l-r$. From (22), taking into account (24), we find

$$cm.\ B_j(s)+j+1\leq\max\{M+m,\ M-1+j\},\quad j=\overline{m,\ l-1}, \tag{38}$$

whence, in view of $l-k\geq 0$

$$\max_{m\leq j\leq l-1} cm.\ B_j(s)\leq M-1\leq M+l-k-1. \tag{39}$$

Therefore, based on (24) and (39)

$$\psi(s)\leq M+l-k-1, \tag{40}$$

i.e. the theorem is proved.

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A CLASS OF SYSTEMS OF LINEAR FREDHOLM INTEGRAL EQUATIONS OF THE THIRD KIND WITH THE DEGENERATE MATRIX KERNELS

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On the basis of the new approach it is shown that solutions for a class of systems of linear integral equations of the third kind with degenerate kernels are equivalent to solving systems of linear algebraic equations. The questions of existence and uniqueness of the solution for this system are studied.

Keywords: Solutions, systems, linear, equations, integral, algebraic, Fredholm, the third kind, is equivalent.

Жаңы ыкманын негизинде аргументтери бөлүштүрүлгөн ядролу Фредгольмдун үчүнчү түрдөгү сызыктуу интегралдык теңдемелеринин системасынын бир классын чыгаруу сызыктуу алгебралык теңдемелердин системасын чыгарууга эквиваленттүү экендиги көрсөтүлгөн. Бул системанын чыгарылыштарынын жашашы жана жалгыздыгы изилденген.
Урунтуу сөздөр: Чыгарылыштары, системалар, сызыктуу, теңдемелер, интегралдык, алгебралык, Фредгольм, үчүнчү түр, эквиваленттүү.

На основе нового подхода показано, что решения для одного класса систем линейных интегральных уравнений Фредгольма третьего рода с вырожденными ядрами эквивалентно решению систем линейных алгебраических уравнений. Изучены вопросы существования и единственности решения для этой системы.
Ключевые слова: Решения, систем, линейных, уравнений, интегральных, алгебраических, Фредгольма, третьего рода, эквивалентно.

Let consider the following systems

$$P(x)u(x) = \lambda \sum_{j=1}^{m} A_j(x) \int_a^b B_j(y)u(y)dy + f(x),\ x \epsilon [a,b], \qquad (1)$$

where $P(x) -$ a known continuous function on $[a,b]$, $A_j(x)$ and $B_j(x)$ - $n \times n -$ dimensional known continuous matrix functions on $[a,b]$ $(j = 1,\dots,m)$, $f(x) = \big(f_i(x)\big) - n -$ a dimensional known continuous vector function on $[a,b]$, $u(x) = \big(u_i(x)\big) - n -$ dimensional unknown continuous vector function on $[a,b]$, $\lambda -$ real parameter, $a < b, P(x_l) = 0, x_l \epsilon [a,b]$, $l = 1,2,\dots,k$.

Many questions for integral equations have been investigated in $[1 - 12]$. In particular, in [3], regularizing operators according to M.M. Lavrentiev are constructed for solving linear integral Fredholm equations of the first kind. In [5-6], for systems of nonlinear Volterra integral equations of the third kind and for systems of linear Fredholm integral equations of the third kind, uniqueness theorems are proved and regularizing operators according to M.M. Lavrentiev are constructed. In this paper, we prove the uniqueness and existence theorems of the solution for systems of integral equations (1). Denote by $C_n[a,b] -$ the space of all n-dimensional vector functions with elements from $C[a,b]$. For vectors $u = (u_1,\dots,u_n)^T$, $v = (v_1,\dots,v_n)^T \epsilon R^n$ we define the scalar product by the formula

$$< u,v >= u_1v_1 + \cdots + u_nv_n .$$

Throughout this paper we assume that

$$P(x) = \prod_{e=1}^{k} P_l(x),\ P_l(x_l) = 0,\ P_l(x)\,\epsilon\ C[a,b], \tag{2}$$

$$P_l(x) \neq 0 \ \text{ for } \ x\,\epsilon\,[a,b] \ \text{ and } \ x \neq x_l,\ l = 1,\dots,k.$$

Assume that the following conditions hold:

a) For all $l = 1,\dots,k$ and $j = 1,\dots,m$ $A_{l,j}(x) - n \times n -$ dimensional matrix functions are continuous matrix functions on $[a,b]$, where

$$A_{0,j}(x) = A_j(x), A_{l,j}(x)\frac{1}{P_l(x)}\left[A_{l-1}(x) - A_{l-1,j}(x_l)\right], \qquad x\,\epsilon\,[a,b];$$

б) For all $l = 1,\dots,k$ $F_l(x) -\ n -$ dimensional vector functions are continuous vector functions on $[a,b]$, where

$$F_0(x) = f(x), \qquad F_l(x) = \frac{1}{P_l(x)}[F_{l-1}(x) - F_{l-1}(x_l)], \qquad x\,\epsilon\,[a,b].$$

Theorem. Let the conditions (2), (a) and (b) be satisfied. Then

1) if the system of linear algebraic equations

$$\begin{cases} \lambda \sum_{j=1}^{m} A_j(x_1)C_j + f(x_1) = 0, \\ \lambda \sum_{j=1}^{m} A_{1,j}(x_2)C_j + F_1(x_2) = 0, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \lambda \sum_{j=1}^{m} A_{k-1,j}(x_k)C_j + F_{k-1}(x_k) = 0, \\ C_j = \lambda \sum_{j=1}^{m}\left[\int_a^b B_j(y)\,A_{k,l}(y)dy\right]C_j + \\ + \int_a^b B_j(y)\,F_k(y)dy,\ j = 1,\dots,m, \end{cases} \tag{3}$$

relative to unknown vectors $C_j = (C_{j1},\dots,C_{jn})^T, j = 1,\dots,m$ has a unique solution, then systems of integral equations (1) in space $C_n[a,b]$ has a unique solution that can be represented as

$$u(x) = \lambda \sum_{j=1}^{m} A_{k,j}(x)C_j + F_k(x),\ x\,\epsilon\,[a,b]; \tag{4}$$

2) if the system of linear algebraic equations (3) are incompatible, then the system of integral equations (1) in space $C_n[a,b]$ doesn't have a solution;
3) if the system of linear algebraic equations (3) has an infinite number of solutions depending on q parameters, then the system of integral equations (1) in space

$C_n[a,b]$ has an infinite number of solutions depending on q parameters. In this case, the total solution of the system (1) is determined by the formula (4).

Proof. First, let $u(t) \epsilon C_n[a,b]$ is the solutions of system (1). Then, setting $x = x_1$ from (1) we have

$$\lambda \sum_{j=1}^{m} A_j(x_1) \int_a^b B_j(y)\, u(y)dy + f(x_1) = 0. \qquad (5)$$

Subtracting (5) from (1) we get

$$\prod_{l=1}^{k} P_l(x)\, u(x) = \lambda \sum_{j=1}^{m} [A_j(x) - A_j(x_1)] \int_a^b B_j(y)\, u(y)dy + f(x) - f(x_1).$$

Hence, taking into account conditions a) and b) yields

$$\prod_{l=2}^{k} P_l(x)\, u(x) = \lambda \sum_{j=1}^{m} A_{1,j}(x) \int_a^b B_j(y)\, u(y)dy + F_1(x),\ x \epsilon [a,b]. \quad (6)$$

If k=1, then

$$\prod_{l=2}^{k} P_l(x) = 1,\ x \epsilon [a,b].$$

In the case when $k \geq 2$ setting $x = x_2$ from (6) we have

$$\lambda \sum_{j=1}^{m} A_{1,j}(x_2) \int_a^b B_j(y)\, u(y)dy + F_1(x_2) = 0. \qquad (7)$$

Subtracting (7) from (6) and taking into account conditions a) and b), yields

$$\prod_{l=3}^{k} P_l(x)\, u(x) = \lambda \sum_{j=1}^{m} A_{2,j}(x) \int_a^b B_j(y)\, u(y)dy + +F_2(x),\ x \epsilon [a,b]. \qquad (8)$$

If $k = 2$, then

$$\prod_{l=3}^{k} P_l(x) = 1,\ x \epsilon [a,b].$$

In the case when $k \geq 3$, continuing this process, we make sure that the solution of the system (1) $u(x)$ satisfies the condition (3) and is determined by the formula (4).

Conversely, let $u(x) \epsilon C_n[a,b]$ satisfies the condition (3) and is determined by the formula (4). Multiplying (4) by $P_k(x)$ and taking into account condition (3) we get

$$P_k(x)u(x) = \lambda \sum_{j=1}^{m} A_{k-1,j}(x)C_j + F_{k-1}(x),\ x \epsilon [a,b]. \qquad (9)$$

Further, multiplying (9) by $P_{k-1}(x)$ and taking into account (3) we have

$$P_{k-1}(x)\,P_k(x)u(x) = \lambda \sum_{j=1}^{m} A_{k-2,j}(x)C_j + F_{k-2}(x),\ x \epsilon [a,b]. \qquad (10)$$

Continuing this process with respect to system (10) and taking into account condition (3), we see that $u(x)$ is a solution of the system (1). The theorem is proved.

Example. Consider the system

$$x(x-1)\begin{pmatrix} u_1(x) \\ u_2(x) \end{pmatrix} = \lambda\begin{pmatrix} 1 & 2 \\ x & 3 \end{pmatrix}\int_0^1 \begin{pmatrix} y^2 & 0 \\ 0 & y \end{pmatrix}\begin{pmatrix} u_1(y) \\ u_2(y) \end{pmatrix} dy + \begin{pmatrix} \propto_1 x^2 + \propto_2 x + \propto_3 \\ \beta_1 x^3 + \beta_2 x + \beta_3 \end{pmatrix},\ x \epsilon [0,1], \qquad (11)$$

where $\lambda, \propto_1, \propto_2, \propto_3, \beta_1, \beta_2, \beta_3$ are real parameters. It is easy to verify that for system (11) conditions (2), a) and b) are satisfied for

$$a = 0, b = 1, n = 2, m = 1, k = 2, x_1 = 0, x_2 = 1, P_1(x) = x, P_2(x) = x - 1,$$

$$A_{0,1}(x) = A_1(x) = \begin{pmatrix} 1 & 2 \\ x & 3 \end{pmatrix},\ B_1(y) = \begin{pmatrix} y^2 & 0 \\ 0 & y \end{pmatrix},$$

$$A_{1,1}(x) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},\ A_{2,1}(x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$F_0(x) = f(x) = \begin{pmatrix} \propto_1 x^2 + \propto_2 x + \propto_3 \\ \beta_1 x^3 + \beta_2 x + \beta_3 \end{pmatrix},\ F_1(x) = \begin{pmatrix} \propto_1 x + \propto_2 \\ \beta_1 x^3 + \beta_2 \end{pmatrix},$$

$$F_2(x) = \begin{pmatrix} \propto_1 \\ \beta_1(x+1) \end{pmatrix}, x \epsilon [a,b].$$

Then for system (11) conditions (3) are written in the following form:

$$C_1 = \begin{pmatrix} c_{11} \\ c_{12} \end{pmatrix} = \int_0^1 B(y)\begin{pmatrix} u_1(y) \\ u_2(y) \end{pmatrix} dy,$$

$$\begin{cases} \lambda\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} c_{11} \\ c_{12} \end{pmatrix} + \begin{pmatrix} \propto_3 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \lambda\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} c_{11} \\ c_{12} \end{pmatrix} + \begin{pmatrix} \propto_1 + \propto_2 \\ \beta_1 + \beta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} c_{11} \\ c_{12} \end{pmatrix} = \int_0^1 \begin{pmatrix} y^2 & 0 \\ 0 & y \end{pmatrix}\begin{pmatrix} \propto_1 \\ \beta_1(y+1) \end{pmatrix} dy = \begin{pmatrix} \frac{\propto_1}{3} \\ \frac{5}{6}\beta_1 \end{pmatrix}. \end{cases} \qquad (12)$$

From (4) we have

$$\begin{pmatrix} u_1(x) \\ u_2(x) \end{pmatrix} = F_2(x) = \begin{pmatrix} \propto_1 \\ \beta_1(x+1) \end{pmatrix},\ x \epsilon\ [0,1]. \qquad (13)$$

From conditions (12) we have that the system (11) has a unique solution in the space $C_2[0,1]$, determined by formula (13) if and only if

$$\propto_2 = -\propto_1, \propto_3 = -\lambda\left(\frac{\propto_1}{3} + \frac{5}{3}\ \beta_1\right), \beta_2 = -\frac{\propto_1}{3}\ \lambda - \ \beta_1,\ \ \beta_3 = -\frac{5}{2}\ \beta_1\lambda\,. \qquad (14)$$

If at least one of equalities (14) is violated, then system (11) has no solution in the space $C_2[0,1]$.

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MSC 45A05, 45B 05

UNIQUENESS OF SOLUTIONS FOR CERTAIN LINEAR EQUATIONS OF THE THIRD KIND WITH TWO VARIABLES

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Бул макалада терс эмес квадраттык формалар усулунун, функционалдык анализдин усулдарынын жардамы менен үчүнчү түрдөгү эки өзгөрүлмөлүү сызыктуу интегралдык тендемелердин чечимдеринин жалгыздыгы далилденди.
Негизги сөздөр: үчүнчү түрдөгү эки өзгөрүлмөлүү сызыктуу интегралдык тендемелер, жалгыздык.

В данной работе, с помощью методом неотрицательных квадратичных форм, методам функционального анализа доказывается единственность решений линейных интегральных уравнений третьего рода с двумя независимыми переменными.
Ключевые слова: Линейные интегральные уравнения, третьего рода, с двумя независимыми переменными, единственность.

In the present article the theorem about uniqueness of the linear integral equations of the third kind two independent variables, with method of nonnegative quadratic forms and functional analysis methods.
Key words: linear inteqral equations, third kind, two variables, uniqueness.

The relevance of the problem is due to the needs in development of new approaches for the regularization and uniqueness of the solution of linear integral equations of the third kind with two independent variables. Integral and operator equations of the first kind with two independent variables arise in theoretical and applied problems.Works of A.N. Tikhonov, M.M. Lavrentyev and B.K. Ivanov, in which a new concept of correctness of setting such targets is given, different from the classical, shown tool for research of ill-posed problems, which stimulated the interest to the integral equations that are of great practical importance. At the present time has been rapidly developing theory and applications of ill-posed problems. One of the classes of such ill-posed problems are integral equations of the third kind with two independent variables.As of approximate solutions of such problems, stable to small variations of the initial data, we use the solutions derived by the method of regularization. In this article we prove uniqueness theorem for such equations in families of sets of correctness's. For the decision of tasks of the used methods of functional analysis and method of nonnegative quadratic forms. The results of the work are new.

The integral equations of the first and thrid kind were studied in [1–8]. More specifically, fundanental results for Fredholm integral equations of the first kind were obtained in [6], where regularizing operators in the sense of $M.M.Lavrentyev$ were constructed for solutions of linear Fredholm inteqral equations of the first kind. For linear Volterra integral equations of the first kind and third kinds with smooth kernels, the existence of a multiparameter family of solution was proved in [7]. The regularization and uniqueness of solutions to systems of nonlinear Volterra integral equations of the first kind were investigated in [4].In this work we shall study the problems of uniqueness and stability of solution of the integral equation

$$Ku = f(t,x), (t,x) \in G = \{(t,x) \in R^2 : t_0 \le t \le T, a \le x \le b\}, \quad (1)$$

where

$$Ku \equiv m(t,x)u(t,x) + \int_a^x P(t,x,y)u(t,y)dy + \int_{t_0}^t Q(t,x,s)u(s,x)ds + \int_{t_0}^T \int_a^b C(t,x,s,y)u(s,y)dyds, \quad (2)$$

$P(t,x,y)$ and $Q(t,x,s)$ are given functions, respectively on the domains

$$G_1 = \{(t,x,y) : t_0 \le t \le T, \ a \le y \le x \le b\},$$
$$G_2 = \{(t,x,s) : t_0 \le s \le t \le T, \ a \le x \le b\},$$

$C(t,x,s,y)$, $m(t,x)$, $f(t,x)$ are given functions.

Assume that the following conditions are satisfied:

$$(i).P(t,b,a)\geq 0 \quad t\in[t_0,T], P(t,b,a)\in C[t_0,T],$$

$$m(t,x)\geq 0 \; for\; all\; (t,x)\in G$$

$$P'_y(t,y,a)\leq 0 \; for\; all\,(t,y)\in G,\; P'_y(t,y,a)\in C(G),$$

$$P'_z(s,b,z)\geq 0 \; for\; all\; (s,z)\in G,\; P'_z(s,b,z)\in C(G),$$

$$P''_{zy}(s,y,z)\leq 0 \; for\; all\; (s,y,z)\in G_1,\; P''_{zy}(s,y,z)\in C(G_1).$$

$$(ii).Q(T,y,t_0)\geq 0 \; for\; all\; y\in[a,b],\; Q(T,y,t_0)\in C[a,b],$$

$$Q'_s(s,y,t_0)\leq 0 \; for\; all\; (s,y)\in G, Q'_s(s,y,t_0)\in C(G),$$

$$Q'_\tau(T,y,\tau)\geq 0 \; for\; all\; (y,\tau)\in G,\; Q'_{\psi(\tau)}(T,y,\tau)\in C(G),$$

$$Q''_{\tau s}(s,y,\tau)\leq 0 \; for\; all\; (s,y,\tau)\in G_2, Q''_{\tau s}(s,y,\tau)\in C(G_2).$$

iii). At least one of the following conditions holds:

$$(a)P'_y(s,y,a)<0 \; for\; almost\; all \quad (s,y)\in G;$$

$$(b)P'_z(s,b,z)>0 \quad for\; almost\; all\; (s,z)\in G;$$

$$(c)Q'_s(s,y,t_0)<0 \quad for\; almost\; all \quad (s,y)\in G;$$

$$(d)Q'_\tau(T,y,\tau)>0 \; for\; almost\; all \quad (y,\tau)\in G;$$

$$(e)\; P''_{zy}(s,y,z)<0 \quad for\; almost\; all \quad (s,y,z)\in G_1;$$

$$(f)\; Q''_{\tau s}(s,y,\tau)<0 \quad for\; almost\; all\; (s,y,\tau)\in G_2;$$

$$(h)\; m(t,x)>0 \; for\; almost\; all\,(t,x)\in G.$$

(iv). $C(t,x,s,y)\in L_2(G^2)$ and

$$\frac{1}{2}\left[C(t,x,s,y)+C(s,y,t,x)\right]=\sum_{i=1}^{m}\lambda_i\varphi_i(t,x)\varphi_i(s,y),$$

$$C(t,x,s,y)=\sum_{i=1}^{m}\lambda_i\varphi_i(t,x)\varphi_i(s,y),\; m\leq\infty,\; 0\leq\lambda_i,\; i=1,2,...,m \tag{3}$$

where $\{\varphi_i(t,x)\}$ is an orthonormal sequence of eigen functions from $L_2(G)$ and $\{\lambda_i\}$ is the sequence of corresponding nonzero eigenvalues of the Fredholm integral operator C generated by the kernel $\frac{1}{2}\left[C(t,x,s,y)+C(s,y,t,x)\right]$ with the elements

$\{\lambda_i\}$ arranged in decreasing order of their absolute values. If $C(t,x,s,y)=0$ for all $(t,x,s,y)\in G^2$, we assume that $\lambda_1=\lambda_2=...=\lambda_m=0.$

Theorem 1. Let conditions (i)-(iv) be satisfied. Then the solution of the equation (1) is unique in $L_2(G)$.

Proof. Taking the multiplication of both sides of the equation (1) with u(t, x), integrating the results on G, we obtain

$$\int_a^b\int_{t_0}^T m(s,y)u^2(s,y)dyds+\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)u(s,z)u(s,y)dzdyds+$$
$$+\int_a^b\int_{t_0}^T\int_{t_0}^s Q(s,y,\tau)u(\tau,y)u(s,y)d\tau dsdy+$$
$$+\int_a^b\int_{t_0}^T\int_{t_0}^T\int_a^b C(s,y,\tau,z)u(\tau,z)u(s,y)dzd\tau dsdy=\int_a^b\int_{t_0}^T f(s,y)u(s,y)dsdy. \quad (4)$$

Integrating by parts and using the Dirichlet formula

$$\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)u(s,z)u(s,y)dzdyds=$$
$$=-\int_{t_0}^T\int_a^b\int_a^y P(s,y,z)\frac{\partial}{\partial z}\left(\int_z^y u(s,v)dv\right)dz\ u(s,y)dyds=$$
$$=\frac{1}{2}\int_{t_0}^T\int_a^b\left[P(s,y,a)\frac{\partial}{\partial y}\left(\int_a^y u(s,v)dv\right)^2\right]dyds+$$
$$+\frac{1}{2}\int_{t_0}^T\int_a^b\int_z^b P_z'(s,y,z)\frac{\partial}{\partial y}\left(\int_z^y u(s,v)dv\right)^2\varphi ydzds=$$
$$=\frac{1}{2}\int_{t_0}^T P(s,b,a)\left(\int_a^b u(s,v)dv\right)^2 ds-$$
$$-\frac{1}{2}\int_{t_0}^T\int_a^b P_y'(s,y,a)\left(\int_a^y u(s,v)dv\right)^2 dyds+$$
$$+\frac{1}{2}\int_{t_0}^T\int_a^b P_z'(s,b,z)\left(\int_z^b u(s,v)dv\right)^2 dzds-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}\int_{a}^{y}P''_{zy}(s,y,z)\left(\int_{z}^{y}u(s,\nu)d\nu\right)^2 dzdyds. \qquad (5)$$

Similarly integrating by parts and using the Dirichlet formula analogically we have

$$\int_{a}^{b}\int_{t_0}^{T}\int_{t_0}^{s}Q(s,y,\tau)u(\tau,y)u(s,y)d\nu dsdy =$$

$$=\frac{1}{2}\int_{a}^{b}Q(T,y,t_0)\left(\int_{t_0}^{T}u(\xi,y)d\xi\right)^2 dy-$$

$$-\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}Q'_s(s,y,t_0)\left(\int_{t_0}^{s}u(\xi,y)d\xi\right)^2 dsdy+$$

$$+\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}Q'_\tau(T,y,\tau)\left(\int_{\tau}^{T}u(\xi,y)d\xi\right)^2 d\tau dy-$$

$$-\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}\int_{t_0}^{s}Q''_{\tau s}(s,y,\tau)\left(\int_{\tau}^{s}u(\xi,y)d\xi\right)^2 d\tau dsdy. \qquad (6)$$

Using the Dirichlet formula we have

$$\int_{a}^{b}\int_{t_0}^{T}\int_{a}^{b}\int_{t_0}^{T}C(t,x,s,y)u(s,y)u(t,x)dsdydtdx =$$

$$=\int_{a}^{b}\int_{t_0}^{T}\int_{a}^{b}\int_{t_0}^{t}C(t,x,s,y)u(s,y)u(t,x)dsdydtdx =$$

$$=\int_{a}^{b}\int_{t_0}^{T}\int_{a}^{b}\int_{t}^{T}C(t,x,s,y)u(s,y)u(t,x)dsdydtdx =$$

$$=\int_{a}^{b}\int_{t_0}^{T}\int_{a}^{b}\int_{t_0}^{T}\left[C(t,x,s,y)+C(s,y,t,x)\right]u(s,y)u(t,x)dsdydtdx. \qquad (7)$$

Taking into account (5), (6), (7) and (4) from (5) we obtain

$$\int_{t_0}^{T}\int_{a}^{b}m(s,y)u^2(s,y)dyds+\frac{1}{2}\int_{t_0}^{T}P(s,b,a)\left(\int_{a}^{b}u(s,\nu)d\nu\right)^2 ds-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}P'_y(s,y,a)\left(\int_{a}^{y}u(s,\nu)d\nu\right)^2 dyds+$$

$$+\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}P'_z(s,b,z)\left(\int_{z}^{b}u(s,v)\,dv\right)^2 dzds-$$

$$-\frac{1}{2}\int_{t_0}^{T}\int_{a}^{b}\int_{a}^{y}P''_{zy}(s,y,z)\left(\int_{z}^{y}u(s,v)\,dv\right)^2 dzdyds+$$

$$+\frac{1}{2}\int_{a}^{b}Q(T,y,t_0)\left(\int_{t_0}^{T}u(\xi,y)\,d\xi\right)^2 dy-$$

$$-\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}Q'_s(s,y,t_0)\left(\int_{t_0}^{s}u(\xi,y)\,d\xi\right)^2 dsdy+$$

$$+\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}Q'_\tau(T,y,\tau)\left(\int_{\tau}^{T}u(\xi,y)\,d\xi\right)^2 d\tau dy-$$

$$-\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}\int_{t_0}^{s}Q''_{\tau s}(s,y,\tau)\left(\int_{\tau}^{s}u(\xi,y)\,d\xi\right)^2 d\tau dsdy+.$$

$$+\frac{1}{2}\int_{a}^{b}Q(T,y,t_0)\left(\int_{t_0}^{T}u(\xi,y)\,d\xi\right)^2 dy-$$

$$-\frac{1}{2}\int_{a}^{b}\int_{t_0}^{T}Q'_s(s,y,t_0)\left(\int_{t_0}^{s}u(\xi,y)\,d\xi\right)^2 dsdy+$$

$$+\sum_{i=1}^{m}\lambda_i\left(\int_{a}^{b}\int_{t_0}^{T}\phi_i(s,y)u(s,y)\,dsdy\right)^2=\int_{a}^{b}\int_{t_0}^{T}f(s,y)u(s,y)\,dsdy. \qquad (8)$$

We assume that $f(s,y)\equiv 0.$ In this case, in the thoroughness of conditions (i)-(iv)from uranium (8) it turns out that $u(t,x)=0,\ (t,x)\in G.$.

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MSC 65R20

APPROXIMATE SOLUTION OF NONLINEAR SECOND-ORDER DIFFERENTIAL EQUATIONS OF A DERIVATIVE WITH RESPECT TO AN INCREASING FUNCTION

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In this study, the approximate solution of a second-order nonlinear differential equation with derivative with respect to an increasing function by using the generalized trapezoid method is established and investigated. One example is solved employing the proposed method.
Keywords. Generalized trapezoid rule, nonlinear second-order differential equation, Volterra-Stieltjes integral equation of the second kind, the derivative with respect to an increasing function.

Бул макалада өсүүчү функция боюнча туундулуу сызыктуу эмес экинчи тартиптеги дифференциалдык теңдеменин жакындатылган чыгарылышы жалпыланган трапеция методу менен жакындаштырылып чыгарылды жана изилденди. Сунушталган метод менен мисал чыгарылды.
Ачкыч сөздөр. Өсүүчү функция боюнча туунду, сызыктуу эмес экинчи тартиптеги дифференциалдык теңдеме, Вольтерра-Стилтьестин II түрдөгү интегралдык теңдемеси, жалпыланган трапеция методу.

В данной работе установлено и исследовано приближенное решение нелинейных дифференциальных уравнений второго порядка с производной по возрастающей функции с использованием обобщенного метода трапеций. Показано решение примера с использованием предложенного метода.
Ключевые слова. Производная по возрастающей функции, нелинейное дифференциальное уравнение второго порядка, интегральное уравнение Вольтерра-Стилтьеса второго рода, обобщенный метод трапеций.

The notion of derivative of a function by means of an increasing function was given by Asanov in [1]. In the study [2], linear and nonlinear Volterra- Stieltjes integral equations of the second kind are studies using the concept of the derivative with respect to an increasing function. In the study [5] the numerical solution of linear Volterra-Stieltjes integral equations of the second kind is investigated by using the generalized trapezoid rule. The generalized trapezoid rule is established on the basis of the derivative of function with respect to strictly increasing function defined in [1].

Problem formulation. We study the Cauchy problem for a second-order nonlinear differential equation with derivative with respect to an increasing function:

$$u''_{\varphi(t)}(t) = p(t)u'_{\varphi(t)}(t) + F(t,u(t)) + g(t), \quad (t,u) \in [t_0,T] \times \check{Y}, \qquad (1)$$

$$u(t_0) = \alpha, \qquad u'_{\varphi}(t_0) = \beta, \quad \varphi(t_0) = 0, \qquad (2)$$

where $\varphi(t)$ is a given increasing continuous function on $[t_0,T]$, $g(t)$, $p(t)$, $F(t,u(t))$ are given continuous functions and $u(t)$ is the sought function on $[t_0,T]$.

To solve the Cauchy problem (1)-(2), we first integrate the equation (1) from t_0 to t by the function $\varphi(t)$ and taking into account the condition (2), we have

$$u'_{\varphi}(t) = p(t)u(t) - \int_{t_0}^{t} p'_{\varphi}(s)u(s)d\varphi(s) + \int_{t_0}^{t} F(s,u(s))d\varphi(s) + \int_{t_0}^{t} g(s)d\varphi(s) + \beta - \alpha p(t_0) . \qquad (3)$$

Now we integrate the integral equation (3) from t_0 to t by the function $\varphi(t)$:

$$u(t) = \int_{t_0}^{t} p(s)u(s)d\varphi(s) + \int_{t_0}^{t} [\varphi(t)-\varphi(s)]\left[F(s,u(s)) - p'_{\varphi}(s)u(s)\right]d\varphi(s) + f(t), \; t \in [t_0,T], \qquad (4)$$

where

$$f(t) = [\beta - \alpha p(t_0)]\varphi(t) + \alpha + \int_{t_0}^{t} [\varphi(t)-\varphi(s)]g(s)d\varphi(s).$$

Thus, the Cauchy problem (1)-(2) is reduced to a nonlinear Volterra-Stieltjes integral equation of the second kind (4). The integral equation of form (4) has been solved numerically by the generalized trapezoid rule.

Numerical solution. The solution of the integral equation (4) will be the approximate solution of the Cauchy problem (1)-(2). Now we get the approximate solution of the integral equation (4) by using a generalized trapezoid rule. For this divide the segment $[t_0, T]$ into n parts by the points $t_0 < t_1 < t_2 < ... < t_n = T$. Let

$$h = \frac{T - t_0}{n}\ , \quad t_k = t_0 + kh\,, \quad k = 1, 2, ..., n, \quad n \in \Gamma\,.$$

We substitute $t = t_k$ in the integral equation (4) and consider the following system of equations:

$$u(t_0) = f(t_0), \qquad t \in [t_0, T],$$
$$u(t_k) = \int_{t_0}^{t_k} p(s)u(s)d\varphi(s) + \int_{t_0}^{t_k} [\varphi(t_k) - \varphi(s)]\left[F(s, u(s)) - p'_{\varphi(s)}(s)u(s)\right]d\varphi(s) + f(t_k), \tag{5}$$

where $k = 1, 2, ..., n.$

We rewrite the integrals in (5) as the sum of the series [4]. For the first integral:

$$\int_{t_0}^{t_k} p(s)u(s)d\varphi(s) = \sum_{j=1}^{k} \frac{1}{2}\left[p(t_{j-1})u(t_{j-1}) + p(t_j)u(t_j)\right]\left[\varphi(t_j) - \varphi(t_{j-1})\right] + \sum_{j=1}^{k} R_{1j}^{(n)}(u)\,, \tag{6}$$

where

$$\left|R_{1j}^{(n)}(u)\right| \le \frac{M_1}{12}\left[\varphi(t_j) - \varphi(t_{j-1})\right]^3, \quad M_1 = \sup_{s \in [t_0, T]} \left|\left[p(s)u(s)\right]''_{\varphi(s)}\right|.$$

For the second integral:

$$\int_{t_0}^{t_k} [\varphi(t_k) - \varphi(s)]\left[F(s, u(s)) - p'_{\varphi(s)}u(s)\right]d\varphi(s) =$$
$$= \sum_{j=1}^{k} \frac{1}{2}\Big\{\left[\varphi(t_k) - \varphi(t_{j-1})\right]\left[F(t_{j-1}, u(t_{j-1})) - p'_{\varphi(s)}(t_{j-1})u(t_{j-1})\right] + \tag{7}$$
$$+\left[\varphi(t_k) - \varphi(t_j)\right]\left[F(t_j, u(t_j)) - p'_{\varphi}(t_j)u(t_j)\right]\Big\}\left[\varphi(t_j) - \varphi(t_{j-1})\right] + \sum_{j=1}^{k} R_{2j}^{(n)}(u)\,,$$

where $\left|R_{2j}^{(n)}(u)\right| \le \frac{M_2}{12}\left[\varphi(t_j) - \varphi(t_{j-1})\right]^3, \quad M_2 = \sup_{t,s \in G}\left|\left\{[\varphi(t) - \varphi(s)]\left[F(s, u(s)) - p'_{\varphi(s)}(s)u(s)\right]\right\}''_{\varphi(s)}\right|.$

Substituting (6) and (7) in (5), we have

$$u(t_k)=\sum_{j=1}^{k}\frac{1}{2}\Big[p(t_{j-1})u(t_{j-1})+p(t_j)u(t_j)\Big]\Big[\varphi(t_j)-\varphi(t_{j-1})\Big]+$$

$$+\sum_{j=1}^{k}\frac{1}{2}\Big\{\Big[\varphi(t_k)-\varphi(t_{j-1})\Big]\Big[F(t_{j-1},u(t_{j-1}))-p'_{\varphi(s)}(t_{j-1})u(t_{j-1})\Big]+ \qquad (8)$$

$$+\Big[\varphi(t_k)-\varphi(t_j)\Big]\Big[F(t_j,u(t_j))-p_{\varphi}(t_j)u(t_j)\Big]\Big\}\Big[\varphi(t_j)-\varphi(t_{j-1})\Big]+\sum_{j=1}^{k}\Big(R_{1j}^{(n)}(u)+R_{2j}^{(n)}(u)\Big)+f(t_k),$$

where $k=1,2,...,n.$ Omitting the terms $\sum_{j=1}^{k}R_{1j}^{(n)}(u)$ and $\sum_{j=1}^{k}R_{2j}^{(n)}(u)$ appearing in each equation of system (8) and $u_k\approx u(t_k)$, for $k=1,2,...,n$ we obtain $u_0=f(t_0)$, $\varphi(t_0)=0$,

$$u_k=\sum_{j=1}^{k}\frac{1}{2}\Big[p(t_{j-1})u_{j-1}+p(t_j)u_j\Big]\Big[\varphi(t_j)-\varphi(t_{j-1})\Big]+$$

$$+\sum_{j=1}^{k}\frac{1}{2}\Big\{\Big[\varphi(t_k)-\varphi(t_{j-1})\Big]\Big[F(t_{j-1},u_{j-1})-p'_{\varphi(s)}(t_{j-1})u_{j-1}\Big]+ \qquad (9)$$

$$+\Big[\varphi(t_k)-\varphi(t_j)\Big]\Big[F(t_j,u_j)-p'_{\varphi}(t_j)u_j\Big]\Big\}\Big[\varphi(t_j)-\varphi(t_{j-1})\Big]+f(t_k).$$

Taking into account that $\varphi(t_0)=0$ for $k=1$ from (9) we get

$$\left(1-\frac{1}{2}p(t_1)\varphi(t_1)\right)u_1=\frac{1}{2}\alpha p(t_0)\varphi(t_1)+\frac{1}{2}\big(\varphi(t_1)\big)^2\Big[F(t_0,u_0)-\alpha p'_{\varphi}(t_0)\Big]+f(t_1)\,. \qquad (10)$$

and for $k=2,3,...,n$

$$\left(1-\frac{1}{2}p(t_k)\big[\varphi(t_k)-\varphi(t_{k-1})\big]\right)u_k=\sum_{j=1}^{k-1}\frac{1}{2}\Big[p(t_{j-1})u_{j-1}+p(t_j)u_j\Big]\Big[\varphi(t_j)-\varphi(t_{j-1})\Big]+$$

$$+\frac{1}{2}p(t_{k-1})u_{k-1}\big[\varphi(t_k)-\varphi(t_{k-1})\big]+\sum_{j=1}^{k-1}\frac{1}{2}\Big\{\Big[\varphi(t_k)-\varphi(t_{j-1})\Big]\Big[F(t_{j-1},u_{j-1})-p'_{\varphi}(t_{j-1})u_{j-1}\Big]+ \qquad (11)$$

$$+\Big[\varphi(t_k)-\varphi(t_j)\Big]\Big[F(t_j,u_j)-p'_{\varphi}(t_j)u_j\Big]\Big\}\Big[\varphi(t_j)-\varphi(t_{j-1})\Big]+$$

$$+\frac{1}{2}\big[\varphi(t_k)-\varphi(t_{k-1})\big]\Big[F(t_{k-1},u_{k-1})-p'_{\varphi}(t_{k-1})u_{k-1}\Big]\big[\varphi(t_k)-\varphi(t_{k-1})\big]+f(t_k).$$

Let us assume that

$$\mu=\sup_{k=1,2,...,n}\frac{1}{2}p(t_k)\big[\varphi(t_k)-\varphi(t_{k-1})\big]<1\,.$$

Then the system of equations (11) has a unique solution which is given by the formulas

$$u_1=\frac{1}{1-\frac{1}{2}p(t_1)\varphi(t_1)}\left\{\frac{1}{2}\alpha p(t_0)\varphi(t_1)+\frac{1}{2}\big(\varphi(t_1)\big)^2\Big[F(t_0,u_0)-\alpha p'_{\varphi}(t_0)\Big]+f(t_1)\right\}. \qquad (12)$$

$$u_k = \frac{1}{1-\frac{1}{2}p(t_k)\left[\varphi(t_k)-\varphi(t_{k-1})\right]}\left\{\sum_{j=1}^{k-1}\frac{1}{2}\left[p(t_{j-1})u_{j-1}+p(t_j)u_j\right]\left[\varphi(t_j)-\varphi(t_{j-1})\right]+\right.$$

$$+\frac{1}{2}p(t_{k-1})u_{k-1}\left[\varphi(t_k)-\varphi(t_{k-1})\right]+\sum_{j=1}^{k-1}\frac{1}{2}\left\{\left[\varphi(t_k)-\varphi(t_{j-1})\right]\left[F(t_{j-1},u_{j-1})-p'_\varphi(t_{j-1})u_{j-1}\right]+\right. \quad (13)$$

$$\left.+\left[\varphi(t_k)-\varphi(t_j)\right]\left[F(t_j,u_j)-p'_\varphi(t_j)u_j\right]\right\}\left[\varphi(t_j)-\varphi(t_{j-1})\right]+$$

$$\left.+\frac{1}{2}\left[\varphi(t_k)-\varphi(t_{k-1})\right]^2\left[F(t_{k-1},u_{k-1})-p'_\varphi(t_{k-1})u_{k-1}\right]+f(t_k)\right\},$$

where $k=2,3,...,n.$

Example. Solve the following Cauchy problem for a second-order nonlinear differential equation with derivative with respect to an increasing function:

$$u''_\varphi(t)+\sqrt{t}u'_\varphi(t)+\frac{(1+t^2)u}{1+u^2}=2+3t, \quad t\in[0,1] \quad (14)$$

$$\varphi(t)=\sqrt{t}, \quad u(0)=0, \quad u'_\varphi(0)=0, \quad (15)$$

It is easily seen that $u(t)=t$ is the exact solution of the Cauchy problem (14)-(15).

We find the approximate solution of this Cauchy problem using the proposed method. For this problem $t_0=0,\ T=1,\quad \alpha=0,\ \beta=0,\quad p(t)=-\sqrt{t},\quad \varphi(t)=\sqrt{t},$ $F(t,u)=-\frac{(1+t^2)u}{1+u^2},\quad g(t)=2+3t$ and using the formula (4) it is reduced the following Volterra-Stieltjes integral equation:

$$u(t)=-\int_0^t\sqrt{s}u(s)d(\sqrt{s})+\int_0^t\left[\sqrt{t}-\sqrt{s}\right]\cdot\left(-\frac{(1+s^2)u(s)}{1+(u(s))^2}\right)d(\sqrt{s})+f(t), \quad (16)$$

where

$$f(t)=\int_0^t\left(\sqrt{t}-\sqrt{s}\right)\left(2+3s\right)d\left(\sqrt{s}\right).$$

Let $n=20,$ then $h=\frac{1-0}{20}=0.05$. The numeric solution of the integral equation (16) by the generalized trapezoid method and its comparison with the exact solution are shown in the table below:

The nodes t_k	Real value at t_k $u(t_k)$	Approx. value at t_k u_k	The error at t_k $\|u(t_k)-u_k\|$
0.05	0.05000000000	0.04939024390	0.00060975610
0.1	0.1000000000	0.09929953941	0.00070046059
0.15	0.1500000000	0.1492544207	0.0007455793
0.2	0.2000000000	0.1992283133	0.0007716867
0.25	0.2500000000	0.2492126104	0.0007873896
0.3	0.3000000000	0.2992033895	0.0007966105
0.35	0.3500000000	0.3491985174	0.0008014826
0.4	0.4000000000	0.3991966968	0.0008033032
0.45	0.4500000000	0.4491970779	0.0008029221
0.5	0.5000000000	0.4991990730	0.0008009270
0.55	0.5500000000	0.5492022600	0.0007977400
0.6	0.6000000000	0.5992063249	0.0007936751
0.65	0.6500000000	0.6492110313	0.0007889687
0.7	0.7000000000	0.6992161960	0.0007838040
0.75	0.7500000000	0.7492216772	0.0007783228
0.8	0.8000000000	0.7992273622	0.0007726378
0.85	0.8500000000	0.8492331626	0.0007668374
0.9	0.9000000000	0.8992390073	0.0007609927
0.95	0.9500000000	0.9492448414	0.0007551586
1	1	0.9992506194	0.0007493806

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A CLASS OF INVERSE PROBLEMS FOR A PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS OF THE FIRST ORDER

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In this paper, the inverse problem for a first-order integro-differential equation with private derivatives is reduced by the method of an additional argument to a system of integral equations. Further the existence and uniqueness theorem for the solution of the inverse problem is proved by the method of the principle of contraction mappings.

Key words: inverse problems, integro-differential equations, partial derivatives, first order.

Бул макалада биринчи тартиптеги жекече туундулуу интегро-дифференциалдык тендемелерге коюлган тескери маселе кошумча аргументтер ыкмасы менен интегралдык тендемелер системасына келтирилген. Андан ары, тескери маселенин жашашы жана жалгыздыгы жонундогу теорема, кысып чагылдыруу принцип ыкмасы менен далилделген.

Урунттуу сөздөр: тескери маселелер, интегро-дифференциалдык тендемелер, айрым туундулар, биринчи тартип.

В данной работе методом дополнительного аргумента обратная задача для интегро-дифференциального уравнения с частными производными первого порядка сводится к системе интегральных уравнений. Далее, методом принципа сжатых отображений доказаны теорема существования и единственности решения обратной задачи.

Ключевые слова: обратные задачи, интегро-дифференциальные уравнения, частные производные, первый порядок.

The integro-differential equation is considered

$$u_t(t,x)+u(t,x)u_x(t,x)=\int_0^t K(\nu)u(t-\nu,x)d\nu,\ x\in R,\ t\in[0,T], \qquad (1)$$

with the initial condition

$$u(0,x)=\varphi(x),\ x\in R, \qquad (2)$$

and with overriding

$$u(t,x_0)=g(t),\ t\in[0,T], \qquad (3)$$

where $\varphi(x)$, $g(t)$- where, $u(t,x)$, $K(t)$- unknown functions. The matching conditions $\varphi(x_0) = g(0)$ are satisfied.

In works [1] the method of additional argument is used to study partial differential equations. Various inverse problems are investigated in [2-5]. In works

[2-3] inverse problems of mathematical physics are investigated by the method of an additional argument. Here, using the method of an additional argument and the principle of contracted mappings, we prove existence and uniqueness theorems for the solution of the inverse problem (1)-(3), the definition of $u(t, x)$, $K(t)$.

Let us introduce the following notation:

$\overline{C}^{\gamma 1,\gamma 2,\gamma 3,\dots\gamma n}(\vartheta_T)$ - is the space of functions having continuous and bounded derivatives with respect to the i-argument up to order γi in the domain ϑ_T, $D=\{(s,t);0\le s\le t\le T\}$, $G_T=\{(t,x);0\le t\le T,x\in R\}$, $Q_T=\{(s,t,x);0\le s\le t\le T,x\in R\}$, $Sup_{t\in[0,T]}|g(t)|=M$, $Sup_{t\in[0,T]}|g'(t)|=M_1$, $Sup_{t\in[0,T]}|g''(t)|=M_2$, $Sup_{x\in R}|\varphi(x)|=\Phi$, $Sup_{x\in R}|\varphi'(x)|=\Phi_1$, $Sup_{x\in R}|\varphi''(x)|=\Phi_2$, $Sup_{x\in R}|\varphi'''(x)|=\Phi_3$.

Suppose the following conditions are met:

а) $\varphi(x)\in \overline{C}^3(R),\quad g(t)\in C^2[0,T]$,

б) $g(0)\ge\alpha>0$.

In (1) replacing t by ρ and x by $p(\rho,t,x)$, where

$$p(\rho,t,x)=x-\int_\rho^t u(\tau,p(\tau,t,x))d\tau,\ p(t,t,x)=x,\ p_\rho(\rho,t,x)=u(\rho,p(\rho,t,x)). \qquad (5)$$

Further, integrating over ρ from 0 до s, we get:

$$u(s,p(s,t,x))=\varphi(x-\int_0^t u(\tau,p(\tau,t,x))d\tau)+\int_0^s\int_0^\rho K(\nu)u(\rho-\nu,x-\int_\rho^t u(\tau,p(\tau,t,x))d\tau)d\nu d\rho. \quad (6)$$

In (6), setting $s=t$, we have:

$$u(t,x)=\varphi(x-\int_0^t u(\tau,p(\tau,t,x))d\tau)+\int_0^t\int_0^\rho K(\nu)u(\rho-\nu,x-\int_\rho^t u(\tau,p(\tau,t,x))d\tau)d\nu d\rho. \qquad (7)$$

In (7), setting $x=x_0$, we have:

$$g(t)=\varphi(x_0-\int_0^t u(\tau,p(\tau,t,x_0))d\tau)+\int_0^t\int_0^\rho K(\nu)u(\rho-\nu,x_0-\int_\rho^t u(\tau,p(\tau,t,x_0))d\tau)d\nu d\rho. \quad (8)$$

From (8) we take the derivative with respect to t twice, and we have:

$$g'(t)=-\varphi'(x_0-\int_0^t u(\tau,p(\tau,t,x_0))d\tau)\left\{g(t)+\int_0^t u_x(\tau,p(\tau,t,x_0))p_t(\tau,t,x_0)d\tau\right\}+\int_0^t K(\nu)g(t-\nu)d\nu-$$

$$-\int_0^t\int_0^\rho K(\nu)u_x(\rho-\nu,x_0-\int_\rho^t u(\tau,p(\tau,t,x_0))d\tau)\{g(t)+\int_\rho^t u_x(\tau,p(\tau,t,x_0))p_t(\tau,t,x_0)d\tau\}d\nu d\rho, \quad (9)$$

$$g''(t)=\varphi''(x_0-\int_0^t u(\tau,p(\tau,t,x_0))d\tau)\left\{g(t)+\int_0^t u_x(\tau,p(\tau,t,x_0))p_t(\tau,t,x_0)d\tau\right\}^2+K(t)\varphi(x_0)-$$

$$-\varphi'(x_0-\int_0^t u(\tau,p(\tau,t,x_0))d\tau)\{g'(t)-u_x(t,x_0)g(t)+\int_0^t u_{xx}(\tau,p(\tau,t,x_0))p_t^{\,2}(\tau,t,x_0)d\tau+$$

$$+\int_0^t u_x(\tau,p(\tau,t,x_0))p_{tt}(\tau,t,x_0)d\tau\}-\int_0^t K(\nu)u_x(t-\nu,x_0)g(t)d\nu+\int_0^t K(\nu)g'(t-\nu)d\nu+$$

$$+\int_0^t\int_0^\rho K(\nu)u_{xx}(\rho-\nu,x_0-\int_\rho^t u(\tau,p(\tau,t,x_0))d\tau)\left\{g(t)+\int_\rho^t u_x(\tau,p(\tau,t,x_0))p_t(\tau,t,x_0)d\tau\right\}^2 d\nu d\rho-$$

$$-\int_0^t\int_0^\rho K(\nu)u_x(\rho-\nu,x_0-\int_\rho^t u(\tau,p(\tau,t,x_0))d\tau)\{g'(t)-u_x(t,x_0)g(t)+ \quad (10)$$

$$+\int_0^t u_{xx}(\tau,p(\tau,t,x_0))p_t^{\,2}(\tau,t,x_0)d\tau+\int_0^t u_x(\tau,p(\tau,t,x_0))p_{tt}(\tau,t,x_0)d\tau\}d\nu d\rho.$$

From (7) we take the derivative with respect to *x* and setting $x=x_0$, we have:

$$u_x(t,x_0)=\varphi'(x_0-\int_0^t u(\tau,p(\tau,t,x_0))d\tau)\left\{1-\int_0^t u_x(\tau,p(\tau,t,x_0))p_x(\tau,t,x_0)d\tau\right\}+$$

$$+\int_0^t\int_0^\rho K(\nu)u_x(\rho-\nu,x_0-\int_\rho^t u(\tau,p(\tau,t,x_0))d\tau)\left\{1-\int_\rho^t u_x(\tau,p(\tau,t,x_0))p_x(\tau,t,x_0)d\tau\right\}d\nu d\rho. \quad (11)$$

In (5), (6), setting $x = x_0$, we get:

$$p(\rho,t,x_0)=x_0-\int_\rho^t u(\tau,p(\tau,t,x_0))d\tau, \quad (12)$$

$$u(s,p(s,t,x_0))=\varphi(x_0-\int_0^t u(\tau,p(\tau,t,x_0))d\tau)-\int_0^s\int_0^\rho K(\nu)u(\rho-\nu,x_0-\int_\rho^t u(\tau,p(\tau,t,x_0))d\tau)d\nu d\rho. \quad (13)$$

From (12) we take the derivative with respect to *t* twice, we get:

$$p_t(s,t,x_0)=-g(t)-\int_s^t u_x(\tau,p(\tau,t,x_0))p_t(\tau,t,x_0)d\tau, \quad (14)$$

$$p_{tt}(s,t,x_0)=-g'(t)+u_x(t,x_0)g(t)-\int_s^t u_{xx}(\tau,p(\tau,t,x_0))p_t^{\,2}(\tau,t,x_0)d\tau-\int_s^t u_x(\tau,p(\tau,t,x_0))p_{tt}(\tau,t,x_0)d\tau. \quad (15)$$

From (12) we take the derivative with respect to *x* and setting $x = x_0$, we obtain:

$$p_x(s,t,x_0) = 1 - \int_s^t u_x(\tau, p(\tau,t,x_0))p_x(\tau,t,x_0)d\tau. \quad (16)$$

From (6) we take the derivative with respect to *x* twice and setting $x = x_0$, we have:

$$\begin{aligned} u_x(s, p(s,t,x_0)) &= \varphi'(x_0 - \int_0^t u(\tau, p(\tau,t,x_0))d\tau)\left\{1 - \int_0^t u_x(\tau, p(\tau,t,x_0))p_x(\tau,t,x_0)d\tau\right\} + \\ &+ \int_0^s\int_0^\rho K(\nu)u_x(\rho-\nu, x_0 - \int_\rho^t u(\tau, p(\tau,t,x_0))d\tau)\left\{1 - \int_\rho^t u_x(\tau, p(\tau,t,x_0))p_t(\tau,t,x_0)d\tau\right\}d\nu d\rho. \end{aligned} \quad (17)$$

$$\begin{aligned} u_{xx}(s, p(s,t,x_0)) &= \varphi''(x_0 - \int_0^t u(\tau, p(\tau,t,x_0))d\tau)\left\{1 - \int_0^t u_x(\tau, p(\tau,t,x_0))p_x(\tau,t,x_0)d\tau\right\}^2 - \\ &- \varphi''(x_0 - \int_0^t u(\tau, p(\tau,t,x_0))d\tau)\left\{\int_0^t u_{xx}(\tau, p(\tau,t,x_0))p_x^{\,2}(\tau,t,x_0)d\tau + \int_0^t u_x(\tau, p(\tau,t,x_0))p_{xx}(\tau,t,x_0)d\tau\right\} - \\ &- \int_0^s\int_0^\rho K(\nu)u_{xx}(\rho-\nu, x_0 - \int_\rho^t u(\tau, p(\tau,t,x_0))d\tau)\left\{1 - \int_\rho^t u_x(\tau, p(\tau,t,x_0))p_x(\tau,t,x_0)d\tau\right\}^2 d\nu d\rho + \\ &+ \int_0^s\int_0^\rho K(\nu)u_x(\rho-\nu, x_0 - \int_\rho^t u(\tau, p(\tau,t,x_0))d\tau)\left\{\int_\rho^t u_{xx}(\tau, p(\tau,t,x_0))p_x^{\,2}(\tau,t,x_0)d\tau + \int_\rho^t u_x(\tau, p(\tau,t,x_0))p_{xx}(\tau,t,x_0)d\tau\right\}d\nu d\rho \end{aligned} \quad (18)$$

By virtue of condition b), solving (10) with respect to *K(t)* we have:

$$\begin{aligned} K(t) &= \frac{1}{\varphi(x_0)}\Bigg\{g''(t) - \varphi''(x_0 - \int_0^t u(\tau, p(\tau,t,x_0))d\tau)\left\{g(t) + \int_0^t u_x(\tau, p(\tau,t,x_0))p_t(\tau,t,x_0)d\tau\right\}^2 + \\ &+ \varphi'(x_0 - \int_0^t u(\tau, p(\tau,t,x_0))d\tau)\{g'(t) - u_x(t,x_0)g(t) + \int_0^t u_{xx}(\tau, p(\tau,t,x_0))p_t^{\,2}(\tau,t,x_0)d\tau + \\ &+ \int_0^t u_x(\tau, p(\tau,t,x_0))p_{tt}(\tau,t,x_0)d\tau\} + \int_0^t K(\nu)u_x(t-\nu, x_0)g(t)d\nu - \int_0^t K(\nu)g'(t-\nu)d\nu - \\ &- \int_0^t\int_0^\rho K(\nu)u_{xx}(\rho-\nu, x_0 - \int_\rho^t u(\tau, p(\tau,t,x_0))d\tau)\left\{g(t) + \int_\rho^t u_x(\tau, p(\tau,t,x_0))p_t(\tau,t,x_0)d\tau\right\}^2 d\nu d\rho + \\ &+ \int_0^t\int_0^\rho K(\nu)u_x(\rho-\nu, x_0 - \int_\rho^t u(\tau, p(\tau,t,x_0))d\tau)\{g'(t) - u_x(t,x_0)g(t) + \int_0^t u_{xx}(\tau, p(\tau,t,x_0))p_t^{\,2}(\tau,t,x_0)d\tau + \\ &+ \int_0^t u_x(\tau, p(\tau,t,x_0))p_{tt}(\tau,t,x_0)d\tau\}d\nu d\rho\Bigg\}. \end{aligned} \quad (19)$$

System (11), (12), (13), (14), (15), (16), (17), (18), (19) define a closed-loop system for finding the unknowns $u_x(t,x_0)$, $p(s,t,x_0)$, $u(s,p(s,t,x_0))$, $p_t(s,t,x_0)$, $p_{tt}(s,t,x_0)$,

$p_x(s,t,x_0)$, $u_x(s,p(s,t,x_0))$, $u_{xx}(s,p(s,t,x_0))$, $K(t)$. Substituting the value of $K(t)$ from relations (19) into equation (6) and setting $s=t$ we obtain the function $u(t,x)$.

Thus, inverse problem (1) - (3) is equivalent to the following system:

$$V(s,t,x)=BV(s,t,x), \tag{20}$$

Introduce space

$$Y = C[0,T]\times C(D)\times C(D)\times C(D)\times C(D)\times C(D)\times C(D)\times C(D)\times C[0,T].$$

For any $V(s,t,x)\in Y$ we introduce the norm:

$$\|V(s,t,x)\|_Y = Sup_{t\in[0,T]}|u_x(t,x_0)| + Sup_D|p(s,t,0)| + Sup_D|u(s,p(s,t,x_0)| + Sup_D|p_t(s,t,x_0)| +$$

$$+Sup_D|p_{tt}(s,t,x_0)| + Sup_D|p_x(s,t,x_0)| + Sup_D|u_x(s,p(s,t,x_0)| + Sup_D|u_{xx}(s,p(s,t,x_0)| + Sup_{t\in[0,T]}|K(t)|.$$

Let's construct successive approximations:

$$V^0(s,t,x) = \begin{pmatrix} u_x^0(t,x_0) \\ p^0(s,t,x_0) \\ u^0(s,p(s,t,x_0)) \\ p_t^0(s,t,x_0) \\ p_{tt}^0(s,t,x_0) \\ p_x^0(s,t,x_0) \\ u_x^0(s,p(s,t,x_0)) \\ u_{xx}^0(s,p(s,t,x_0)) \\ K^0(t) \end{pmatrix} = \begin{pmatrix} \varphi'(x_0) \\ x_0 \\ \varphi(x_0) \\ -g(0) \\ -g(0)+\varphi'(x_0)g(0) \\ 1 \\ \varphi'(x_0) \\ \varphi''(x_0) \\ \dfrac{1}{\varphi(x_0)}\left\{-\varphi''(x_0)g^2(0)+g''(0)-\varphi'(x_0)\left(g'(0)-\varphi'(x_0)g(0)\right)\right\} \end{pmatrix}, \tag{21}$$

for n=1,2,… $V^n(s,t,x)=BV^{n-1}(s,t,x)$.

Let $R=\|V^0(s,t,x)\|_Y$, then we take a ball of radius 2R . i.e.

$U_{2R}=\{V(s,t,x)\in Y: \|V(s,t,x)\|_Y\leq 2R\}$.

There is T>0, such that for any $V(s,t,x)\in U_{2R}$ the following inequality holds: $\|BV(s,t,x)\|_Y\leq 2R$.

There is T>0, such that for any $V^*(s,t,x), V(s,t,x) \in U_{2R}$, the following inequalities hold: $\|BV^*(s,t,x) - BV(s,t,x)\|_Y\leq q\|V^*(s,t,x) - V(s,t,x)\|_Y$, $0<q<1$, where q is a known positive constant depending on T,R.

It is seen that for any n, $V^n(s,t,x)\in U_{2R}$, since $V^0(s,t,x)\in U_R$.

For the elements $V^0(s,t,x)\in U_R$, $V^1(s,t,x)\in U_{2R}$ the following inequality holds:

$$\| BV^1(s,t,x) - BV^0(s,t,x) \|_Y \leq C_2,$$

where C_2- is a known positive constant depending on T, R.

We construct a functional series from the terms of sequence (21) as follows

$$V^0(s,t,x) + \sum_{i=0}^{\infty} (V^i(s,t,x) - V^{i-1}(s,t,x)) . \tag{22}$$

Estimating each term of series (22), we obtain a majorizing numerical series:

$$R + C_2 \sum_{i=1}^{\infty} q^{i-1} , \; 0<q<1. \tag{23}$$

Passing to the limit in (21) for $n \to \infty$, and using the continuity of the operator $V(s,t,x)=BV(s,t,x)$, we obtain that the element $V(s,t,x)$ is a solution to the system of nonlinear integral equations.

By virtue of contracting mappings, systems (11), (12), (13), (14), (15), (16), (17), (18), (19) have a unique solution $V(s,t,x) \in U_{2R}$.

Thus, the following is proved.

Lemma 1. There exists T>0 such that under conditions a), b), system (11), (12), (13), (14), (15), (16), (17), (18), (19) has a unique solution $V(s,t,x) \in U_{2R}$.

Lemma 2. If a vector function $V(s,t,x)$ is a solution to system (11), (12), (13), (14), (15), (16), (17), (18), (19), then the functions $u(t,x)$, $K(t)$ satisfy problem (1) - (3) and vice versa.

The proof of Lemma 2 is similar to [3].

Theorem. If conditions a), b) are satisfied, then there is T>0 such that the inverse problem, (1) - (3) has a unique solution $\{u(t,x), K(t)\}$, from the class $\bar{C}^{1,1}([0,T]\times R)\times C^1 [0,T]$.

The proof follows from the proofs of Lemmas 1, 2.

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A PRIORI ESTIMATES OF SOLUTIONS OF THE CAUCHY PROBLEM FOR A QUASI-LINEAR PARABOLIC EQUATION

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In this article we have various assumptions on the modulus of continuity we investigate properties and deduce interior a priori estimates of solutions of a multi-dimensional singularly perturbed parabolic equation with several small parameters

Key words: Quasiliner, parabolic equation, degenerate problem, solution, several lines, asymptotic expansion, function, continuous derivatives, point, generally speaking, standard algorithms, hogt's, estimetes.

Бул макалада биз үзгүлтүксүздүктүн модулу жөнүндө ар кандай божомолдорду жасайбыз, касиеттерин иликтейбиз жана бир нече кичинекей параметрлери бар көп өлчөмдүү сингулярдуу бузулган параболалык теңдеме боюнча чечимдердин ички априордук бааларын чыгарабыз.

Урунттуу сөздөр: Квазисызыктуу, параболалык теңдеме, кубулган теңдеме, чечим, бир нече сызыктар, асимптотикалык ажыроо, функция, үзгүлтүксүз түзөтүлөр жана стандарттык алгоритмдер, туунду, чекит.

В данной статье мы делаем различные предположения о модуле непрерывности, исследуем свойства и выводим внутренние априорные оценки решений многомерного сингулярно возмущенного параболического уравнения с несколькими малыми параметрами.

Ключевые слова: Квазилинейное, параболическое уравнение, вырожденная задача, решение, несколько линий, асимптотическое разложение, функция, непрерывные поправки и стандартные алгоритмы, хогтсовские, эстимейтовые.

Consider the Cauchy problem

$$Lu \equiv \varepsilon_i \frac{\partial^2 u}{\partial {x_i}^2} - \frac{d}{dx_i}\varphi_i(t,x,u) - \psi(t,x,u) - \frac{\partial u}{\partial t} = 0, \qquad (1)$$

$$u|_{t=0} = u_0(x). \qquad (2)$$

Here $x = (x_1, x_2, \ldots, x_n)$ is a point of space $\mathcal{R}^n$, $\Pi_T = (0,T] \times \mathcal{R}^n$, the functions $\varphi_i(t,x,u)$ and $\psi(t,x,u)$ are defined and continuous for all $(t,x,u) \in \overline{\Pi}_T \times \mathcal{R}^1$ along with their partial derivatives in the variables x_k and u up to some order, $\varepsilon_i \in (0,1]$, $u_0(x)$ is some bounded measurable function,

$$\frac{d}{dx_i}\varphi_i(t,x,u) \equiv \sum_{i=1}^{n}\left[\frac{\partial \varphi_i(t,x,u)}{\partial x_i} + \frac{\partial \varphi_i(t,x,u)}{\partial u}\frac{\partial u}{\partial x_i}\right].$$

In the equation (1) and everywhere below if either term has two or more same indices, then this means summation over all these indices from 1 to n.

Introduce the following notation: (x, t) are points of the space $\mathcal{R}^n$;

$$|x| = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2};$$

$$x_{(c,i)} = (x_1, \ldots, x_{i-1}, c_i, x_{i+1}, \ldots x_n)\,; \quad g_{(c,i)}(t, x) = g_i\big(t, x_{(c,i)}\big);$$

$$\int_a^b g(x,t)\,dx = \int_{a_1}^{b_1}\int_{a_2}^{b_2}\ldots\int_{a_n}^{b_n} g(t, x_1, x_2 \ldots, x_n)\,dx_1\,dx_2 \ldots dx_n\,;$$

$$\int_a^b g(t,x)\,dx_{(c,i)} = (b_i - a_i)^{-1}\int_a^b g_{(c,i)}(t,x)\,dx\,;$$

ε is a vector with the coordinates $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$, $\varepsilon_0 = \min_{1\le i\le n}\varepsilon_i$; $\varepsilon^\gamma = \varepsilon_1^\gamma \varepsilon_2^\gamma \ldots \varepsilon_n^\gamma$.

By M, M_k, $k = 1, 2, \ldots$ we denote independent of ε constants, if the value of these constant is unessential for our further reasoning.

In deducing estimates for the solution of the problem (1), (2), the solution will be assumed to be bounded everywhere in $\overline{\Pi}_T$ by a constant m_0. Moreover, we will assume if needed that for $(t,x) \in \overline{\Pi}_T$ and $|v(t,x)| \le m_0$ the following estimates are valid:

$$|\varphi_i(t,x,v)| \le m_i\,; |\varphi'_{iv}(t,x,v)| \le m_{i,v}\,; |\varphi''_{ivv}(t,x,v)| \le m_{i,vv};$$

$$\left|\varphi'_{ix_k}(t,x,v)\right| \le p_{i,k}\,; \left|\varphi''_{ix_k x_s}(t,x,v)\right| \le p_{i,k,s}\,; \left|\varphi''_{iv\,x_k}(t,x,v)\right| \le p_{i,k,v}\,;$$

$$|\psi(t,x,v)| \le r;\ |\psi'_v(t,x,v)| \le r_v;\ \left|\psi'_{x_k}(t,x,v)\right| \le r_k.$$

Let $\mathfrak{f}_a(z)$ be an infinitely differentiable function of one variable, defined for $z \in (-\infty, \infty)$ and satisfying $\mathfrak{f}_a(z) \equiv 1$ for $|z - a| \le 1$, $\mathfrak{f}_a(z) \equiv 0$ for $|z - a| \ge 2$,

$0 \le f_a(z) \le 1$.

Consider the function $f(x)=f_{b_1}(\varepsilon_1^{-1}x_1)\, f_{b_2}(\varepsilon_2^{-1}x_2)\ldots f_{b_n}(\varepsilon_n^{-1}x_n)$, where b is a point of $\mathcal{R}^n$.

The function $v(t,x)= u(t,x)\, f(x)$ satisfies the equation

$$L_1\upsilon \equiv \varepsilon_i \frac{\partial^2 \upsilon}{\partial x_i^2} - \frac{\partial v}{\partial t} = -2\varepsilon_i \frac{\partial u}{\partial x_i}\frac{\partial f}{\partial x_i} - \varepsilon_i\, u \frac{\partial^2 f}{\partial x_i^2} + f \frac{d}{dx_i}\varphi_i(t,x,u) + f\,\psi(t,x,u) \qquad (3)$$

and the initial condition

$$v(0,x)=v_0(x)= f(x)\, u_0(x). \qquad (4)$$

Consider the modulus of continuity of $u(t,x)$ with respect to the spatial variables. To this end it suffices to estimate as auxiliary, we will get the final estimate the difference $u(t,x) - u(t,x_{(y,j)})$ which will be done in two steps. First we will obtain a preliminary estimate (see the inequality (5)) and then, using this estimate as auxiliary, we will get the final estimate.

In obtaining the auxiliary estimate, the points x, y will be assumed to belong to the cube $b_i - \varepsilon_i \le x_i, y_i \le b_i + \varepsilon_i$, $i = 1, 2, \ldots, n$. In this case $v(t,x)= u(t,x)$, $v(t,y)= u(t,y)$, and therefore

$$u(t,x)- u(t,x_{(y,j)})= \int_{b-2\varepsilon}^{b+2\varepsilon}\left[G(t,x,z,0) - G(t,x_{(y,j)},z,0)\right] v_0(z)\, dz +$$

$$+\int_0^t \int_{b-2\varepsilon}^{b+2\varepsilon}\left[G(t,x,z,r) - G\left(t,x_{(y,j)},z,r\right)\right]\left[\varphi_i \frac{\partial f}{dz} + \varepsilon_i u \frac{\partial^2}{\partial z_i^2} - f\psi\right] dzdr +$$

$$+\int_0^t d\,r \int_{b-2\varepsilon}^{b+2\varepsilon}\left\{\frac{\partial}{\partial z_i}\left[G(t,x,z,r) - G(t,x_{(y,j)},z,r)\right]\right\}(1-\delta_{i,j})\left[\varphi_i f + 2u\varepsilon_i \frac{\partial f}{\partial z_i}\right] dz +$$

$$+\int_0^t \int_{b-2\varepsilon}^{b+2}\left\{\frac{\partial}{\partial z_j}\left[G(t,x,z,r) - G(t,x_{(y,j)},z,r)\right]\right\}\left[\varphi_i f + 2u\varepsilon_i \frac{\partial f}{\partial z_i}\right] dzdr =$$

$$= A_1 + A_2 + A_3 + A_4,$$

where $G(t,x,z,r) = \varepsilon^{-1/2}\,[4\pi(t-r)]^{-n/2}\exp\{-(x_i - z_i)^2/4[\varepsilon_i(t-r)]\}$ is the fundamental solution of the heat conductivity equation, $\delta_{i,j}$ is the Kronecker symbol: $\delta_{i,j}= 0$ for $i \ne j$, $\delta_{i,i}= 1$.

The integrals A_1 and A_2 are estimated directly:

$$|A_1| \le m_0 \left| \int_{y_j}^{x_j} d\xi \int_{b-2\varepsilon}^{b+2\varepsilon} \left| \frac{\partial}{\partial z_j} G(t, x_{(\xi,j)}, z, 0) \right| dz \right| \le M_1 m_0 \varepsilon_j^{-1/2} t^{-1/2} |x_j - y_j| ;$$

$$|A_2| \le M_2(\varepsilon_0^{-1}\overline{m}_0 + \varepsilon_0^{-1} m_0 + r)\, \varepsilon_j^{-1/2} t^{1/2} |x_j - y_j|, \text{ where } \overline{m}_0 = \max_{1\le i\le n} m_i.$$

When estimating the integral A_3 we first assume that the inequality $y_j \le x_j$ is fulfilled and represent it as a sum $A_3 = A_{3,1}+A_{3,2}+A_{3,3}+A_{3,4}+A_{3,5}+A_{3,6}$ in such a way that the interval $(b_j -2\varepsilon_j , b_j +\varepsilon_j)$ of integration with respect to the variable x_j be partitioned by the points $2y_j -x_j$; y_j ; $2^{-1}(x_j + y_j)$; x_j ; $2x_j - y_j$ into 6 segments. Since for $(b_j -2\varepsilon_j \le z_j \le 2y_j - x_j$ the inequalities $0 \le x_j - y_j \le y_j - z_j$ are fulfilled, the estimate for the integral $A_{3,1}$ can be obtained in the form

$$\left|A_{3,1}\right| \le M_4(m_0+\overline{m}_0) \left|x_j - y_j\right|^{\gamma} \int_0^t dr \int_{b-2\varepsilon}^{b+2\varepsilon} G(t,x,z,r) \frac{x_i - z_i}{\varepsilon_i \varepsilon_j (t-r)^2} dz_{(x,j)} \times$$

$$\int_{b_j-2\varepsilon_j}^{b_j+2\varepsilon_j} (y_j - z_j)^{2-\gamma} \exp\left[\frac{(y_j - z_j)^2}{4\varepsilon_{j(t-r)}}\right] dz_j \le M_5(1-\gamma)^{-1}(m_0+\overline{m}_0)\varepsilon_0^{-1/2} t^{(1-\gamma)/2} \left|x_j - y_j\right|^{\gamma},$$

$0 < \gamma < 1$. The integral $A_{3,6}$ is estimated analogously.

The variable z_j in the integral $A_{3,2}$ satisfies the inequalities $2y_j - x_j \le z_j \le y_j$ from which we obtain $0 \le y_j - z_j \le x_j - y_j$. Hence $\exp\{-(x_j - y_j)^2/[4\varepsilon_j (t - r)]\} \le \exp\{-(y_j - z_j)^2/[4\varepsilon_j (t - r)]\}$, and therefore

$$\left|A_{3,2}\right| \le M_6(m_0+\overline{m}_0) \int_0^t dr \int_{b-2\varepsilon}^{b+2\varepsilon} G(t,x,z,r)\, dz_{(x,j)} \int_0^1 \left|x_j - y_j\right| \times$$

$$\left\{\int_{2y_j-x_j}^{y_j} \left\{\frac{y_j - z_j}{2\varepsilon_j(t-r)} \exp\left[-\frac{(y_j - z_j)^2}{4\varepsilon_j(t-r)}\right] \exp\left[\frac{\theta^2 (x_j - y_j)^2}{4\varepsilon_j(t-r)}\right]\right\} dz_j\right\} d\theta.$$

From the equality $x_j - y_j = (x_j - y_j)^{\gamma}\theta^{\gamma-1}[[\theta(x_j - y_j)]^{1-\gamma}$ we have

$$\left|A_{3,2}\right| \le M_6(m_0+\overline{m}_0) \left|x_j - y_j\right|^{\gamma} \int_0^t dr \int_{b-2\varepsilon}^{b+2\varepsilon} G(t,x,z,r)\, dz_{(x,j)} \int_0^1 \theta^{\gamma-1} \times$$

$$\{\int_{2y_{j-x_j}}^{y_j} \frac{y_j - z_j}{2\varepsilon_j(t-r)} [\theta(x_j - y_j)]^{1-\gamma} \exp$$

$$\left[-\frac{(y_j - z_j)^2}{4\varepsilon_j(t-r)} - \frac{\theta^2 (x_j - y_j)^2}{4\varepsilon_j(t-r)}\right] dz_j + \int_{2y_{j-x_j}}^{y_j} \frac{[\theta(x_j - y_j)]^{2-\gamma}}{2\varepsilon_j(t-r)} \exp$$

$$\left[-\frac{(y_j - z_j)^2}{4\varepsilon_j(t-r)}\right] dz_j \exp\left[-\frac{\theta^2 (x_j - y_j)]^2}{4\varepsilon_j(t-r)}\right]\} d\theta \le$$

$$\le M_7(m_0+\overline{m}_0)[\gamma(1-\gamma)]^{-1} \varepsilon_j^{-\gamma/2} \varepsilon_0^{-1/2} t^{(1-\gamma)/2} \left|x_j - y_j\right|^{\gamma}.$$

The estimate for the integral $A_{3,5}$ is analogous to that of the integral $A_{3,2}$.

From $y_j \leq z_j \leq (x_j + y_j)/2$ there follow the inequalities $0 \leq z_j - y_j \leq (x_j - y_j)/2 \leq x_j - z_j$ and from $(x_j + y_j)/2 \leq z_j \leq x_j$ the inequalities $0 \leq x_j \leq z_j \leq (x_j + y_j)/2 \leq z_j - y_j$. By virtue of these relations, the integrals $A_{3,3}$ and $A_{3,4}$ are estimated just in the same way as the integrals $A_{3,2}$ and $A_{3,5}$.

As for the integral A_4, we partition it into 6 summands $A_{4,k}$, $1 \leq k \leq 6$ and estimate each summand by using the same techniques as for the corresponding part of the integral A_3. Thus we can consider that the intermediate estimate for the modulus of continuity of the function $u(t,x)$ is obtained:

$$|u(t,x) - u(t,x_{(y,j)})| \leq M\Big[m_0 \varepsilon_j^{-1/2} t^{-1/2} |x_j - y_j| + (\varepsilon_0^{-1} m_0 + \varepsilon_0^{-1} \overline{m}_0 + r_0) \times$$

$$\varepsilon_j^{-\frac{1}{2}} t^{-\frac{1}{2}} |x_j - y_j| + (1-\gamma)^{-1} \gamma^{-1} (m_0 + \overline{m}_0) \varepsilon_j^{-\gamma/2}\, \varepsilon_0^{-1/2} t^{(1-\gamma)/2} (x_j - y_j)^{\gamma}\Big]. \quad (5)$$

We use the estimate (5) for the determination of the estimate of the difference $u(t,x)$- $u(t,x_{(y,j)})$. The points x, y will now be assumed to belong to the cube $b_i - 2^{-1}\varepsilon_i \leq x_i, y_i \leq b_i + 2^{-1}\varepsilon_i$, $1 \leq i \leq n$. It can be easily seen that the estimates for the integrals A_1, A_2 can remain unchanged. We rewrite the sum $A_3 + A_4$ as

$$\int_0^t dr \int_{b-2\varepsilon}^{b+2\varepsilon} \left\{\frac{\partial}{\partial z_i}\left[G(t,x,z,r) - G(t,x_{(y,j)},z,r)\right]\right\} \left(\varphi_i f + 2u\varepsilon_i \frac{\partial f}{\partial z_i}\right) dz =$$

$$= \int_0^t dr \int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)} \int_{b_j-2\varepsilon_j}^{b_j-\varepsilon_j} (\dots)\, \partial z_j + \int_0^t \int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(x,j)} \int_{b_j-\varepsilon_j}^{b_j+2\varepsilon_j} (\dots) dz_j +$$

$$\int_0^t dr \int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(x,j)} \int_{b_j-\varepsilon_j}^{b_j-\varepsilon_j} (\dots)\, dz_j = B_1 + B_2 + B_3.$$

In the integral B_1, the inequalities

$$\sum_{i=1}^{n} \frac{(x_i - z_{ji})^2}{4\varepsilon_i (t-r)} \geq \frac{\varepsilon_j}{16(t-r)}, \qquad \frac{(y_j - z_j)^2}{4\varepsilon_i (t-r)} + \sum_{i \neq j} \frac{(y_j - z_j)^2}{4\varepsilon_i (t-r)} \geq \frac{\varepsilon_j}{16(t-r)}$$

Are fulfilled, and therefore

$$|B_1| \leq M_9 (m_0 + \overline{m}_0) |x_j - y_j| \int_0^t dr \int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)}$$

$$\int_{b_j-2\varepsilon_j}^{b_j+2\varepsilon_j} G(t,x,z_{(x+\varepsilon/2,j)},r)\left[\frac{|x_i-z_i|\,|x_j-z_j|}{4\varepsilon_i\varepsilon_j(t-r)^2}+\frac{\delta_{i,j}}{2\varepsilon_j(t-r)}\right]dz_j \le M_{10}(m_0+$$

$$+\bar{m}_0)\left|x_j-y_j\right|(\varepsilon_j^{-1/2}t^{-1/2}+\varepsilon_j^{-3/2}t^{1/2}\,\varepsilon_j^{-1/2}\varepsilon_0^{-1/2})\exp\{-\varepsilon_j/(16t)\}.$$

It is obvious that the same estimate is valid for the integral B_2 as well. Let us now pass to the estimation of the integral B_3. We write it as

$$B_3=\int_0^t dr\int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)}\int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j}\left\{\frac{\partial}{\partial z_i}\left[G(t,x,z,r)-G(t,x_{(y,j)}\,z,r)\right]\right\}(\varphi_i f+$$

$$+2u\varepsilon_i\frac{\partial f}{\partial z_i})\,dz_j=\int_0^t dr\int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)}\int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j}\left\{\frac{\partial}{\partial z_i}\left[G(t,x,z,r)-\right.\right.$$

$$\left.\left.-G(t,x_{(y,j)}\,z,r)\right]\right\}\varphi_i dz_j+\int_0^t dr\int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)}\int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j}\left\{\frac{d}{dz_i}\left[G(t,x,z,r)-\right.\right.$$

$$\left.\left.-G(t,x_{(y,j)}\,z,r)\right]\right\}\times 2u\varepsilon_i\frac{\partial f}{\partial z_i}dz_j = B_{3,1}+B_{3,2}.$$

Since

$$\varphi_i(r,z,u(r,z))=\varphi_i\left(r,z_{(\xi,j)},u(r,z_{(\xi,j)})\right)+\left[\varphi_i(r,z,u(r,z_{(\xi,j)}))-\right.$$

$$\left.-\varphi_i(r,z_{(\xi,j)},u(r,z_{(\xi,j)}))\right]+\left[\varphi_i(r,z,u(r,z))-\varphi_i(r,z,u(r,z_{(\xi,j)}))\right],$$

the integral $B_{3,1}$ can be represented in the form of three summands denoted by $B_{3,1,1}$, $B_{3,1,2}$ and $B_{3,1,3}$ respectively. For the inequality $b_j\text{ -}2^{-1}\varepsilon_j\le x_j,\ y_j\le b_j+2^{-1}\varepsilon_j$, the integral $B_{3,1,1}$ is estimated directly:

$$|\int_0^t dr\int_{y_j}^{x_j} d\xi_j\int_{b-2\varepsilon}^{b+2\varepsilon}[\varphi_i(r,z_{(\xi,j)}\ ,u(r,z_{(\xi,j)}))f(z)+2\varepsilon_j u f'_{x_i}]\,dz_{(\xi,j)}\times$$

$$\times\int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j}\frac{\partial^2}{\partial z_i\,\partial z_j}G(t,x_{(\xi,j),}z,r)dz_j|\le M_{11}\bar{m}_0|x_j-y_j|\varepsilon_0^{-1/2}\varepsilon_j^{-1/2}t^{1/2}\exp\{\text{-}\varepsilon_j/16t)\}.$$

Using the inequality

$$|\varphi_i(r,z,u(r,z_{(\xi,j)}))\text{ - }\varphi_i(r,z_{(\xi,j)},u(r,z_{(\xi,j)}))\,|\le p_{i,j}\,|\xi_j-z_j|.$$

For the summand $B_{3,1,2}$, we have

$$|B_{3,1,2}|M_{14}p_{0,j}\int_0^t dr\int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)}\int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j} dz_j\int_{y_j}^{x_j} G(t,x_{(\xi,j)}z,t)\times\left[\frac{|\xi_j-z_j|}{\varepsilon_j(t-r)}\delta_{i,j}+\right.$$

$$\left.+\frac{|\xi_j-z_j|^2|x_i-z_i+\delta_{i,j}(\xi_j-x_j)|}{\varepsilon_i\varepsilon_j(t-r)^2}\right]d\xi_j\le M_{15}p_{0,j}\,\varepsilon_0^{-1/2}t^{1/2}|x_j-y_j|,$$

where $p_{0,k}=\max_{1\le i\le n}p_{i,k}$. To the integral $B_{3,1,3}$ we apply the intermediate estimate (5).

Since

$$|\varphi_i(r,z,u(r,z)) - |\varphi_i\left(r,z,u\left(r,z_{(\xi,j)}\right)\right)| \leq M_{16} m_{i,u}\{[m_0 \varepsilon_j^{-1/2} t^{-1/2} +$$

$$+ (m_0 + \ \bar{m}_0 + \bar{r})\, \varepsilon_j^{-1/2} \varepsilon_0^{-1} t^{1/2}]$$

$$|\xi_j - z_j|^\gamma + \gamma^{-1}(1-\gamma)^{-1}(m_0 + + \bar{m}_0) \varepsilon_j^{-\gamma/2} \varepsilon_0^{-1/2} t^{(-1-\gamma)/2} |\xi_j - z_j|^\gamma\},$$

the integral $B_{3,1,3}$ is estimated as

$$|B_{3,1,3}| \leq M_{17} m_{0,u} \sum_{i=1}^{n} dr \int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)} \int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j} \ dz_j | \int_{y_j}^{x_j} \{[m_0\, \varepsilon_j^{-1/2} r^{-1/2} +$$

$$+(\varepsilon_0^{-1} m_0 + \varepsilon_0^{-1} \bar{m}_0 \ + r)\, \varepsilon_j^{-1/2} t^{1/2} |\xi_j - z_j|] +$$

$$+\gamma^{-1}\,(1\text{-}\,\gamma)^{-1}\,(m_0 + \bar{m}_0)\, \varepsilon_j^{-\gamma/2} \varepsilon_0^{-1/2} r^{(1-\gamma)/2} |\xi_j - z_j|^\gamma\} G''_{z_i z_j}(t, x_{(\xi,j)}, z, r) d\,\xi_j| \,,$$

where $m_{0,u} = \max_{1 \leq i \leq n} m_{i,u}$. Evidently, without any difficulties we can obtain

$$\int_0^t r^\alpha dr \int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)} \int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j} \ |\xi_j - z_j| \ \left| \frac{\partial^2}{\partial z_i z_j} \ G(t, x_{(\xi,j)}, z, r) \right| dz_j \leq$$

$$\int_0^t r^\alpha \, dr \int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)} \int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j} \ G(t, x_{(\xi,j)}, z, r) \left[\frac{|x_i - z_i||\xi_j - z_j|^2}{\varepsilon_i \varepsilon_i (t-r)^2} + \frac{\delta_{i,j} |\xi_j - z_j|}{\varepsilon_i (t-r)} \right] dz_j \leq$$

$$M_{20} \varepsilon_0^{-1/2} t^{1/2 + \alpha},$$

$$\int_0^t r^\alpha dr \int_{b-2\varepsilon}^{b+2\varepsilon} dz_{(z,j)} \int_{b_j-\varepsilon_j}^{b_j+\varepsilon_j} \ G(t, x_{(\xi,j)}, z, r) [\frac{|x_i - z_i||\xi_j - z_j|^{1+\gamma}}{\varepsilon_i \varepsilon_j (t-r)^2} + \frac{\delta_{i,j} |\xi_j - z_j|^\gamma}{\varepsilon_j (t-r)}] \, dz_j \leq$$

$$M_{21} \varepsilon_0^{-1/2} \varepsilon_j^{(\gamma-1)/2} t^{\alpha + \gamma/2}.$$

Consequently, for the integral $B_{3,1,3}$ we have the estimate

$$|B_{3,1,3}| \ \leq M m_{0u} [m_0 \varepsilon_0^{-1/2} \varepsilon_j^{-1/2} + (m_0 + \bar{m}_0)\, \varepsilon_0^{-1} \varepsilon_j^{-1/2} t^{1/2} + (m_0 + \bar{m}_0 +$$

$$+\bar{r})\, \varepsilon_0^{-3/2} \varepsilon_j^{-1/2} t]\, |x_j - y_j|.$$

The integral $B_{3,2}$ can be estimated in the same manner as the integral $B_{3,1,3}$. Combining all the obtained inequalities, we can write out the estimate for the modulus of continuity of the function $u(t,x)$ with respect the variable x_j:

$$|\, u(t,x) - u\left(t, x_{(y,j)})| \leq M\{m_0 \varepsilon_j^{-\frac{1}{2}} t^{-\frac{1}{2}} + \ m_0 \varepsilon_0^{-\frac{1}{2}} \varepsilon_j^{-\frac{1}{2}} [m_{0,u}(m_0 + \bar{m}_0) \times\right.$$

$$\left. \times\, \varepsilon_0^{-1} \varepsilon_j^{-\frac{1}{2}} + (\varepsilon_0^{-1} m_0 + \varepsilon_0^{-1} \bar{m}_0 + r) \varepsilon_j^{-\frac{1}{2}} \right) +$$

$$+p_{0,0}\varepsilon_0^{-1/2}]t^{1/2}+m_{0,u}(\varepsilon_0^{-1}m_0+\varepsilon_0^{-1}\bar{m}_0+r)\,\varepsilon_0^{-1/2}\varepsilon_j^{-1/2}t\}|x_j-y_j|.$$

From the last inequality, in particular, it follows that one can weaken the requirements imposed on the initial data of the problem as follows:

$$|\varphi_{ix_k}(t,x,v)| \le p_{0,k} \le \varepsilon_k^{-1}\bar{p}_{0,k},\ |\psi(t,x,v)| \le r \le \varepsilon_0^{-1}\,\bar{r}.$$

Then the latter inequality can be written as

$$|\,u(t,x)-u\big(t,x_{(y,j)}\big)| \le M\{m_0(\varepsilon_j^{-1/2}t^{-1/2}+\varepsilon_0^{-1/2}\varepsilon_j^{-1/2})+$$

$$+[\,m_{0u}(m_0+\bar{m}_0)\,\varepsilon_j^{-1/2}+(m_0+\bar{m}_0+\bar{r})\,\varepsilon_j^{-1/2}+\bar{p}_{0,j}\varepsilon_j^{-1/2}]\,\varepsilon_0^{-1}t^{1/2}+$$

$$+m_{0u}(m_0+\bar{m}_0+\bar{r})\,\varepsilon_0^{-3/2}\varepsilon_j^{-1/2}t\}|x_j-y_j|. \qquad (6)$$

Thus we have proved the following assertion.

Theorem 1. Let $u_0(x)$ be a bounded measurable function. If the functions $\varphi_i(t,x,u)$ are bounded for all

$$(t,x)\in\bar{\Pi}_T, |\,u(t,x)| \le m_0,\ \varepsilon_0|\,\psi(t,x,u)| \le \bar{r},\ \ \varepsilon_j\left|\varphi'_{ix_j}(t,x,u)\right| \le \bar{p}_{0,j},$$

$|\varphi_{iu}(t,x,u)| \le m_{0u}$ where the constant $\bar{r}, \bar{p}_{0,j},\ m_{0,u}$ are independent of ε , then for the modulus of continuity of the function $u(t,x)$ with respect to the variable x_j the estimate (6) is fulfilled. If $t\in[0,T]$, then this estimate can be written in form

$$|u(t,x)-u(t,x_{(y,j)}| \le M\varepsilon_j^{-\frac{1}{2}}(t^{-\frac{1}{2}}+\varepsilon_0^{-\frac{1}{2}}+\varepsilon_0^{-1}t^{\frac{1}{2}}+\varepsilon_0^{-\frac{3}{2}}t)|x_j-y_j|, \qquad (7)$$

while if $t\in(0,\varepsilon_0]$, then

$$|u(t,x)-u(t,x_{(y,j)}| \le M\varepsilon_j^{-1/2}(t^{-1/2}+\varepsilon_0^{-1/2})\,|x_j-y_j|, \qquad (8)$$

The constants M depend only on the upper bounds of the functions $|u(t,x)|$,| $\varphi_i(t,x,u)|$, $\varepsilon_0|\,\psi(t,x,u)|$, $\varepsilon_j|\,\varphi'_{ix_j}(t,x,u)|$, $|\varphi_{iu}(t,x,u)|$ in the cylinder $N_b=\{(t,x)|0< t\le T, b_i-2\varepsilon_i \le x_i \le b_i+2\varepsilon_i\}$.

Remark 1. Theorem 1 remains valid if instead of the differentiability of the functions $\varphi_i(t,x,u)$ with respect to the variables x_j we require the fulfillment of the Lipschitz condition with respect to those variables. Moreover,our reasoning is also true if the functions $\varphi_i(t,x,u)$satisfy the Hölder condition with respect to the variables x_j, u, respectively with the exponents λ_1, λ_2, $0 < \lambda_1$, $\lambda_2 < 1$; not that naturally the right-hand sides of the inequalities(7), and (8) somehow change.

In the previous subsection we have required that the functions $\varphi_i(t, x, u)$ possess the bounded first order derivatives with respect to the variables x_j, u. If, however, these functions and the function $\psi(t, x, u)$ are more smooth with respect to the above-mentioned variables, then the estimate (7) may be essentially improved.

Theorem 2. Let the conditions of Theorem 1 be fulfilled. Let, moreover, the functions $\varphi_i(t, x, u)$ have the second derivatives with respect to the variables x_j, u and the function $\psi(t, x, u)$has the first derivatives with respect to the same variables. If everywhere in the strip $\{ (t, x) | \varepsilon_0 \le t \le T, -\infty \le x_i \le \infty\}$ these derivatives satisfy the conditions $|\varphi''_{ix_jx_k}(t, x, u)| \le \varepsilon_j^{-1}\varepsilon_k^{-1}p_{0,j,k}$, $|\varphi''_{ix_ju}(t, x, u)| \le \varepsilon_j^{-1}q_{0,j}$, $|\varphi''_{iuu}(t, x, u)| \le \bar{m}_{0uu}$ $|\psi'_{x_j}(t, x, u)| \le \varepsilon_j^{-1} \le \varepsilon_0^{-1}r_0$, $|\psi'_u(t, x, u)| \le \varepsilon_0^{-1}r_u$ where the constants $p_{0,j,k}$, $q_{0,j}$, $\bar{m}_{0uu}$, r_0, r_u remain unchanged when ε changes, then everywhere in the strip Π_T the estimate (8) is valid for the function $u(t, x)$, and the constant M does not depend on the values of the functions $u(t, x)$, $\varphi_i(t, x, u)$, and $\psi(t, x, u)$ outside the cylinder N_b.

Proof. Let the function $f_a(z)$, defined by us in the first section, satisfy the condition $[f'_a(z)]^2[f_a(z)]^{-1} \le M$ for all $\alpha - 2 \le z \le \alpha + 2$; obviously, such functions do exist. Consider the function $v_1(t, x)$= f(x) $[u^2(t, x) + \beta_j u'_{x_j}(t, x)]$, where the constant β_j is chosen from the condition β_j= min { $\sqrt{\varepsilon_j\varepsilon_0}$; $\varepsilon_j\bar{m}_{0uu}^{-1}$ }. Let the function $v_1(t, x)$ at some point $P_0(t_0, x_0)$ of the strip $\varepsilon_0 \le t \le T$ reach the greatest positive value. If $t_0 = \varepsilon_0$, then the assertion of Theorem 2 follows from

Theorem 1. Suppose $t_0 > \varepsilon_0$. For $t = t_0, x = x_0$ the equality $f(u^2 + \beta_j u'_{x_j})'_{x_k}=$ = - $f'_{x_k}(u^2 + \beta_j u'_{x_j})$ is fulfilled, and therefore at the above-mentioned point we have the relation.

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MSC 49M37

DETERMINING THE SIZE OF THE LOAN AND THE COMPOSITION OF THE BREEDS OF ANIMALS WITH AN INDICATIVE PLAN OF PRODUCTION

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In this article, a mathematical model has been developed for the problem of determining the optimal loan size and the composition of productive breeds of animals on the farm with an indicative plan for the production of livestock products according to the criterion of the minimum total costs. An algorithm for its solution is presented. The performance of the model is shown in a numerical example.

Key words: mathematical model, optimal size, credit, economy, indicative plan, production, algorithm, example.

Бул макалада насыянын оптималдуу өлчөмүн жана малдын продуктуларын өндүрүүнүн индикативдик планы менен чарбада өндүрүмдүү породалардын курамын аныктоо маселеси боюнча математикалык модель иштелип чыгып, минималдуу жалпы чыгымдардын критерийине ылайык келтирилген. Аны чечүүнүн алгоритми келтирилген. Моделдин көрсөткүчтөрү сандык мисалда келтирилген.

Урунттуу сөздөр: математикалык модель, оптималдуу өлчөм, кредит, экономика, индикативдик план, өндүрүш, алгоритм, мисал.

В этой статье разработана математическая модель задачи определение оптимального размера кредита и состава продуктивных пород животных в хозяйстве при индикативном плане производства продукции животноводства по критерию минимума суммарных затрат. Изложен алгоритм ее решения. Работоспособность модели показана на числовом примере.
Ключевые слова: математическая модель, оптимальный размер, кредит, хозяйство, индикативный план, производство, алгоритм, пример.

Formulation of the problem. Let a farm with sown areas of various categories (irrigated, rainfed, etc.) in the amount s_k, $k \in K$ planned to receive a loan at $\alpha\%$ per annum for τ years and produce livestock products in an amount not less than b^h, $h \in H$ by optimizing animal breeds in the farm, where h - the type of livestock products produced on the farm.

It is assumed that for each type of animal breed, the productivity and the corresponding daily ration are known, as well as the additional consumption per head in the amount of q_l^h, $l \in L$, $h \in H$, depending on the breed.

Also known is the yield and consumption for growing crops for each category of the sown area used by the farm in the ration of animal feeding.

It is required to determine the optimal composition of animals y_l^h on the farm and the size of the received credit z, allowing to ensure the production of livestock products in the planned volume, with minimal total costs.

The mathematical model of the problem can be presented in the form.

$$\sum_{k \in K} a_{kj} x_{kj} = \sum_{h \in H} \sum_{l \in L} \alpha_{lj}^h y_l^h, \quad j \in J_0, \tag{1}$$

$$\sum_{l \in L} \theta_l^h y_l^h \geq b^h, \quad h \in H, \tag{2}$$

$$\sum_{l \in L} \sum_{h \in H} q_l^h y_l^h = z \geq 0, \tag{3}$$

$$x_{kj} \geq 0, \quad k \in K, \quad j \in J_0, \tag{4}$$

$$y_l^h \geq 0, \quad l \in L, \quad h \in H \text{ - integer}, \tag{5}$$

where $x = \{ x_{kj} \geq 0, \ k \in K, \ j \in J_0 \}$, $y = \{ y_l^h \geq 0$ - integer, $h \in H, \ l \in L \}$,

j – index of the type of agricultural products crop production used in the daily ration of animal feeding, $j \in J_0 = \{1,2,\ldots,n\}$;

J_0 - many types of crop production aimed at animal feed, $j \in J_0$;

k – index of the type of the category of cultivated areas on the farm, $k \in K$;

K- many types of crop area categories, $K=\{1,2,\ldots,p\}$;

h – index of the type of livestock products produced on the farm, $h \in H$;

H – many types of livestock products, $H=\{1,2,\ldots, \bar{H}\}$;

l – index of the type of breed of animal on the farm, $l \in L$;

L- many types of animal breeds, $L=\{1,2,\ldots, \bar{L}\}$;

The parameters are known:

s_k – the size of the cultivated area of the k-th category on the farm, $k \in K$;

a_{kj} – the yield of the j -th type of crop on the k-th category of the sown area of the farm, $k \in K$, $j \in J_0$;

α_{lj}^h - the annual need for the j-th type of crop production in accordance with the daily feeding ration per one animal of the l-th breed in the production of the h-th type of product, where

$$\alpha_{lj}^h = \beta_{jl}^h \gamma_{jl}^h, \quad j \in J_0, \quad l \in L, \quad h \in H; \tag{6}$$

β_{jl}^h - the share of the j-th crop production in the daily ration per one animal of the l-th breed on the farm for the production of the h-type of product, $j \in J_0$, $l \in L$, $h \in H$;

γ_{jl}^h - the number of days in the ration of feeding the j-th type of crop production for the l-th breed of animal in the production of the h-th type of product, $j \in J_0$, $l \in L, h \in H$;

θ_l^h - the volume of production of the h -th type received by the farm from one animal of the l-th breed, $l \in L$, $h \in H$;

b^h – the planned volume of production of the h type of animal husbandry produced by the farm, $h \in H$;

c_{kj} – costs per unit of size of the k-th category of the sown area for the j-th type of crop, $j \in J_0$, $k \in K$;

c_l^h - annual consumption per one animal of the *l*-th breed in the production of the h-th type of livestock products, $h \in H$, $l \in L$;

q_l^h - one-time consumption per one animal of the l-th breed, depending on the type of product h, $l \in L, h \in H$;

λ - payback period of the loan;

α - interest rate of the loan.

Variables sought:

x_{kj} - the size of the k-th category of the sown area allocated for the j-th type of culture, $j \in J_0, k \in K$;

y_l^h - - the number of animals of the l-th breed in the farm for the production of the h-th type of product, $h \in H, l \in L;$

z – the size of the loan received by the farm.

Objective function (1) determines the minimum total consumption of the farm for growing fodder crops, for keeping an animal for the production of products in the planned volume and for an annual payment for a loan;

Constraints (2) determine that the total size of the cultivated area of the farm allocated for fodder crops in each category should not exceed the size of the cultivated area of this category;

Constraint (3) shows that the volume of agricultural production products of each type for feed should be equal to the volume of needs of the farm for domestic needs (for feed);

Constraint (4) requires that the volume of livestock production for each species must not be less than the planned volume of production of these products;

Equality (5) determines the amount of credit received by the farm;

Constraint (6) requires that the variables are not negative;

Constraint (7) requires that the value of variables must be an integer.

Algorithm for solving the problem. Calculations start with determining α_{lj}^h, $j \in J_0, l \in L, h \in H$ according to equality (8).

Using known data $a_{kj}, c_{kj}, s_k, k \in K, j \in J_0$ и $\theta_l^h, c_l^h, b^h, h \in H, l \in L$, a numerical model of the form (1)-(7) is formulated and solved. The solution algorithm ends.

From the solution of the problem, the quantitative composition of animals $y=\{ y_l^h , h\in H, l\in L\}$ is determined on the farm for the production of livestock products in each direction and the size of sown areas for each type of agricultural crop for feed $x=\{ x_{kj},\ k\in K,\ \ j\in J_0\}$ at minimum total costs.

Let's check the performance of the mathematical model and the algorithm for solving the problem using a numerical example.

Example. Let the farm has sown areas of S = 366 ha, of which irrigated s_1 = = 280 ha, and rainfed s_2 = 86 ha.

The main activity of the farm is the production of livestock products: milk and beef meat and plans to produce 125 tons of milk and 25 tons of beef meat.

The farm has the opportunity to choose the production composition of animals from two breeds of dairy production and from two breeds of meat production.

The following are known: - the daily ration for feeding dairy cows of the first type of breed with a milk yield of 3600 kg (Table 1), i.e. ($\theta_{l=1}^{h=1}$ = 3600 кг);

- daily ration for feeding dairy cows of the second type of breed with a milk yield of 4500 kg (Table 2), i.e. ($\theta_{l=2}^{h=1}$ = 4500 кг);

- the daily ration for feeding cows with a live weight of 300 kg - the first type of breed for meat (Table 3), i.e. ($\theta_{l=1}^{h=2}$ = 300 кг);

- the daily ration for feeding cows with a live weight of 450 kg - the first type of breed for meat (Table 3), i.e. ($\theta_{l=2}^{h=2}$ = 450 кг):

Table 1

The ration of feeding dairy cows of the first type of breed with a milk yield of 3600 kg of milk

	Feed name		Daily ration kg (1 head)		Fodder units	Total units	Total for 1 head qty	Number of days
1	Lucerne (hay)		4		0,5	2,0	720,0	180
2	straw	wheat	3	1	0,2	0,6	180.0	180
		barley		2			360.0	
3	haylage		6		0,3	1,8	1080,0	180
4	The conc.	wheat	2,4	0,3	1	2,4	109,5	365
		barley		1,5			547,5	

	feed	corn		0,6			219,0	
5	Siloge (corn)		10		0,3	3,0	1800,0	180
6	Mineral feed		0,010		-	-	-	365
7	salt		0,030		-	-	-	365
8	Grazing feed		40		-	-	7200,0	180
	Green feed							
	Total		-		-	9,8	-	-

Table 2

The ration of feeding dairy milk cows of the second breed with milk yield 4500 kg of milk

	The name of feed		Daily ration (1 animal)		Days Qty	Per year (1 animal)
1.	Lucerne (hay)		10 kg		180	1800 kg
2.	Straw	Wheat	3 kg	1 kg	180	180 kg
		Barley		2 kg		360 kg
3.	Hay		8 kg		180	1440 kg
4.	The concentrate on of the feed	Barley	3 kg	2 kg	365	730 kg
		Wheat		0,5 kg		182,5 kg
		Grain (corn)		0,5 kg		182,5 kg
5.	Siloge (corn)		12 kg		180	2160 kg
6.	Mineral. feed		-		365	3,6 kg
7.	salt		-		365	10,8 kg
8.	Grain (corn)		50 kg		180	9000 kg
	Green feed					

Table 3

Daily ration for feeding cattle (bulls, heifers) with live weight 300 kg

	Name of feed		Daily ration kg (1 nimal),		Feed unit	General unit	Total per 1 animal	Days qty
1.	Lucerne (hay)		3		0,5	1,5	540,0	180
2.	Straw	Wheat	2	0,5	0,2	0,4	90,0	180
		Barley		1,5			270,0	
3.	Hay		6		0,3	1,8	1080,0	180
4.	Conc. feed	Barley	1,5	0,5	1		182,5	
		Wheat		0,5		1,5	182,5	365

		Grain		0,5			182,5	
5.	Siloge (corn)		5,0		0,3	1,5	900,0	180
6.	Mineral feed		0,010		-	-	-	365
7.	Salt		0,030		-	-	-	365
8.	Grazing feed Green feed		30		-	-	5400,0	180
	Total		-		-	6,7	-	-

Table 4

Daily ration for feeding cattle (bulls, heifers) for meat with a live weight of 450 kg

	Name of feed		Daily ration (1 head)		Days qty	Per year (1 head)
1.	Lucerne (hay)		5 kg		365	1825 kg
2.	Straw (Barley)		1 kg		365	365 kg
3.	Hay		10 kg		180	1800 kg
4.	The concentrate on of the feed	Barley	3 kg	1 kg	365	365 kg
		Grain (corn)		1 kg		365 kg
		Wheat		1 kg		365 kg
5.	Siloge (corn)		10 kg		180	1800 kg
6.	Mineral feed		-		365	-
7.	Salt		-		365	-
8.	Grazing feed Green feed		30 kg		180	5400 kg

- productivity of agricultural crops on irrigated fields (I) and rainfed (II), included in the feeding ration, a_{kj}, k=1,2, j=1,2,…,7, Table 5.

Table 5

	wheat	barley	Hay (Lucerne)	Hay	Green feed	Siloge (corn)	Grain (corn)
	1	2	3	4	5	6	7
I	2070.0	1962.2	2380.0	6281.0	5730.0	12340.0	20280.0
II	1500.0	0	1700.0	0	0	0	0

- costs of growing crops per unit of size (I) and (II) fields, $|c_{kj}|_{2,7}$ Table 6.

Table 6

	1	2	3	4	5	6	7
I	2279.0	1096.0	2618.0	5071.0	9225.0	13574.0	7743.0
II	2000.0	1000.0	2000.0	5000.0	9000.0	13000.0	7000.0

In addition, the consumption for the maintenance of one animal of each breed is known for the production of milk and meat, respectively $c_{l=1}^{h=1}$ = 50040.0 soms, $c_{l=2}^{h=1}$ = 55080.0 soms, $c_{l=1}^{h=2}$ = 18030.0 soms, $c_{l=2}^{h=2}$ = 25020.0 soms.

Also known: - additional costs for one cattle, depending on the breed, i.e. $q_l^h = (q_1^1,\ q_2^1,\ q_1^2,\ q_2^2) = (45000.0,\ 50000.0,\ 15000.0,\ 15000.0)$;

- loan term = 10 years;
- loan interest rate α = 6% per annum.

It is required to determine the optimal composition of dairy and meat animals on the farm and the size of the loan that ensures the planned volume of milk and meat production at the lowest total cost.

For the mathematical formalization of the problem, we determine the annual feed requirement α_{lj}^h for one animal of each breed in production, $j \in J_0$, $l \in L$, $h \in H$.

Using the daily feeding ration, we will determine the annual need for each type of agricultural product included in the feed for one dairy cow with a milk yield of 3600 kg of milk and one dairy cow with a milk yield of 4500 kg. We will also determine the annual need for each type of agricultural products for fodder for one cow of the first and second types of breed for meat (see Table 7).

Table 7

Annual feed requirement per animal, depending on on breed and productivity

Name stern	Feed requirement per dairy cow		Feed requirement per cow for meat	
	1 breed with 3600 kg milk yield	2 breed with 3600 kg milk yield	1 breed with a live weight of 300 kg	2 species of live weight breed 450 kg
1.wheat	289,5	362,5	272,5	365,0
2.barley	907,5	1090,0	452,5	730,0
3. perennial grass 3.1. hay (alfalfa)	720,0	1800,0	540,0	1825,0
3.2. haylage	1080,0	1440,0	1080,0	1800,0
3.3. Green feed	7200,0	9000,0	5400,0	5400,0
4. Corn 4.1. silage	1800,0	2160,0	900,0	1800,0
4.2.grain	219,0	182,5	182,5	365,0

We formulate a numerical model of the problem.

Find a minimum

$$L(x,y)=2279.0x_{11}+1096.0x_{12}+2618.0x_{13}+5071.0x_{14}+9225.0x_{15}+13574.0x_{16}+7743.0x_{17}+$$
$$+2000.0x_{21}+1000.0x_{22}+2000.0x_{23}+5000.0x_{24}+9000.0x_{25}+13000.0x_{26}+7000.0x_{27}+$$
$$+50040.0\,y_1^1+55080.0\,y_2^1+18030.0\,y_1^2+25020.0\,y_2^2+\frac{(1+0,06\times 10)z}{10} \quad (7)$$

with conditions of

$$\sum_{j=1}^{7} x_{1j}=x_1\le 280, \qquad \sum_{j=1}^{7} x_{2j}=x_2\le 86, \quad (8)$$

$$2070,0x_{11}+1500,0x_{21}=289,5\,y_1^1+362,5\,y_2^1+272,5\,y_1^2+365,0\,y_2^2,$$
$$1962,0x_{12}+0x_{22}=907,5\,y_1^1+1090,0\,y_2^1+452,5\,y_1^2+730,0\,y_2^2,$$
$$2380,0x_{13}+1700,0x_{23}=720,0\,y_1^1+1800,0\,y_2^1+540,0\,y_1^2+1825,0\,y_2^2,$$
$$6281,0x_{14}+0x_{24}=1080,0\,y_1^1+1440,0\,y_2^1+1080,0\,y_1^2+1800,0\,y_2^2,$$
$$5730,0x_{15}+0x_{25}=7200,0\,y_1^1+9000,0\,y_2^1+5400,0\,y_1^2+5400,0\,y_2^2,$$
$$12340,0x_{16}+0x_{26}=1800,0\,y_1^1+2160,0\,y_2^1+900,0\,y_1^2+1800,0\,y_2^2,$$
$$20280,0x_{17}+0x_{27}=219,0\,y_1^1+182,5\,y_2^1+182,5\,y_1^2+265,0\,y_2^2, \quad (9)$$

$$3600\,y_1^1+4500\,y_2^1\ge 125000,$$
$$300\,y_1^2+450\,y_2^2\ge 25000, \quad (10)$$

$$45000,0\,y_1^1+50000,0\,y_2^1+15000,0\,y_1^2+15000,0\,y_2^2=z\ge 0, \quad (11)$$

$$x_{kj}\ge 0, \quad k=1,2, \quad j=1,2,\ldots,7, \quad (12)$$

$$y_l^h\ge 0, \quad l=1,2, \quad h=1,2. \quad (13)$$

Let us write down the numerical model of problem (9)-(15) in the form of a table 8.

Table 8

Representation of the condition of problem (3.9)-(3.15) in the form of a table

x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{21}	x_{22}	x_{23}	x_{24}
1	1	1	1	1	1	1				
							1	1	1	1
2070.0							1500.0			
	1962.0							0		
		2380.0							1700.0	
			6281.0							0

				5730.0						
					12340.0					
						20280.0				
2279.0	1096.0	2618.0	5071.0	9225.0	13574.0	7743.0	2000.0	1000.0	2000.0	5000.0

Continuation of Table 8

x_{25}	x_{26}	x_{27}	y_1^1	y_2^1	y_1^2	y_2^2	z		
								≤	280
1	1	1						≤	86
			-289,5	-362,5	-272,5	-365,0		=	0
			-907,5	-1090,0	-452,5	-730,0		=	0
			-720,0	-1800,0	-540,0	-1825,0		=	0
			-1080,0	-1440,0	-1080,0	-1800,0		=	0
0			-7200,0	-9000,0	-5400,0	-5400,0		=	0
	0		-1800,0	-2160,0	-900,0	-1800,0		=	0
		0	-219,0	-182,5	-182,5	-265,0		=	0
			3600,0	4500,0				≥	125000
					300,0	450,0		≥	25000
			45000,0	50000,0	15000,0	15000,0	-1	=	0
9000,0	13000,0	7000,0	50040,0	55080,0	18030,0	25020,0	0,16	→	min

Having solved the problem (7)-(13), by the method in [1], we determine the optimal plan for the production of livestock products (milk in the amount of 125100 kg, meat - 25050 kg), the distribution of acreage for fodder crops

x={ $x_{11}=14,7$; $x_{12}=36,2$; $x_{13}= 63,1$; $x_{14}=22,3$; $x_{15}= 96,4$; $x_{16}=13,0$; $x_{17}=1,0$}

and the composition of animals in the dairy and meat sector

$$y=\{ y_1^1 = 1; \quad y_2^1 = 27; \quad y_1^2 = 1; \quad y_2^2 = 55 \}.$$

The loan received by the farm is 2235000.0 soms, and the annual payment for the loan is 357600.0 soms. With such a plan, the total costs are equal

$$L(x,y) = 4713653.0 \text{ soms.}$$

Output. From the optimal solution, it follows that for the production of the planned volume of products in the amount of 125 tons of milk and 25 tons of meat, the farm must have 27 cows of the 2 type and in the amount of one cow of the 1 type for milk production. For meat production, the farm must have 55 cows of the 2 breed and one cow of the 1 breed.

At the same time, out of 280 hectares of irrigated and 86 hectares of rainfed sown areas, the farm uses only 246.7 hectares of irrigated sown areas for agricultural crops for fodder, i.e. 14.7 hectares for wheat; 36.2 hectares for barley; only 181.8 hectares for perennial grass, of which 63.1 hectares for hay (alfalfa); 22.3 hectares for haylage; 96.4 ha is used as green forage; 14.0 hectares are used for corn, of which 13.0 hectares are used for silage and 1 hectare for corn grain.

The total expenses for growing crops for feed and caring for animals to obtain 125.1 tons of milk, 25.05 tons of meat, including the annual loan payment in the amount of 357600 soms, amounted to 4713653.0 soms.

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THE PROBLEM OF DETERMINING THE SIZE OF THE LOAN AND THE SELECTION OF PRODUCTIVE BREEDS OF ANIMALS

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The work developed a two-level mathematical model of the problem of optimizing the choice of a productive breed of animals, which allows to ensure the planned volume of production and the size of the loan, as well as to distribute the unused sown area of the farm for vegetable growing. The algorithm for solving the problem is illustrated with a numerical example.

Key words: mathematical model, sown area, production, yield, consumption, income, credit.

Өндүрүштүн пландаштырылган көлөмүн жана насыянын көлөмүн камсыз кылууга, ошондой эле пайдаланылбай жаткан айдоо аянтын бөлүштүрүүгө мүмкүнчүлүк берген, өндүрүмдүү жаныбарлардын тукумун тандоону оптималдаштыруу маселесинин эки деңгээлдүү математикалык модели иштелип чыккан. Жашылча естуруучу чарба. Маселени чечүү алгоритми сандык мисал менен чагылдырылган.

Урунттуу сөздөр: математикалык модель, айдоо аянты, өндүрүш, түшүмдүүлүк, керектөө, киреше, кредит.

В работе разработана двухуровневая математическая модель задачи оптимизации выбора продуктивной породы животных, позволяющих обеспечить запланированный объем продукции и размера кредита, а также распределить неиспользованную посевную площадь хозяйства под культуру овощеводства. Алгоритм решения задачи проиллюстрирован на числовом примере.
Ключевые слова: математическая модель, посевная площадь, производство, урожай, потребление, доход, кредит.

Problem statement. For the production of planned livestock products in volume a^h, $h \in H$ the household needs a loan in the amount $z \le D$ под α% for τ years (where D - maximum loan amount) to renew the composition of animals with more productive breeds.

To create working conditions depending on the breed of animals and the corresponding daily ration for one animal, a one-time financial expense is required in the amount of $\varepsilon_l^h, l \in L, h \in H$.

The farm has irrigated sowing areas of s_0 ha. x_0 ha of them the household uses it to grow crops for feed, $(s_0$-$x_0)$ ha - uses for vegetable crops sold on the sales market, where x_0 - sought quantity.

Also known: the yield of agricultural crops for animal feed and vegetable crops, as well as the costs of growing them.

It is required to determine the optimal composition of animals $y_l^h, l \in L, h \in H$, that allows to ensure the production of products in the planned volume and the size of the financial loan z, as well as the volumes of vegetable products sold v_j, $j \in J_1$, delivering maximum net income to the farm.

The mathematical model of the problem can be presented in the form.

To find the maximum

$$G(\bar{x}, v) = \sum_{j \in J_1} (d_j v_j - c_j x_j) + \{\sum_{h \in H} \sum_{l \in L} c^h \theta_l^h \bar{y}_l^h - \min_{x,y} L(x, y)\} \tag{1}$$

under conditions

$$\sum_{j \in J_1} x_j = s_0 - \bar{x}_0, \tag{2}$$

$$a_j x_j = v_j, \quad j \in J_1, \tag{3}$$

$$x_j \geq 0, \quad v_j \geq 0, \ j \in J_1, \tag{4}$$

where

$$\overline{x} = \{ x_j : \quad j \in J_1 \}, \quad v = \{ v_j : \ j \in J_1 \},$$

$\overline{x}_0, \quad \overline{y}_l^h$ – optimal solution to the problem:

$$L(x,y) = \sum_{j \in J_0} c_j x_j + \sum_{h \in H} \sum_{l \in L} c_l^h y_l^h + (1 + \alpha\lambda) z / \lambda \to min \tag{5}$$

under conditions

$$\sum_{j \in J_0} x_j = x_0 \leq s_0, \tag{6}$$

$$a_j x_j = \sum_{h \in H} \sum_{l \in L} \alpha_{jl}^h y_l^h, \quad j \in J_0, \tag{7}$$

$$\sum_{l \in L} \theta_l^h y_l^h \geq a^h, \quad h \in H, \tag{8}$$

$$\sum_{l \in L} \sum_{h \in H} \varepsilon_l^h y_l^h = z \leq D, \tag{9}$$

$$x_j \geq 0, \quad j \in J_0, \tag{10}$$

$$y_l^h \geq 0, \quad l \in L, \quad h \in H \text{ - integer}, \tag{11}$$

$$x_0 \geq 0, \quad z \geq 0, \tag{12}$$

where

$$x = \{x_j: \ j \in J_0\}, \quad y = \{ y_l^h : h \in H, l \in L\}.$$

J – is the set of types of crop and vegetable products produced in the household, $J = J_0 \cup J_1$, $J_0 \cap J_1 = \emptyset$;

J_0 – is the set of types of crop products for feed $j \in J_0$;

J_1 – is the set of types of vegetable products, $j \in J_1$;

h – is index of the type of livestock products produced on the farm, $h \in H$;

H – is the set of types of livestock products, $H = \{1, 2, \ldots, \overline{H}\}$;

l – is an animal breed index, $l \in L$;

L – is the set of types of animal breeds, $L = \{1, 2, \ldots, \overline{L}\}$;

j – is index of the type of crop and vegetable production produced on the farm, $j \in J$;

s_0 – is the size of the cultivated area on the farm;

a_j – is yield of j-type of culture on the farm, $j \in J$;

α_{jl}^{h} – is an annual demand for j type crop products, in accordance with the daily feed ration for one animal l breeds in production of h type of product, where

$$\alpha_{jl}^{h} = \beta_{jl}^{h}\gamma_{jl}^{h}, \quad j \in J_0, \quad l \in L, \quad h \in H; \qquad (13)$$

β_{jl}^{h} – is the share of the j-th crop production in the daily ration for one animal the l-th breed on the farm for the production of the h-th type of product, $j \in J_0$, $l \in L$, $h \in H$;

γ_{jl}^{h} – is the number of days in the ration of feeding the j-th type of crop production for the l-th breed of animal in the production of the h-th type of product, $j \in J_0$, $l \in L$, $h \in H$;

θ_{l}^{h} – is the volume of production of the h-type received by the farm from one animal l-th breed, $l \in L$, $h \in H$;

a^{h} – is the planned volume of production of the h-type of livestock products produced by the farm, $h \in H$;

d_j – is realizable price of the j-th type of vegetable production, $j \in J_1$;

c^{h} – the realizable price per unit volume of the h -th type of animal husbandry product, $h \in H$;

c_j – are costs per unit of sown area for the j-th type of crop, $j \in J_0$;

c_{l}^{h} – is an annual consumption per one animal of the l-th breed in the production of the h-th type of livestock products, $h \in H$, $l \in L$;

ε_{l}^{h} – is one-time financial expense to create production conditions for one animal of the l-th breed in the production of the h-type of product, $l \in L$, $h \in H$;

λ – payback period of the loan; α – interest rate of the loan.

Variables sought:

x_j – is the size of the cultivated area allotted for the j-th type of crop, $j \in J_0$;

v_j – is the volume of production of vegetable products of the j-th type, $j \in J_1$;

y_l^h – is the number of animals of the l-th breed on the farm for the production of the h-th type of product, $h \in H$, $l \in L$;

x_0 – is the number of animals of the l-th breed on the farm for the production of the h-th type of product,

z – is the size of the loan received by the farm.

Objective function (1) determines the maximum net consumption of the farm from the production and sale of livestock and vegetable products;

Equality (2) determines that the total size of the sown area allotted for each type of vegetable growing should be equal to the size of the sown area of the farm not occupied by fodder crops;

Equality (3) determines the volume of vegetable production received from the sown area of farms not occupied by fodder crops;

Constraint (4) does not require the sought variables to be negative; Objective function (5) determines the minimum total costs of a farm for growing forage crops and keeping animals, as well as payment for a loan. Constraint (6) requires that the total sown area allotted for fodder crops should not exceed the size of the available sown area of the farm;

Constraint (7) requires that the volume of products produced for each type of feed should be equal to the volume of animal needs contained in the farm;
Condition (8) requires that the volume of livestock production for each species must not be less than the planned volume of production of this product;

Equality (9) determines the size of the loan received by the economy, should not exceed its maximum size of issue;

Constraint (10), (12) does not require the variables to be negative, but condition (11) requires that the value of the variables must be an integer.

Algorithm for solving the problem. Calculations begin with determining the values of the parameters α_{jl}^h, $j \in J_0$, $l \in L$, $h \in H$ according to equality (13).

Further, using the known data a_j, c_j, $j \in J_0$, θ_l^h, c_l^h, ε_l^h, a^h, $h \in H$, $l \in L$ и s_0, D, α, λ a partially integer model of the problem of the form (5) - (12) is constructed.

In what follows, a numerical model of the problem of the form (1) - (4) is formulated using known data a_j, c_j, d_j, $j \in J_1$, θ_l^h, c^h, $l \in L$, $h \in H$ and data $\min_{x,y} L(x, y)$, $(s_0 - \bar{x}_0)$, $\bar{y}_l^h$, $l \in L$, $h \in H$ from the optimal solution to problem (5) - (12).

Problem (5) - (12) and (1) - (4) are solved sequentially. The solution method ends.

From the solution of these problems, the quantitative composition of animals is determined $y=\{ y_l^h, \quad h \in H, \ l \in L\}$ on the farm providing the volume of production of livestock products according to the planned volume, the size of the sown area for each type of crop for feed $x=\{ x_j, \ j \in J_0\}$ and vegetable crops $\bar{x} =\{ x_j, \ j \in J_1\}$, sold in the sales market, as well as the size of the financial loan received by the farm, delivering the maximum net income $z \le D$. Let's check the performance of the mathematical model of the problem using a numerical example.

Example. Let the farm has a sown area of S = 600 hectares and has planned to produce livestock products: 125 tons of milk and 25 tons of meat, and the farm plans to allocate the sown area not occupied by fodder crops for vegetable crops (onions, potatoes). To meet this volume of production, the farm planned to update its production composition of productive animals with more efficient breeds. For this purpose, the farm needs a financial loan of the maximum size, which is no more than 50 million soms for 10 years at 6% per annum.

The farm is faced with the choice of the best breeds from the following two breeds of animals for the production of milk and meat in the corresponding known daily ration of feeding in [1].

Known for the dairy direction: the first type of breed is dairy cows with a milk yield of 3600 kg, i.e. ($\theta_{l=1}^{h=1}$ = 3600 kg);

the second type of breed is dairy cows with a milk yield of 4500 kg, ie. ($\theta_{l=2}^{h=1}$ = 4500 kg);

- In the direction of meat: the first type of breed is bulls (heifers) with a live weight of 300 kg, i.e. ($\theta_{l=1}^{h=2}$ = 300 kg);

- In the direction of meat: the first type of breed is bulls (heifers) with a live weight of 300 kg, i. ($\theta_{l=2}^{h=2}$ = 450 kg)

In addition, the following are known: - the yield of agricultural crops for feed, vegetable crops and the costs of their cultivation, as well as the selling price (Table 1).

Table 1

	wheat	barley	hay (alfalfa)	haylage	Green feed	Silage (corn)	Grain (kukur.)	onion	Potatoes
	1	2	3	4	5	6	7	8	9
a_j *(kg)*	2070.0	1962.2	2380.0	6281.0	5730.0	12340.0	20280.0	4500.0	7000.0
c_j *(som)*	2279.0	1096.0	2618.0	5071.0	9225.0	13574.0	7743.0	25000	20000
d_j *(som)*	14.0	12.0	5.0	-	-	-	8.0	15.0	20.0

- expenses for the maintenance of one animal of each breed in the production of milk and meat, respectively $c_{l=1}^{h=1}$=50040.0 soms, $c_{l=2}^{h=1}$=55080.0 soms, $c_{l=1}^{h=2}$=18030.0 soms, $c_{l=2}^{h=2}$=25020.0 soms and the selling price of milk $c^{h=1}$=25 som/kg and meat $c^{h=2}$=130 som/kg.

- one-off costs per animal depending on the breed, i.e.

$$\varepsilon_l^h = (\varepsilon_1^1,\ \ \varepsilon_2^1,\ \varepsilon_1^2,\ \ \varepsilon_2^2) = (45000.0,\ 50000.0,\ 15000.0,\ 15000.0);$$

- loan term λ = 10 year;

- loan interest rate α = 6 % per year.

It is required to determine the optimal composition of animals for dairy and meat production, allowing to ensure the planned volume of milk and meat production and the size of the loan, as well as the plan for the production of vegetable products so that the net income of the farm would be maximum.

Let us formulate a numerical model of the problem. Using a daily feeding ration for the listed breeds in the dairy and meat direction given in work [1], we will determine by the formula (13) the annual need for each type of agricultural products included in the composition of feed for one dairy cow with a milk yield of 3600 kg of milk and one dairy cow with a milk yield of 4500 kg (Table 2).

Table 2

Annual feed requirement per animal, depending on

on breed and productivity

Name of stern	Feed requirement for one dairy cow		Feed requirement per cow for meat	
	1 вида породы с удоем 3600 кг	2 вида породы с удоем 4500 кг	1 вида породы с живым весом 300 кг	2 вида породы с живым весом 450 кг
1. wheat	289,5	362,5	272,5	365,0
2. barley	907,5	1090,0	452,5	730,0
3. perennial herbs 3.1. hay (alfalfa)	720,0	1800,0	540,0	1825,0
3.2. haylage	1080,0	1440,0	1080,0	1800,0
3.3. Green feed	7200,0	9000,0	5400,0	5400,0
4.corn 4.1. silage	1800,0	2160,0	900,0	1800,0
4.2.grain	219,0	182,5	182,5	365,0

Next, we will determine the net income of the farm from the production of vegetable and livestock products.

Then the numerical model of the problem according to (1) - (4) and (5) - (12) will be written in the form.

To find the maximum

$$G(x,v) = 15.0v_1 + 20.0v_2 - 25000.0x_8 - 20000.0x_9 + 25(3600\bar{y}_1^1 + $$
$$+4500\bar{y}_2^1) + 130(300\bar{y}_1^2 + 450\bar{y}_2^2) - \min_{x,y} L(x,y) \qquad (14)$$

under conditions

$$\sum_{j\in J_1} x_j = 600 - \bar{x}_0, \quad J_1=\{8,9\}, \qquad (15)$$

$$4500.0x_8 = v_8, \;\; 7000.0x_9 = v_9, \qquad (16)$$

$$x_j \geq 0, \quad v_j \geq 0, \quad j \in J_1, \qquad (17)$$

where

x = { x_j: $j\in J_1$}, v= { v_j: $j\in J_1$}, a $\bar{x}_0$, $\bar{y}_l^h$, $l\in L=\{1,2\}$, $h\in H=\{1,2\}$ – optimal solution to the problem:

To find the maximum

$$L(x,y) = 2279.0x_1+1096.0x_2+2618.0x_3+ 5071.0x_4+9225.0x_5+13574.0x_6+$$
$$+7743.0x_7+50040.0\,y_1^1+ 55080.0\,y_2^1+18030.0\,y_1^2+25020.0\,y_2^2+ 0,16z \quad (18)$$

under conditions

$$\sum_{j\in J_0} x_j = x_0 \le 600, \quad J_0 = \{1,2,...,7\}, \quad (19)$$

$$2070.0x_1 = 289.5\,y_1^1+362.5\,y_2^1+272.5\,y_1^2+365.0\,y_2^2,$$
$$1962.0x_2 = 907.5\,y_1^1+1090.0\,y_2^1+452.5\,y_1^2+730.0\,y_2^2,$$
$$2380.0x_3 = 720.0\,y_1^1+1800.0\,y_2^1+540.0\,y_1^2+1825.0\,y_2^2,$$
$$6281.0x_4 = 1080.0\,y_1^1+1440.0\,y_2^1+1080.0\,y_1^2+1800.0\,y_2^2,$$
$$5730.0x_5 = 7200.0\,y_1^1+9000.0\,y_2^1+5400.0\,y_1^2+5400.0\,y_2^2,$$
$$12340.0x_6 = 1800.0\,y_1^1+2160.0\,y_2^1+900.0\,y_1^2+1800.0\,y_2^2,$$
$$20280.0x_7 = 219.0\,y_1^1+182.5\,y_2^1+182.5\,y_1^2+265.0\,y_2^2, \quad (20)$$

$$3600.0\,y_1^1+4500.0\,y_2^1 \ge 125000.0,$$
$$300.0\,y_1^2+450.0\,y_2^2 \ge 25000.0, \quad (21)$$

$$45000.0\,y_1^1+50000.0\,y_2^1+15000.0\,y_1^2+15000.0\,y_2^2=z\le 50000000.0, \quad (22)$$

$$x_j\ge 0, \quad j\in J_0, \quad (23)$$

$$y_l^h \ge 0, \quad l \in L = \{1,2\}, \quad h \in H = \{1,2\}, \quad (24)$$

$$z\ge 0,\ x_0 \ge 0. \quad (25)$$

We solve problem (18) - (25) by the method described in [2].

We will obtain an optimal plan for the production of livestock products (milk - 125100 kg, meat -25050 kg), a plan for the distribution of sown areas for fodder crops (ha) x={ $x_1=14.7;$ $x_2=36.2;$ $x_3= 63.1;$ $x_4=22.3;$ $x_5= 96.4;$ $x_6=13.0$; $x_7=1.0$}, plan - vegetable culture $\bar{x} = \{x_8 = 0;\ x_9 = 353.3\}$, the composition of the animals kept in the dairy and meat farm y = { $y_1^1 = 1;$ $y_2^1 = 27;$ $y_1^2 = 1;$ $y_2^2 = 55$ }, the size of the loan received by the household is z = 2235000.0 soms for ten years at 6% per annum.

From the optimal solution, it follows that the following products were produced: milk in the amount of 125100 kg, meat - in 25050 kg.

The composition of the breeds of animals kept - in the dairy direction in the amount of 1 cows of the 1st breed and in the amount of 27 cows of the 2nd breed, and in the meat direction in the amount of 1 cow of the 1st breed, and in the amount of 55 cows of the 2nd breed. Of the available 600 hectares of sown area, 246.7 hectares are used for agricultural crops for fodder, the remaining 353.3 hectares for potatoes.

Taking into account all income and expenses, as well as the annual payment for a loan in the amount of 357600.0 soms, the net income of the farm is 44066347.0 soms.

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MSC 49M37

OPTIMIZATION OF PRODUCTION OF AN AGRICULTURAL COOPERATIVE WITH FULFILLMENT OF THE COMMITMENT PLAN

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The paper formulates a mathematical model for determining the optimal size of the cultivated area of farms of the cooperative for each type of culture and the amount of financial credit when fulfilling the plan of the obligation.

Keywords: mathematical model, sown area, financial credit, cooperative, optimal size.

Бул жумушта милдеттенменин планын аткарууда өсүмдүктөрдүн ар бир түрү боюнча кооперативдин чарбаларынын айдоо аянттарынын оптималдуу өлчөмүн жана каржылык насыянын көлөмүн аныктоонун математикалык модели иштелип чыккан.

Уруунтуу сөздөр: математикалык модель, айдоо аянты, каржылык насыя, кооператив, оптималдуу өлчөм.

В работе сформулирована математическая модель определения оптимального размера посевной площади хозяйств кооператива под каждый вид культуры и объема финансового кредита при выполнении плана обязательства.

Ключевые слова: математическая модель, посевная площадь, финансового кредит, кооператив, оптимального размер.

Formulation of the problem. Let an agricultural cooperative consisting of p farms with sown areas s_k,, k∈K = {1,2, ..., p} planned to grow j∈ J_0= {1,2, ..., n} types of agricultural products for a processing enterprise A_0 under a contract and the wholesale purchase price of a unit of volume of each type of product is established.

It is assumed that the yield of each type of crop in each farm is known, the rate of consumption of mineral fertilizers per unit area for each type of crop and the wholesale price of their purchase, as well as the interest rate of the loan.

It is required to determine the optimal size of the sown area for each type of crop, as well as the amount of financial credit for a year so that the volume of agricultural production produced by the cooperative under the contract is fully fulfilled and the profit of the agricultural cooperative is maximum.

The mathematical model of the problem can be written as.

Find the maximum

$$D(x,z) = \sum_{k\in K}\{\sum_{j\in J_0} c_{j0} b_{kj} x_{kj} - (1+\alpha) z_k\} \quad (1)$$

under conditions

$$\sum_{j\in J_0} x_{kj} \le s_k, \quad k\in K, \quad (2)$$

$$\sum_{k\in K} b_{kj} x_{kj} = a_{j0}, \quad j \in J_0, \quad (3)$$

$$\sum_{j\in J_0} \left(c_{kj}^{-} + \sum_{r\in R} c_k^r a_{kj}^r\right) x_{kj} = z_k, \quad k\in K, \quad (4)$$

$$\sum_{k\in K} z_k \le Q, \quad (5)$$

$$x_{kj} \ge 0, \quad k\in K, \quad j\in J_0, \quad (6)$$

$$z_k \ge 0, \quad k\in K, \quad (7)$$

where $c_{kj}^{-} = c_{kj0} + c_{kj}$, $k\in K$, $j \in J_0$, $c_k^r = c^r + c_k^{-r}$, $k\in K$, $r\in R$,

$x=\{x_{kj}:\ j\in J_0,\ k\in K,\quad z=\{z_k:\ k\in K\}$.

Here j- is the index of the type of agricultural product, $j\in J_0 = \{1,2,\dots,n\}$;

J_0 - many indices of the type of agricultural products;

r - the index of the type of mineral fertilizers used by the farm for growing agricultural crops, r ∈ R={1,2,…,R^{-}};

R- many types of mineral fertilizers used by the farm;

k- cooperative farms index, $k \in K = \{1,2,\dots,p\}$;

K- many indices of farms in the cooperative.

The known parameters are:

c_{j0}- the purchase price of a unit of weight for the j-th type of agricultural product under the contract, $j \in J_0$;

c_{kj0}- transportation costs for the transportation of crops from a unit of the sown area of agricultural crops of the j-th type from the k-th farm to the processing enterprise, $k \in K,\ \ j \in J_0$;

c_k^r- consumption for the purchase and transportation of a unit volume of r-type mineral fertilizers, where $c_k^r = c^r + c_k^{-r}$, $r \in R$, $k \in K$;

c_k^{-r}- transportation costs for the transportation of a unit of weight of the r-type of mineral fertilizers in the k-th farm, c^r - purchase price;

s_k - the size of the sown area of the k-th farm, $k \in K$;

a_{j0} - the volume of agricultural products of the j-th type produced by the cooperative for the processing enterprise according to the contract, $j \in J_0$;

a_{kj}^r- the consumption rate of the r-type of mineral fertilizer per unit of sown area for the j-type of crop in the k-th farm, $r \in R,\ \ j \in J_0,\ \ k \in K$;

b_{kj}- the yield of the j-th type of agricultural crop per unit of sown area in the k-th farm of the cooperative, $j \in J_0,\ \ k \in K$;

c_{kj} - the costs of growing the j-th type of agricultural crop per unit of sown area in the k-th farm of the cooperative, $j \in J_0,\ \ k \in K$;

α - loan interest rate;

Q - the maximum allowable amount of a loan.

Variables sought:

x_{kj} - the size of the sown area for the j-th type of crop in the k-th farm, $j \in J_0, k \in K$;

z_k- the size of the loan received by the k-th farm at an interest rate for growing crops.

Objective function (1) determines the maximum net income of the farms of the cooperative;

Constraint (2) requires that the total sown area for each type of crop on the farm should not exceed the available sown area;

Equality (3) determines that the volume of products of each type produced by the cooperative must correspond to the planned volume under the contract;

Equality (4) determines the size of the financial credit received by the farm;

Constraint (5) requires that the total amount of financial credit received by farms should not exceed the maximum possibility of issuing;

Constraint (6), (7) requires the non-negativity of the variables.

Model (1) - (7) can be represented in the form of Table 1.

Table 1

x_{11}	x_{12}	…	x_{1n}	x_{21}	x_{22}	…	x_{2n}	…	x_{p1}	x_{p2}	…	x_{p1}	z_1	z_2	…	z_p		
1	1	…	1														$\leq$	S_1
				1	1	…	1										$\leq$	S_1
		…				…		…						…	…		…	…
									1	1	…	1					$\leq$	S_p
b_{11}				b_{21}					b_{p1}								=	a_{10}
	b_{12}				b_{22}					b_{p2}							=	a_{20}
		…				…					…			…	…		…	…
			b_{1n}				b_{2n}	…				b_{pn}					=	a_{n0}
q_{11}	q_{12}	…	q_{1n}										-1				=	0
				q_{21}	q_{22}	…	q_{2n}							-1			=	0
		…				…		…						…	…		…	…
								…	q_{p1}	q_{p2}	…	q_{pn}		…	…	-1	=	0
d_{11}	d_{12}	…	d_{1n}	d_{21}	d_{22}	…	d_{2n}	…	d_{p1}	d_{p2}	…	d_{p1}	-(1+α)	-(1+α)	…	-(1+α)	→	max

where

$$d_{kj} = c_{j0}b_{kj} - c_{kj}^{-},\ k \in K,\ j \in J_0,\ q_{kj} = c_{kj}^{-} + \sum_{r \in R} c_k^r a_{kj}^r,\ k \in K,\ j \in J_{0},$$

$$c_{kj}^{-} = c_{kj0} + c_{kj}, \quad k \in K, \quad j \in J_0.$$

Algorithm for solving the problem. We start the calculations by determining the values of the parameters d_{kj}, q_{kj}, c_{kj}^{-}, $k \in K$, $j \in J_0$. Using known data S_k, $k \in K$, a_{j0}, $j \in J_0$, b_{kj}, $k \in K$, $j \in J_0$, we construct a numerical model of the problem according to (1) - (7). Next, we will use the PPP LP developed in the EMM laboratory [1] and solve the problem. The calculation algorithm ends.

From the solution of the formulated numerical model (1) - (7), we determine the optimal plan for the distribution of sown areas for each type of crop, the amount of financial credit received by each farm and the amount of the cooperative's net income when fulfilling the plan of the obligation.

Let's give a numerical example. Let the agricultural cooperative in the region consist of 3 farms with sown areas (ga) $S_k = (S_1, S_2, S_3) = (2000, 1500, 1500)$.

An agreement "on the production of agricultural products" was concluded between the processing enterprise and the agricultural cooperative of the region for the current year for three types of agricultural products in the following amount: for the first type of agricultural products 10000.0 tons, for the second type of agricultural products 10000.0 tons, for the third type of agricultural products 10000.0 tons and set the wholesale purchase price of the processing company per unit of weight for each type of product продукции $c_{j0} = (c_{10}, c_{20}, c_{30})$ = (15 som/kg, 12 som/kg, 10 som/kg).

Known: yield for each type of crop of each farm, Table 2.

Table 2.

Name Farms	Name of crops		
	Agricultural № 1	Agricultural № 2	Agricultural № 3
Farms № 1	10 tons / ga	15 tons / ga	6 tons / ga
Farms № 2	8 tons / ga	12 tons / ga	6 tons / ga
Farms № 3	12 tons / ga	15 tons / ga	10 tons / ga

- the costs of mineral fertilizers and resources, as well as their purchase prices (Table 2), used for a unit of sown area;
- costs of cultivation, the rate (costs) of mineral fertilizers and resources used per unit of sown area, as well as their purchase prices, Table 2;
-the interest rate of the loan α = 6% per annum;
- the maximum amount of a loan is Q = 450000000.0 soms.

It is required to determine the optimal size of the sown area for each type of crop of each farm and the corresponding size of the financial loan so that the volume of the cooperative's production according to the contract is fulfilled, while the profit is maximized.

Based on the above data, the mathematical model of the problem according to (1) - (7) is presented in the following form.

It is required to maximize the function

$$D(x,z) = 150000x_{11} + 180000x_{12} + 60000x_{13} + 120000x_{21} + \\ +144000x_{22} + 60000x_{23} + 120000x_{31} + 180000x_{32} + \\ + 100000x_{33} - 1{,}06(z_1 + z_2 + z_3) \qquad (8)$$

under conditions

$$\sum_{j=1}^{3} x_{1j} \le 2000, \quad \sum_{j=1}^{3} x_{2j} \le 1500, \quad \sum_{j=1}^{3} x_{3j} \le 1500, \qquad (9)$$

$$10000x_{11} + 8000x_{21} + 12000x_{31} = 10000000,$$
$$15000x_{12} + 12000x_{22} + 15000x_{32} = 10000000,$$
$$6000x_{13} + 6000x_{23} + 10000x_{33} = 10000000, \qquad (10)$$

$$43150x_{11} + 45150x_{12} + 12300x_{31} = z_1,$$
$$47150x_{21} + 52150x_{22} + 17300x_{23} = z_2,$$
$$45150x_{31} + 47150x_{32} + 17300x_{33} = z_3, \qquad (11)$$

$$\sum_{k=1}^{3} z_k \le 450000000, \qquad (12)$$

$$x_{kj} \ge 0, \quad k=1,2,3, \quad j=1,2,3, \qquad (13)$$

$$z_k \ge 0, \quad k=1,2,3. \qquad (14)$$

where $x=\{x_{kj}:\ k = 1,2,3,\ \ j = 1,2,3,\ \ z = \{z_k:\ k\mathrm{k} = 1,2,3\}$.

Mathematical model (8) - (14) can be presented in the form of table 3.

Table 3.

x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	z_1	z_2	z_3		
1	1	1										≤	2000
			1	1	1							≤	1500
						1	1	1				≤	1500
10000			8000			12000						=	10000000
	15000			12000			15000					=	10000000
		6000			6000			12000				=	10000000
43150	45150	12300							-1			=	0
			47150	52150	17300					-1		=	0
						45150	47150	17300			-1	=	0
									1	1	1	≤	450000000
150000	150000	60000	120000	144000	60000	120000	150000	100000	-1,06	-1,06	-1,06	→	max

From the solution of problem (8)-(14), we will determine the optimal plan for the distribution of the sown areas of each farm for each type of crop No.1, No. 2 and No.3 x={$x_{11} = 1000.0$, $x_{12} = 666.7$, $x_{13} = 1000.0$, $z_1 = 73250000.0$, $z_2 = 0$, $z_3 = 173000000.0$}, the volume of a financial loan for a year at α = 6%, as well as the profit of the cooperative, i.e. Dx, z = 274 million soms.

From this optimal solution, it follows that in order to fulfill the plan of the obligation, the cooperative should allocate 1000.0 hectares from the first farm for the first type of crop, 666.7 ha for the second type of agricultural crop, and 1000.0 hectares of sown area for the third type of crop from the third farm. At the same time, the cooperative receives 90550000.0 soms of a loan at α = 6% per annum. Of these amounts, 73250000 soms corresponds to the first farm, and 17300000.0 soms to the second farm of the cooperative. For the fulfillment of the plan of the obligation of the cooperative, the participation of the second farm was ineffective. With this plan, the maximum profit of the cooperative is 274.0 million soms.

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MSC 49M37

DETERMINATION OF THE OPTIMAL COMPOSITION OF PRODUCTIVE ANIMALS AND THE SIZE OF THE RECEIVED LOAN BY THE CRITERION OF THE MAXIMUM OF NET INCOME

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In this article, a mathematical model and an algorithm for solving the problem of choosing the optimal breed of animals in the farm have been developed, allowing to produce the planned volume of production and the size of the financial loan received, as well as the volumes of vegetable products sold, at which the maximum net income would be achieved. The performance of the model is shown with a numerical example.

Key words: mathematical model, algorithm, volume, credit, vegetable growing, products, net income.

Бул макалада өндүрүштүн пландалган көлөмүн жана алынган каржылык насыянын көлөмүн, ошондой эле көлөмүн чыгарууга мүмкүндүк берген чарбада жаныбарлардын оптималдуу тукумун тандоо маселесин чечүүнүн алгоритми иштелип чыккан. сатылган жашылча-жемиштер, бул учурда максималдуу таза киреше алынмак. Моделдин көрсөткүчтөрү сандык мисалда көрсөтүлгөн.

Урунттуу сөздөр: математикалык модель, алгоритм, көлөм, насыя, жашылча өстүрүү, продукция, таза киреше.

В этой статье разработана математическая модель и алгоритм решения задачи выбора оптимальной породы животных в хозяйстве позволяющих производить запланированный объем продукции и размер получаемого финансового кредита, а также объемы реализуемой продукции овощеводства, при котором достигало бы максимальный чистый доход. Работоспособность модели показана на числовом примере.

Ключевые слова: математическая модель, алгоритм, объем, кредит, овощеводство, продукция, чистый доход.

Formulation of the problem. To fulfill the indicative plan for the production of livestock products in the amount of a^h, $h \in H$, the farm has the opportunity to obtain a financial loan $z \le D$ at a rate of interest per annum for years (where D is the maximum loan amount) and to update the composition of animals with more productive breeds.

It is assumed that for each type of animal breed, productivity and the corresponding daily feeding ration are known, as well as a one-time consumption per animal in the amount $q_l^h, l \in L, h \in H$ depending on the breed and the type of product obtained.

In addition, the farm has irrigated areas of s ha. Of these, the farm uses x hectares for growing agricultural crops for animal feed, and (s-x) hectares for vegetable crops sold on the market.

Also known: the yield and costs of growing crops for animal feed, and the costs of growing vegetables sold on the market.

It is required to determine the optimal composition of animals $y_l^h, l \in L, h \in H$, allowing to ensure the production of products in the planned volume and the size of the financial loan z, as well as the volumes of vegetable products sold $v_j,\ j \in J_1$, delivering the maximum net income to the farm.

The mathematical model of the problem can be presented in the form.

Find the maximum

$$L(x,v) = \sum_{h \in H} c^h x^h + \sum_{j \in J_1} \bar{c}_j v_j - \sum_{j \in J} c_j x_j - \sum_{h \in H} \sum_{l \in L} c_l^h y_l^h - (1+\alpha\lambda) z / \lambda \qquad (1)$$

при условиях

$$\sum_{j \in J} x_j = s, \qquad (2)$$

$$a_j x_j = \sum_{h \in H} \sum_{l \in L} \alpha_{lj}^h y_l^h, \quad j \in J_0, \qquad (3)$$

$$a_j x_j = v_j, \quad j \in J_1, \qquad (4)$$

$$\sum_{l \in L} \theta_l^h y_l^h = x^h \geq a^h, \quad h \in H, \qquad (5)$$

$$\sum_{l \in L} \sum_{h \in H} \varepsilon_l^h y_l^h = z \leq D, \qquad (6)$$

$$x_j \geq 0, \quad v_j \geq 0, \ j \in J_0 \cup J_1, \qquad (7)$$

$$x^h \geq 0, \quad h \in H, \qquad (8)$$

$$y_l^h \geq 0, \quad l \in L, \quad h \in H \text{ - integer}, \qquad (9)$$

$$z \geq 0, \qquad (10)$$

where x={ $x^h \geq 0,\ h \in H$}, v={ $v_j \geq 0,\ j \in J_1$}, $J = J_0 \cup J_1, \quad J_0 \cap J_1 \neq 0$,

J – many types of crop and vegetable products produced on the farm;

J_0 – many types of crop production for feed, $j \in J_0$;

J_1 - many types of vegetable products, $j \in J_1$;

h – index of the type of livestock products produced on the farm, $h \in H$;

H – many types of livestock products, $H=\{1,2,\ldots, \bar{H}\}$;

l – index of the species of animal breeds, $l \in L$;

L- many types of animal breeds, $L=\{1,2,\ldots, \bar{L}\}$;

j – index of the type of crop and vegetable production produced on the farm $j \in J$;

s – the size of the cultivated area on the farm;

a_j – the yield of the j -th type of crop on the farm, $j \in J$;

α_{lj}^{h} - the annual need for the j-th type of crop production, in accordance with the daily feeding ration for one animal of the l-th breed in the production of the h-type of products, where

$$\alpha_{jl}^{h} = \beta_{jl}^{h} \gamma_{jl}^{h}, \quad j \in J_0, \quad l \in L, \quad h \in H; \qquad (11)$$

β_{jl}^{h} - the share of the j-th crop production in the daily ration per one animal of the l-th breed on the farm for the production of the h-th type of product, $j \in J_0$, $l \in L$, $h \in H$;

γ_{jl}^{h} - the number of days in the ration of feeding the j-th type of crop production for the l-th breed of animal in the production of the h-th type of product, $j \in J_0$, $l \in L$, $h \in H$;

θ_{l}^{h} - the volume of production of the h -th type received by the farm from one animal of the l-th breed, $l \in L$, $h \in H$;

a^h – indicative production plan of the h -th type of livestock products produced by the farm, $h \in H$;

$\bar{c}_j$ - the realizable price of the j-th type of vegetable production, $j \in J_1$;

c^h- realizable price per unit volume of the h -th type of livestock product, $h \in H$;

c_j – costs per unit of sown area for the j-th type of crop, $j \in J_0$;

c_l^h - annual consumption per one animal of the l-th breed in the production of the h-th type of livestock products, $h \in H, l \in L$;

ε_l^h - one-time consumption per one animal of the l-th breed in the production of the h-th type of product, $l \in L$, $h \in H$; λ - payback period of the loan; α - interest rate of the loan.

Variables sought:

x_j – the size of the sown area allotted for the j-th type of culture, $j \in J_0$;

y_l^h - the number of animals of the l-th breed on the farm for the production of the h-th type of product, $h \in H, l \in L;$

x^h - the volume of production of animal husbandry of the h -th type, $h \in H;$

z – the size of the loan received by the farm.

Objective function (1) determines the maximum net consumption of the farm from the production and sale of livestock and vegetable products;

Equality (2) determines that the total size of the sown area allotted for fodder crops and vegetable crops should be equal to the size of the sown area available on the farm;

Equality (3) requires that the volume of production for each species, produced for feed, must be equal to the volume of needs of the animals kept on the farm;

Equality (4) determines the volume of products sold for each type of vegetable growing;

Condition (5) requires that the volume of livestock products produced for each species must be no less than the volume of the indicative plan for the production of these products;

Equality (6) determines the size of the loan received by the household, which should not exceed its maximum size of issue;

Constraint (7, 8, 10) does not require the variables to be negative, but condition (9) requires that the value of the variables must be an integer.

Solution method. Calculations begin with determining the values α_{jl}^{h}, $j \in J_0$, $l \in L$, $h \in H$ according to equality $\alpha_{jl}^{h} = \beta_{jl}^{h}\gamma_{jl}^{h}, \quad j \in J_0, \; l \in L, \; h \in H$.

Further, using the known data a_j, c_j, $j \in J_0$, $\bar{c}_j$, $j \in J_1$, θ_l^h, c_l^h, c^h, ε_l^h, a^h, $h \in H$, $l \in L$ and s, D, α, λ a numerical model is formulated in accordance with (1)-(10) and the problem is solved by the standard method. The solution method ends.

From the solution of the problem, the quantitative composition of animals on the farm is determined $y=\{ y_l^h, \quad h \in H, \; l \in L\}$ ensuring the volume of production of livestock products according to a given indicative plan, the size of sown areas for each type of crop for feed $x=\{ x_j, \; j \in J_0\}$ and vegetable growing sold to the market $\bar{x}=\{ x_j, \; j \in J_1\}$, as well as the size of the financial loan received by the farm that provides the maximum net income. Let's check the performance of the mathematical model of the problem using a numerical example.

Example. Let the farm have a sown area of S = 600 hectares and planned to produce livestock products: 125 tons of milk and 25 tons of meat. For this purpose, the farm needs a financial loan with a maximum amount of up to 50 million soms for 10 years at 6% per annum.

The farm is faced with the selection of the best breeds of animals from the following two known breeds for the production of milk and meat with an appropriate daily ration.

Known for the dairy direction: the first type of breed is dairy cows with a milk yield of 3600 kg (Table 1), i.e. ($\theta_{l=1}^{h=1}$ = 3600 кг);

the second type of breed is dairy cows with a milk yield of 4500 kg (Table 2), i.e. ($\theta_{l=2}^{h=1}$ = 4500 кг);

- In the direction of meat: the first type of breed is bulls (heifers) with a live weight of 300 kg (Table 3), i.e. ($\theta_{l=1}^{h=2}$ = 300 кг);

- the second type of breed - bulls (heifers) with a live weight of 450 kg (Table 4), i.e. ($\theta_{l=2}^{h=2}$ = 450 кг).

Table 1

The ration of feeding dairy cows of the first type of breed with a milk yield of 3600 kg of milk

	Feed name		Daily ration kg (1 head)		Fodder units	Total units	Total for 1 head qty	Number of days
1	Lucerne (hay)		4		0,5	2,0	720,0	180
2	straw	wheat	3	1	0,2	0,6	180.0	180
		barley		2			360.0	
3	haylage		6		0,3	1,8	1080,0	180
4	The conc. feed	wheat	2,4	0,3	1	2,4	109,5	365
		barley		1,5			547,5	
		corn		0,6			219,0	
5	Silo (corn)		10		0,3	3,0	1800,0	180
6	Mineral feed		0,010		-	-	-	365
7	salt		0,030		-	-	-	365
8	Grazing feed		40		-	-	7200,0	180
	Green feed							
	Total		-		-	9,8	-	-

Table 2

The ration of feeding dairy milk cows of the second breed with milk yield 4500 kg of milk

	The name of feed		**Daily ration (1 animal)**		**Days Qty**	**Per year (1 animal)**
1.	Lucerne (hay)		10 kg		180	1800 kg
2.	Straw	Wheat	3 kg	1 kg	180	180 kg
		Barley		2 kg		360 kg
3.	Hay		8 kg		180	1440 kg
4.	The concentrate on of the feed	Barley	3 kg	2 kg	365	730 kg
		Wheat		0,5 kg		182,5 kg
		Grain (corn)		0,5 kg		182,5 kg
5.	Siloge (corn)		12 kg		180	2160 kg
6.	Mineral. feed		-		365	3,6 kg
7.	salt		-		365	10,8 kg
8.	Grain (corn)		50 kg		180	9000 kg
	Green feed					

Table 3

Daily ration for feeding cattle (bulls, heifers) with live weight 300 kg

	Name of feed		**Daily ration kg (1 nimal),**		**Feed unit**	**General unit**	**Total per 1 animal**	**Days qty**
1.	Lucerne (hay)		3		0,5	1,5	540,0	180
2.	Straw	Wheat	2	0,5	0,2	0,4	90,0	180
		Barley		1,5			270,0	
3.	Hay		6		0,3	1,8	1080,0	180
4.	Conc. feed	Barley	1,5	0,5	1	1,5	182,5	365
		Wheat		0,5			182,5	
		Grain		0,5			182,5	

5.	Siloge (corn)	5,0	0,3	1,5	900,0	180
6.	Mineral feed	0,010	-	-	-	365
7.	Salt	0,030	-	-	-	365
8.	Grazing feed Green feed	30	-	-	5400,0	180
	Total	-	-	6,7	-	-

Table 4

Daily ration for feeding cattle (bulls, heifers) for meat with a live weight of 450 kg

	Name of feed		Daily ration (1 head)		Days qty	Per year (1 head)
1.	Lucerne (hay)		5 kg		365	1825 kg
2.	Straw (Barley)		1 kg		365	365 kg
3.	Hay		10 kg		180	1800 kg
4.	The concentrate on of the feed	Barley	3 kg	1 kg	365	365 kg
		Grain (corn)		1 kg		365 kg
		Wheat		1 kg		365 kg
5.	Siloge (corn)		10 kg		180	1800 kg
6.	Mineral feed		-		365	-
7.	Salt		-		365	-
8.	Grazing feed Green feed		30 kg		180	5400 kg

In addition, the following are known: - the yield of agricultural crops for fodder, for vegetable crops and the costs of growing them, as well as the selling price (Table 5).

Table 5

	wheat	barley	alfalfa hay	haylage	Green feed	Silo (corn)	Grain (corn)	onion	potatoes
	1	2	3	4	5	6	7	8	9
a_j *(kg)*	2070.0	1962.2	2380.0	6281.0	5730.0	12340.0	20280.0	4500.0	7000.0
c_j *(soms)*	2279.0	1096.0	2618.0	5071.0	9225.0	13574.0	7743.0	25000	20000
(soms)	14.0	12.0	5.0	-	-	-	8.0	15.0	20.0

- expenses for the maintenance of one animal of each breed in the production of milk and meat, respectively $c_{l=1}^{h=1}$ = 50040.0 soms, $c_{l=2}^{h=1}$ = 55080.0 soms, $c_{l=1}^{h=2}$ = 18030.0 soms, $c_{l=2}^{h=2}$ = 25020.0 soms and the selling price of milk $c^{h=1}$ = 25 soms / kg and meat $c^{h=2}$ = 130 soms / kg.

- additional costs per animal depending on the breed, i.e.

$$\varepsilon_l^h = (\varepsilon_1^1,\ \ \varepsilon_2^1,\ \varepsilon_1^2,\ \ \varepsilon_2^2) = (45000.0,\ 50000.0,\ 15000.0,\ 15000.0);$$

- loan term λ = 10 years;

- loan interest rate α = 6% per annum.

It is required to determine the optimal composition of animals for dairy and meat production, allowing to ensure the planned volume of milk and meat production and the size of the loan, as well as the plan for the production of vegetable products so that the net income of the farm would be maximum.

For the mathematical formalization of the problem, we determine the annual feed requirement α_{lj}^{h} for one animal of each breed in the production of milk and meat products, $j \in J_0$, $l \in L$, $h \in H$.

Using the daily feeding ration, we will determine the annual need for each type of agricultural product included in the feed for one dairy cow with a milk yield of 3600 kg of milk and one dairy cow with a milk yield of 4500 kg. We will also determine the annual need for each type of agricultural products for fodder for one cow of the first and second types of breed for meat (see Table 6).

Table 6

Annual feed requirement per animal, depending from breed and productivity

Name stern	Feed requirement per dairy cow		Feed requirement per cow for meat	
	1 breed with 3600 kg milk yield	2 breed with 3600 kg milk yield	1 breed with a live weight of 300 kg	2 species of live weight breed 450 kg
1.wheat	289,5	362,5	272,5	365,0
2.barley	907,5	1090,0	452,5	730,0
3. perennial grass 3.1. hay (alfalfa)	720,0	1800,0	540,0	1825,0
3.2. haylage	1080,0	1440,0	1080,0	1800,0
3.3. Green feed	7200,0	9000,0	5400,0	5400,0
4. Corn 4.1. silage	1800,0	2160,0	900,0	1800,0
4.2.grain	219,0	182,5	182,5	365,0

We formulate a numerical model of the problem.

Find a minimum

$$L(x,v) = 25.0\,x^1 + 130.0\,x^2 + 15.0\,v_1 + 20.0\,v_2 - \{2279.0x_1 + 1096.0x_2 + 2618.0x_3 +$$

$$+5071.0x_4 + 9225.0x_5 + 13574.0x_6 + 7743.0x_7 + 25000.0x_8 + 20000.0x_9 + 50040.0\,y_1^1 +$$

$$+55080.0\,y_2^1 + 18030.0\,y_1^2 + 25020.0\,y_2^2 + 0{,}16z \qquad (12)$$

with conditions of

$$\sum_{j\in J} x_j = 600, \quad J = J_0 \cup J_1, \quad J_0 = \{1,2,...,7\}, \quad J_1 = \{8,9\}, \qquad (13)$$

$$2070{,}0x_1 = 289{,}5\, y_1^1 + 362{,}5\, y_2^1 + 272{,}5\, y_1^2 + 365{,}0\, y_2^2,$$

$$1962{,}0x_2 = 907{,}5\, y_1^1 + 1090{,}0\, y_2^1 + 452{,}5\, y_1^2 + 730{,}0\, y_2^2,$$

$$2380{,}0x_3 = 720{,}0\, y_1^1 + 1800{,}0\, y_2^1 + 540{,}0\, y_1^2 + 1825{,}0\, y_2^2,$$

$$6281{,}0x_4 = 1080{,}0\, y_1^1 + 1440{,}0\, y_2^1 + 1080{,}0\, y_1^2 + 1800{,}0\, y_2^2,$$

$$5730{,}0x_5 = 7200{,}0\, y_1^1 + 9000{,}0\, y_2^1 + 5400{,}0\, y_1^2 + 5400{,}0\, y_2^2,$$

$$12340{,}0x_6 = 1800{,}0\, y_1^1 + 2160{,}0\, y_2^1 + 900{,}0\, y_1^2 + 1800{,}0\, y_2^2,$$

$$20280{,}0x_7 = 219{,}0\, y_1^1 + 182{,}5\, y_2^1 + 182{,}5\, y_1^2 + 265{,}0\, y_2^2, \qquad (14)$$

$$4500x_8 = v_1, \quad 7000.0x_9 = v_2, \qquad (15)$$

$$3600\, y_1^1 + 4500\, y_2^1 = x^1 \geq 125000,$$

$$300\, y_1^2 + 450\, y_2^2 = x^2 \geq 25000, \qquad (16)$$

$$45000{,}0\, y_1^1 + 50000{,}0\, y_2^1 + 15000{,}0\, y_1^2 + 15000{,}0\, y_2^2 = z \leq 50000000, \qquad (17)$$

$$x_j \geq 0, \quad j \in J_0, \quad v_j \geq 0, \quad j \in J_1, \qquad (18)$$

$$x^h \geq 0, \quad h \in H = \{1,2\}, \qquad (19)$$

$$y_l^h \geq 0, \quad l \in L = \{1,2\}, \quad h \in H \text{ - whole}, \qquad (20)$$

$$z \geq 0. \qquad (21)$$

Let us write down the numerical model of problem (12) - (21) in the form of a table. 7 and solve it by the method given in [1].

Table 7

Condition of problem (12)-(21) in tabular form

$x^{h=1}$	$x^{h=2}$	v_1	v_2	x_1	x_2	x_3	x_4	x_5	x_6	x_7
				1	1	1	1	1	1	1
				2070,0						
					1962,0					
						2380,0				
							6281,0			
								5730,0		
									12340,0	
										20280.0
		-1								
			-1							
-1										
	-1									
1										

	1									
25.0	130.0	15.0	20.0	-2279.0	-1096.0	-2618.0	-5071.0	-9225.0	-13574.0	-7743.0

Continuation of Table 7

x_8	x_9	y_1^1	y_2^1	y_1^2	y_2^2	z		
1	1						=	600
		-289,5	-362,5	-272,5	-365,0		=	0
		-907,5	-1090.0	-452.5	-730.0		=	0
		-720.0	-1800.0	-540.0	-1825.0		=	0
		-1080.0	-1440.0	-1080.0	-1800.0		=	0
		-7200,0	-9000.0	-5400.0	-5400.0		=	0
		-1800.0	-2160.0	-900.0	-1800.0		=	0
		-219.0	-182.5	-182.5	-265.0		=	0
4500.0							=	0
	7000.0						=	0
		3600.0	4500.0				=	0
				300.0	450.0		=	0
							≥	125000.0
							≥	25000.0
		45000.0	50000.0	15000.0	15000.0	-1	=	0
						1	≤	50000000
-25000.0	-20000.0	-50040.0	-55080.0	-18030.0	-25020.0	-0,16	→	max

We will obtain an optimal plan for the production of livestock products (milk - 126900 kg, meat -25200 kg), a plan for the distribution of sown areas for fodder crops x={ $x_1=16,0$; $x_2=35,7$; $x_3= 30,1$; $x_4=20,5$; $x_5= 123,5$; $x_6=11,3$; $x_7=1,1$}, vegetable crops $\bar{x} = \{x_8 = 0;\ x_9 = 361,9\}$, the composition of the animals kept in the dairy and meat farm y={ $y_1^1 = 34$; $y_2^1 = 1$; $y_1^2 = 84$; $y_2^2 = 0$ }, the size of the loan received by the household is z = 2840000.0 soms for ten years at 6% per annum.

From the optimal solution it follows that for the planned volume of production, i.e. milk in the amount of 126,900 kg, meat - 25,200 kg, the farm should contain 34 cows of the 1st breed in the dairy direction and in the amount of one cow of the 2nd breed, and in the meat direction it is enough to contain 84 cows of the 1st breed. Of the available 600 hectares of sown area, 238.1 hectares are used for agriculture for fodder, the remaining 361, 9 hectares for potatoes.

Taking into account all income and expenses, including the annual loan payment in the amount of 454400.0 soms, the net income of the farm is 44590892.0 soms.

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MSC 49M37

MODELING ECONOMIC DEVELOPMENT USING PRODUCTION FUNCTIONS

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The work is devoted to the study of the problems of modeling the domestic economy in the transition period. It is shown that the construction of production functions in a transition economy is in principle possible, but it is necessary to take into account that the use of a standard set of factors (funds and labor) does not allow us to obtain any acceptable results, since official statistics do not provide a market assessment of capital and the actual labor used.

An algorithm for constructing a two-factor production function in a three-sector economy is proposed. The developed algorithm for constructing the production function by sector was tested.

Keywords: production function, elasticity, three-sector model, transition economy.

Бул жумуш өткөөл мезгилдеги ата мекендик экономиканы моделдөө көйгөйлөрүн изилдөөгө арналган. Өткөөл экономика шарттарында өндүрүштүк функцияларды түзүү мүмкүн экени, бирок бул стандарттык факторлорду (фонддор жана эмгек) пайдалануу кандайдыр бир алгылыктуу натыйжаларын алууга мүмкүндүк бербейт экенин эске алуу зарыл, анткени расмий статистика рыноктук капиталдын жана реалдуу колдонулуучу эмгектин баасын бербейт.

Үч секторлуу экономикада эки фактордук өндүрүштүк функцияны куруунун алгоритми сунушталган. Секторлор боюнча өндүрүштүк функцияны куруунун иштелип чыккан алгоритми сыноодон өткөрүлдү.

Урунттуу сөздөр: Өндүрүштүк функция, ийкемдүүлүк, үч долбоордук модель, өткөөл мезгилдин экономикасы.

Работа посвящена исследованию проблем моделирования отечественной экономики в переходном периоде. Показано, что построение производственных функций в условиях переходной экономики в принципе возможно, но необходимо учитывать, что использование стандартного набора факторов (фонды и труд) не позволяют получить сколько-нибудь приемлемых результатов, поскольку данные официальной статистики не дают рыночной оценки капитала и реально используемого труда.

Предложен алгоритм построения двухфакторной производственной функции в трехсекторной экономике. Проведена апробация разработанного алгоритма построения производственной функции по секторам.

Ключевые слова: Производственная функция, эластичность, трехсекторная модель, экономика переходного периода.

1. Introduction. The most important component of the aggregated single-sector model of economic growth is the production function, which relates output to the volume of fixed assets, labor costs, and possibly other factors of production. The aggregated production function can be considered as a tool for both forecasting and retrospective analysis.

The purpose of this study is to study the problems of building production dependencies for the transition economy of the country. The task is to find out the possibility of using the apparatus of production functions for an adequate description of the processes in the domestic economy.

Solving these problems would allow us to conclude that it is possible to use the apparatus of production functions for forecasting in a transition economy.

The apparatus of production functions is quite well developed for developed market economies. At the same time, the conditions of the transition economy bring significant specifics to the problems associated with the construction of production functions. Thus, in a transition economy, it is problematic even to obtain any reliable data on the costs of factors of production - funds and labor. The fact is that in a transition economy, it is difficult, and often impossible, to give a market assessment of production assets.

2. Materials and methods of research. Attempts to construct an aggregated production function for the transition economy encounter serious difficulties due to the specifics of the transition economy as an object of research. There is a need for adequate consideration of this specificity at the level of the analysis methodology used.

The specifics of the transition economy are that the principles of its functioning may differ from the principles of the functioning of the market economy. The absence of effective mechanisms that bring the economy into balance can lead to long-term and significant deviations from the equilibrium.

In the transition economy, it is difficult to use data on the shares of capital and labor as direct (i.e., obtained on the basis of relevant statistical information, rather than through econometric calculations) estimates of factor elasticities. This narrows down the arsenal of adequate methods and makes it particularly difficult to analyze aggregate factor productivity.

All the problems considered lead to a deterioration in the quality of econometric estimates and force us to pay more attention to the preliminary analysis of the data.

The complexity of economic systems, for the description of the functioning of which neoclassical production functions of the CES-function type are used, does not always allow us to assert that the values of the elasticity of labor replacement by capital σ in the considered systems are constant, since this situation is not so common in the real conditions of the functioning of economic systems. Therefore, to model the economic processes of the transition period, we use the Cobb-Douglas production function. For practical modeling, we use a three-sector model.

First, the exogenous and initial endogenous parameters of the model are calculated. These include:

1) The coefficients of elasticity of the Cobb-Douglas functions, as well as the functions themselves, taking into account the coefficients of neutral technological progress.

2) Coefficients of direct material costs.

3) The initial shares of investment and labor resources are determined from the currently available time as parameters of the industry structure in investment resources and data on the labor supply by sector.

4) Initial and stationary values of the stock-to-weight ratio. If the initial values are known to us from the statistics of labor and investment resources in the sectors, the optimal values are on the basis previously found optimal distribution of labor and investment resources through so-called stationary equations of motion:

$$\frac{dk_i}{dt} = -\lambda_i k_i + \frac{s_i}{\theta_i}(1+\gamma_1)\theta_1 f_1(k_1),\ \ i = 0,1,2 \tag{1}$$

where k_i – the stock-weight ratio of the *i*-sector; $\lambda_i,\ s_i,\ \theta_i -$ the parameters of the equation.

Then, based on the study of the behavior of the conjugate variables, as well as at the expense of some additional ones, the total time of the transition process is determined, and the moment of the first switching over investment resources is also found.

To construct the Cobb-Douglas functions, we first need to numerically determine the exogenous components of our model. The calculations are based on available statistical data. First of all, you need to know the production functions of the three sectors of the economy, without which it makes no sense to perform numerical modeling. We will consider the year 2000 as the initial or basic year for constructing production functions (PF). Statistical data of the NSC KR were used in the calculation [1].

The production functions of the three-sector model of the economy have the form

$$F_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i}\ i = 0,1,2. \tag{2}$$

The calculation uses the assumption that the total factor productivity does not change over time. Therefore, the coefficient A_i – is clearly independent of time. Next, we denote by F_{0i}, K_{0i}, L_{0i} – the corresponding values of output, funds and labor of the *i*-th sector in the base year. Then the formula (2) can be represented as:

$$\frac{F_i}{F_{0i}} = \frac{A_i}{A_{0i}}\left(\frac{K_i}{K_{0i}}\right)^{\alpha_i}\left(\frac{L_i}{L_{0i}}\right)^{1-\alpha_i}, i = 0,1,2. \tag{3}$$

Hence, in particular, it turns out that the coefficient of neutral economic progress $\frac{A_i}{A_{0i}} = 1$. Indeed, in our assumptions, A_i does not depend on time.

In order to calculate the estimates α_i of the elasticity coefficients of the production functions α_i, we will build a series of index values based on the data obtained from the aggregation of statistical information of the NSC KR. We will denote the indices as the corresponding variable with a wave, i.e. they will take the form:

$$\tilde{F}_i = \frac{F_i(t)}{F_{0i}}, \qquad \tilde{K}_i = \frac{K_i(t)}{K_{0i}}, \qquad \tilde{L}_i = \frac{L_i(t)}{L_{0i}},$$

here t – is the time period (in our case, a year) for which the corresponding value is measured. Further assume the existence of statistical data in certain error - ξ_i random variable taking positive values, zero logarithm of the mathematical expectation and unit variance, i.e.:

$$M(\ln(\xi_i)) = 0, \qquad D(\ln(\xi_i)) = 1, \qquad i = 0,1,2. \tag{4}$$

Taking into account the introduced notation and the proposed assumptions, equality (3) is written as:

$$\tilde{F}_i = \tilde{K}_i^{\alpha_i}\tilde{L}_i^{1-\alpha_i} + \xi_i \; i = 0,1,2. \tag{5}$$

Since the indices $\tilde{F}_i(t), \tilde{K}_i(t), \tilde{L}_i(t)$ and the random variable ξ_i have positive values, we have the right to take the natural logarithm of both parts of equality (5). As a result, after simple transformations, we get the following expression:

$$ln\tilde{F}_i - ln\tilde{L}_i = \alpha_i\left(ln\tilde{K}_i - ln\tilde{L}_i\right) + \xi_i, \;\; i = 0,1,2, \tag{6}$$

The series of the indices $\tilde{F}_i(t), \tilde{K}_i(t), \tilde{L}_i(t)$ are determined from the available statistical data. So, based on the indices of the physical volume of investments in fixed assets and the index of funds, $\tilde{K}_i$ is obtained as the geometric mean, weighted by the index of investments and the index of fixed assets. For each sector, the weighting is performed with the parameters 0.95 and 0.05.

Using statistical data, the coefficients of the elasticities of the Cobb-Douglas production functions were calculated using regression models in relative terms, $f_i = A_i \cdot k_i^{\alpha_i}$, where $k_i = \frac{K_i}{L_i}$ is the ratio of capital investment to the available labor resources in the sector (if both series are in indices, then their ratio is also in indices).

Then, based on the data on the stock ratios of the sectors in absolute terms and the production capabilities of the sectors (output) in absolute terms, the coefficients of A_i – neutral technological progress were found.

We present the final result of both stages of identification of the Cobb-Douglas coefficients obtained on the basis of data analysis for the period 2000-2019:

as for the financial sector: $-X_0 = 1{,}727 \cdot K_0^{0{,}57} L_0^{0{,}43}$,

for the fund-creating sector $-X_1 = 0{,}48 \cdot K_1^{0{,}674} L_1^{0{,}326}$ (7)

consumer $-X_2 = 0{,}628 \cdot K_0^{0{,}67} L_0^{0{,}33}$.

Table 1. Main characteristics of the parameters of the production function

Критерии	Показатели		
	X	K	L
Промышленность и строительство			
R^2	0,96	0,82	0,76
F	43,21	62,32	9.87
t_1	–0,13	3,2	0,97
t_2	1,24	0,28	–
Сельское хозяйство и добывающая промышленность			
R^2	0,87	0,93	0,45
F	41,32	59,25	4,23
t_1	–0,13	3,2	0,97
t_2	1,24	0,23	–
Торговля и услуги			
R^2	0,78	0,92	0,98
F	37,42	40,32	7,51
t_1	–1,17	7,28	–1,43
t_2	1,64	0,93	2,01

Conclusion

The market mechanism in the transition period cannot provide a desirable scheme for the intersectoral distribution of society's resources, since it operates on the basis of profit - the only important factor in its behavior. The decision to prioritize investments in different sectors may not coincide with the optimal value required for overall socio-economic benefits or costs. Any deviation from this optimum is corrected by the state both through regulatory methods and through direct participation in the economic process.

The government should not ignore the existence of macroeconomic imbalances and in the future it is necessary to coordinate at the state level and implement a number of reforms in the field of managing economic imbalances. Measures should aim to identify economic imbalances at an early stage and take measures to address them in a timely manner.

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MSC 49M37

USING IMITATION MODEL IN BUSINESS

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If the time of sale of the goods is known, then the article shows what profit the businessman will receive for the specified time, if the goods are fully sold. It is considered that the time of sale of the goods is under the normal distribution. Here, the 2-program calculates the time of sale of a given quantity of goods, and the profit of a businessman, if the total volume of goods is known.

Key words: profit, random variables, normal distribution, businessman, average value.

Бул жумушта эгерде товар сатуу мөөнөтү белгилүү болсо, анда бизнесмен, товрды сатып бүткөндөн кийин канча пайда тапканын көрсөтөт. Кардалдардын товар сатып алуу убактысы нормалдык законго баш ийет деп эсептелинет. Ошондой эле эгерде сатыла турган товардын көлөмү белгилүү болсо анда, товар канча убакытта сатылып бүтө турганын билдирет жана андан түшкөн пайда канча болгонун көрсөтөт.

Урунттуу сөздөр: капыстан болгон чоңдук, нормалдык закон, орточо чоңдук, соодагер, тапкан пайда.

Работа показывает, если известно время реализации товара, то бизнесмен получает какой прибыль за укзанное время если товар полностью реализуется.. При этом считается, что время продажи товара подчиняется к нормальному закону. Здесь же 2-программа подсчытывает если известно общий объем товара, то за какое время понностью реализуется данное количество товара, .и бизнесмен получает какой прибыль.

Ключевые слова: прибыль, случайные величины, нормальный закон распределения, бизнесмен, среднее значение.

Today, trade in Kyrgyzstan is one of the developing sectors of the country's economy and covers all types of activities that are directly related to the purchase and sale, exchange of goods for money, money for goods or goods for goods. There are many business entities involved in the trade industry. Officially, commercial entrepreneurship is carried out through - shops, stock exchanges, markets, exhibitions - sales, auctions, trading houses, trading bases. But trade in Kyrgyzstan is carried out mainly in the sphere of small business by economic entities. The basis of commodity-money transactions of purchase - sale or exchange

The shopping centers (malls) and traders buy consumer goods in the market for various purposes and sell them to the customers in order to make a certain profit. This is a normal phenomenon in a market economy.

For shopping centers the profit is usually calculated after the end of the sale using, an automated AIS system.

The purpose of this work is to find out in advance which goods the owner (street vendor or the owner of the shopping center) should buy to make the maximum profit. The article does not include the factors affecting the mark-ups of the goods sold, such as transportation costs, the cost of manufacturing them, and so on. On the other hand, we assume that the goods are fully sold for a certain period and the purchasing time of a certain type of goods is under the normal distribution $(i = \overline{1,n})$, where n is the name of the goods sold.

Problem 1. The time T of the sale of this type of product is known and the amount of the product K sold during this time is unknown. First, we start to solve this problem.

A businessman buys ice cream in bulk at m_1 soms per item and sells it for m_2 soms. The customers come to him at random $\Delta t_i,\ i = \overline{1,n}$ time, get in the line and buy one ice cream. We believe that the buyer can make another additional purchase k_i (0, if there is no additional purchase), and also within the specified time the goods are fully sold. A businessman works T hours a day. The arrival times of buyers and additional amount of purchase are random numbers that follow the normal distribution.

Question. If the above conditions are satisfied, determine the businessman's profit in T hours and find the volume of purchased ice cream at the beginning of the day for its sale.

Suppose after the start of work during the time Δt_1 = {4-10} the 1st customer comes and buys additionally k_1 = {0-2} ice cream. The next customer comes after the first customer in Δt_2 time, etc. businessman works until condition (1) is satisfied

$$\sum_{i=1}^{n} \Delta t_i = T\ h \qquad (1)$$

If condition (1) is satisfied, it is easy to calculate the profit and the morning volume of buying ice cream for sale:

n is the number of buyers in T hours;

$\sum_{i=1}^{n} k_i$ - the sum of additionally purchased ice cream.

The volume of the morning purchase of ice cream for sale is

$$(\text{n} + \sum_{i=1}^{n} k_i). \qquad (2)$$

The profit is:

$$(\text{n} + \sum_{i=1}^{n} k_i) * (m_2 - m_1)\ soms. \qquad (3)$$

where m_1 is the purchasing price of an ice cream unit, m_2 is the selling price of a unit of ice cream.

To implement formulas (1-3), we have compiled a program on *PascalABC.net*.

<u>Numerical example 1.</u>

Input data (parameters: T, m_2*,* m_1*,* t_1*,* t_2*,* Δt_i**,** k_1*).*

Let: T = 8 hours or 480 minutes;

$m_2 = 22;\ m_1 = 16;$

$t_1 = 4; \quad t_2 = 10;\ \Delta t_i = \{4 \div 10\};\ i = \overline{1, n};$

$k_1 = \{0 \div 2\}.$

Output.

After entering the data, the program gave the following output:

The number of ice cream purchased by customers is 127;

The profit is 762 soms.

Note: At the same start of the program with the same data, the program may produce the different results, because Δt_i and k_1 are the random numbers that are under the normal distribution. To get the average results, the program should be run several times with the same data. You can change the values of the input parameters.

Pascal program 1

```
// Бизнес
var a,b,i: integer;
 var k1,k2,p, p1: integer;
  begin
    a:=16;b:= 20;
    writeln('покупка  a=',a);
 writeln('продажа  b=',b);
   k1:=0; k2:=0;
    while k1<=480 do
    begin
        p := Random(3,7);
              i:=i+1;
               k1:=k1+p;
           p1:=Random(0,3);
             k2:=k2+p1;
            writeln(i:5, p:5, k1:5, p1:5, k2:5);
                    //  i:=p+i;
       end;
       i:=i+k2;
       writeln('Прибыль=',i*(b-a));
  writeln('Количество покупки : ',i);
     end.
```

In this example, if the selling time T (8 hours) of the given product (ice cream) is known, then the businessman should buy K = 127 pieces in the morning. ice cream.

Problem 2. If the volume of a given type of product K is known, then find the time T when amount of ice-cream is fully realized

$$\sum_{i=1}^{n} \Delta t_i = T\ h.$$

In order to solve the problem 2, Pascal program 1 can be easily transformed. In this case, we replace T by K, but other parameters remain the same. In this case, program 1 will be written as follows.

Pascal program 2

```
// Бизнес
var a,b,i: integer;
 var k1,k2,p, p1: integer;

  begin
    a:=16;b:= 20;
    writeln('покупка  a=',a);
 writeln('продажа  b=',b);
   k1:=0; k2:=0;
    while k1<=100 do
    begin
        p := Random(3,7);
              i:=i+1;
               k1:=k1+p;
           p1:=Random(0,3);
             k2:=k2+p1;
           writeln(i:5, p:5, k1:5, p1:5, k2:5);
                    //  i:=p+i;
       end;
       i:=i+k2;
       writeln('Прибыль=',i*(b-a));
  writeln('Затраченное время на продажу : ',i);
     end.
```

If you run the program when cost a = 16 and selling price b = 20 soms, then K = 100 pcs. The goods are sold in 49 minutes. The profit will be 196 soms.

A businessman can sell more than one type of goods, but the number of types is known. If a businessman has several types of the goods, we can modify program for solving a new problem.

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CONTENS

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