MSC 54E15

ON WEAKLY SEQUENTIALLY COMPLETE SPACES

Borubaev A.A. [1], Kanetova D.E. [2], Abdurasulova B.S. [3]
[1]*Institute of Mathematics of NAS of KR*
[2]*Scientific Research Medical Social Institute*
[3]*KNU named after J. Balasagun*

The completeness of uniform spaces is an important part of the uniform topology. In the theory of uniform spaces, there are various types of completeness of uniform spaces. For example, completeness, sequentially complete, uniformly Cech completeness etc. In this paper, we study weakly sequentially complete and weakly sequentially Diedonne complete spaces. In particular, a criterion for sequentially Diedonne complete spaces is established.

Keywords: maximal Cauchy filter, weakly sequentially complete spaces, weakly sequentially Diedonne complete spaces.

Толуктук түшүнүк бир калыптуу топологиянын маанилүү бөлүмүн түзөт. Бир калыптуу мейкиндиктер теориясында бир калыптуу мейкиндиктердин толуктугунун түрдүү типтери бар. Мисалы, толуктук, секвенциалдуу толуктук, бир калыптуу Чех боюнча толуктук ж.б. Бул макалада күчсүз секвенциалдуу толуктук жана күчсүз секвенциалдуу Дьедонне боюнча толуктук мейкндиктер изилденет. Күчсүз секвенциалдуу Дьедонне боюнча толуктук мейкндиктин критерийи тургузулган.

Урунттуу сөздөр: Кошинин максималдуу фильтри, күчсүз секвенциалдуу толук мейкиндик, күчсүз секвенциалдуу Дьедонне боюнча толуктук мейкиндик.

Полнота равномерных пространств составляет важную часть равномерной топологии. В теории равномерных пространств существуют различные типы полноты равномерных пространств. Например, полнота, секвенциальная полнота, равномерная полнота по Чеху пространств и.т.д. В настоящей статье изучается слабо секвенциально полные и слабо секвенциально полные по Дъедонне пространства. В частности, установлен критерий слабо секвенциально полных по Дъедонне пространств.

Ключевые слова: максимальный фильтр Коши, слабо секвенциально полные пространства, слабо секвенциально полные по Дъедонне пространства.

A uniform space (X,U) is called weakly complete if every maximal Cauchy filter converges in it [6].

Any complete uniformly space is weakly complete, and the converse is not true in general.

A uniform space (X,U) is called weakly sequentially complete if every maximal Cauchy filter with a countable base converges in it [6].

A mapping $f:X\to Y$ from a topological space X into a topological space Y is called perfect if f is a closed mapping and $f^{-1}y$ is compact for any $y\in Y$ [5], [8].

A mapping $f:(X,U)\to(Y,V)$ from a uniformly space (X,U) to a uniformly space (Y,V) is called doubly uniformly continuous if for any $\alpha\in U$ there exists a $\beta\in V$ such that $f\alpha\succ\beta^{\angle}$, where $\beta^{\angle}=\{\cup\beta_0:\beta_0\subset\beta-finite\}$ [7].

A mapping $f:(X,U)\to(Y,V)$ from a uniformly space (X,U) to a uniformly space (Y,V) is called a precompact mapping if for any $\alpha\in U$ there are $\beta\in V$ and a finite cover $\gamma\in U$ such that $f^{-1}\beta\wedge\gamma\succ\alpha$ [1], [2].

A mapping $f:(X,U)\to(Y,V)$ of a uniformly space (X,U) into a uniformly space (Y,V) is called a uniformly perfect mapping if it is precompact and perfect in the topological sense [3], [4].

Theorem 1. Let $f:(X,U)\to(Y,V)$ be a doubly uniformly continuous mapping. Then if the uniform space (X,U) is weakly sequentially complete, then the uniformly space (Y,V) is also weakly sequentially complete.

Proof. Let (X,U) be a weakly sequentially complete space. Let us show that the space (Y,V) is weakly sequentially complete. Let F be an arbitrary maximal Cauchy filter having a countable base. Denote by ψ the maximal filter in (X,U) containing the family $f^{-1}F=\{f^{-1}P:P\in F\}$. Let show that the maximal filter ψ is a Cauchy filter. Let $\alpha\in U$ be an arbitrary covering. By virtue of the doubly uniformly continuity of the mapping f, there exists $\beta\in V$ such that $f^{-1}\beta\succ\alpha^{\angle}$. Let $E\in\beta$ be a set such that $E\in F$. Then $f^{-1}E\in\psi$. In turn, for $f^{-1}E$ there exists such $A^{\angle}=\bigcup_{i=1}^{n}A_i$, $A_i\in\alpha$, that $f^{-1}E\subset\bigcup_{i=1}^{n}A_i$. Since ψ is the maximum filter and $\bigcup_{i=1}^{n}A_i\in\psi$, then there is a number $i_0\le n$ such that $A_{i_0}\in\psi$. Hence, ψ is a Cauchy filter. Let show that a countable family $f^{-1}B$ is a base for ψ. Let $L\in\psi$ be an arbitrary set. Since $f\psi\subset F$, then $fL\in F$. There is a $N\in B$ such that $N\subset fL$, i.e. $f^{-1}N\subset L$. This means that $f^{-1}B$ is a countable base for ψ. Due to the weak μ-completeness of the space (X,U), the maximal Cauchy filter ψ having a countable base converges to the point x. Then the filter $f\psi$ converges to the point fx. Therefore, fF converges to the point fx. Hence the space

(Y,V) is weakly sequentially complete.

Corollary 1. Let $f:(X,U)\to(Y,V)$ be a doubly uniformly continuous mapping. Then if the uniformly space (X,U) is weakly complete, then the uniformly space (Y,V) is also weakly complete.

Corollary 2. Let $f:(X,U)\to(Y,V)$ be a doubly uniformly continuous mapping. Then if the uniform space (X,U) is complete, then the uniformly space is also complete (Y,V).

Theorem 2. Let $f:(X,U)\to(Y,V)$ be a perfect uniformly continuous mapping. Then if the uniform space (Y,V) is weakly sequentially complete, then the uniformly space (X,U) is also weakly sequentially complete.

Proof. Let (Y,V) be a weakly sequentially complete space. Let us show that the space (X,U) is weakly sequentially complete. Let F be an arbitrary maximal Cauchy filter in (X,U), with a countable base B. Then the filter fF is the basis of some maximal Cauchy filter and, due to the weakly sequential completeness of the space (Y,V), has a point of contact with $y\in Y$. Since the mapping f is perfect, the filter F has a point of contact with $x\in f^{-1}y$. Therefore, the space (X,U) is weakly sequentially complete.

Corollary 3. Let $f:(X,U)\to(Y,V)$ be a uniformly perfect mapping. Then if the uniformly space (Y,V) is weakly sequentially complete, then the uniformly space (X,U) is also weakly sequentially complete.

Corollary 4. Let $f:(X,U)\to(Y,V)$ be a uniformly perfect mapping. Then if the uniformly space (Y,V) is weakly complete, then the uniformly space (X,U) is also weakly complete.

Corollary 5. Let $f:(X,U)\to(Y,V)$ be a perfect mapping. Then if the uniformly space (Y,V) is weakly complete, then the uniform space (X,U) is also weakly complete. The following theorem follows from Theorems 1 and 2.

Theorem 3. Let $f:(X,U)\to(Y,V)$ be a perfect doubly uniformly continuous mapping. If one of the uniformly spaces (X,U) and (Y,V) are weakly sequentially complete, then the other uniformly space is also weakly sequentially complete.

A Tychonoff space X is called a weakly sequentially complete in the sense of Diedonne space if there exists a weakly sequentially complete uniformity on it [6].

In the following theorem, by means of universal uniform structures, the weakly sequential Diedonne completeness of a space is characterized.

Theorem 4. A Tychonoff space X is weakly sequentially complete in the sense of Diedonne if and only if the universal uniformity is weakly sequentially complete.

Proof. Necessity. Diedonne space. Let us show that (X,U_X) is a uniformly space. Let a Tychonoff space X be weakly sequentially complete but with uniformity U_X is weakly sequentially complete. Let F be an arbitrary maximal Cauchy filter in (X,U_X) with a countable base. By virtue of the weakly sequentially completeness in the sense of Diedonne of the space X, there exists a weakly sequentially complete uniformity of U. Since $U_X \supset U$, then F is a Cauchy filter in (X,U). Then F converges to some point x in (X,U). Therefore, the uniform space (X,U_X) is weakly sequentially complete.

Sufficiency. Let a uniform space (X,U_X) be weakly sequentially complete. Then, by the definition of weakly sequential Diedonne completeness of a space, the Tychonoff space X is a weakly sequential Diedonne complete space.

Theorem 5. Let $f: X \to Y$ be a perfect mapping. Then if a topological space Y is weakly sequentially complete in the sense of Diedonne, then a topological space X is also a weakly sequentially complete space in the sense of Diedonne.

Proof. Let Y be a weakly sequentially complete in the sense of Diedonne space. Let show that the space X is weakly sequentially complete in the sense of Diedonne space. Let (Y,V) be a uniformly space whose topology is consistent with the topology of τ_Y, i.e. $\tau_V = \tau_Y$. Then there is such a uniformity U on X that makes the mapping f continuous. Let F be an arbitrary maximal Cauchy filter in (X,U), with a countable base B, where U is a uniformity consistent with the topology τ of the space X. Then the filter fF is the basis of some maximal Cauchy filter and, due to the weakly sequential completeness of the space (Y,V), has a point of contact with $y \in Y$. Since the mapping f is perfect, then the filter F has a point of contact with $x \in f^{-1}y$. Consequently, the space X is weakly sequentially complete in the sense of Diedonne.

REFERENCES

1. Borubaev A.A. Uniformly perfect mappings. Absolutes of uniform spaces // Dokl. Bolg. AN. - 1989. - T. 42, No. 1. - P. 19-23.
2. Borubaev A.A. Uniformly spaces and uniformly continuous mappings. - Frunze: Ilim, 1990. - 172 p.
3. Borubaev A.A. Uniformly spaces. - Bishkek: KGU, 1987. - 145 p.
4. Borubaev A.A. Uniform topology. - Bishkek: Ilim, 2013. - 347 p.
5. Vainshtein I.A. On closed mappings of metric spaces // Dokl. Academy of Sciences of the USSR. - 1947. - T. 57. - P. 319-321.
6. Kanetova D.E. On the completeness of uniform spaces // Vestnik KNU named after J. Balasagun, Special issue. - 2019. - S. 23-27.
7. Pasynkov B.A. On topological groups // Dokl. Academy of Sciences of the USSR. - 1969. - T. 188. - P. 281-282.
8. Engelking R. General topology. – M.: Mir, 1986. – 752 p.

MSC 54E15

CHARACTERIZATION OF SOME PROPERTIES OF TYCHONOFF SPACES BY MEANS OF UNIFORM STRUCTURES

Borubaev A.A. [1], Kanetova D.E. [2], Tadzhimatova D.A. [3]

[1]*Institute of Mathematics of NAS of KR*

[2]*Scientific Research Medical Social Institute*

[3] *Kyrgyz National University named after J. Balasagyn*

This paper characterizes some properties of the compactness type of Tychonoff spaces by means of uniform structure.

Keywords: uniform structure, compact space, countable compact space, Lindelöf space, sequential completeness, pre-Lindelöf space.

Бул илимий макалада тихоновдук мейкиндиктердин айрым компакту типтеринин биркалыптуу структуралары аркылуу мүнөздөмөлөрү берилет.
Урунттуу сөздөр: бир калыптуу структура, компактуу мейкиндик, санактуу компактуу мейкиндик, линделёфтук мейкиндик, секвенциалдуу толуктуулук, пред-Линделёфтук мейкиндик.

В данной статье даются характеристики некоторых свойств типа компактности тихоновских пространств посредством равномерных структур.
Ключевые слова: равномерная структура, компактное пространство, счетно компактное пространтство, линделёфово пространство, секвенциальная полнота, пред-Линделёфово пространство.

In this paper, characterizes are given of the most important classes of the compactness type of Tychonoff spaces with the help of uniform structures.

A uniform space (X,U) is called sequentially complete if every Cauchy filter with a countable base converges in it [1].

A topological space H is called countably compact if each of its open countable covers contains a finite subcover [4].

Lemma 1. If a Tychonoff space X is countably compact, then for every uniform structure U, the uniform space (X,U) is sequentially complete.

Proof. Let the Tychonoff space (X,τ) is countably compact and F be a Cauchy filter in the space (X,U) with a countable base. Then the filter F has a point of contact in the space (X,U).Since in a uniform space the concepts of a point of contact and limit points are equivalent, this implies the convergence of the filter F in (X,U) .Hence, the space (X,U) is sequentially complete.

Lemma 2. If for every uniform structure U the uniform space (X,U) is sequentially complete, then the Tychonoff space X is countably compact.

Proof. Let F be an arbitrary ultrafilter having a countable prebase and (X,U) is any uniform space, such that $\tau_U = \tau$. Then there exists a precompact uniform structure U_P such that $\tau_{U_P} = \tau_U = \tau$. The filter F is a Cauchy filter in (X,U_P). Then it converges to some point in the space (X,U_P). Consequently, the ultrafilter F, which has a countable prebase, converges to some point in the space (X,τ). Hence, the space (X,U) is countably compact.

A uniform space (X,U) is called uniformly paracompact, if for any open cover λ of the space (X,U) there is a sequence of covers $\{\beta_i : i \in N\} \subset U$ that satisfies the condition:

(BP) for each point $x \in X$ there are numbers $n \in N$ and $L \in \lambda$ such that $\beta_n(x) \subset L$ [2].

Various types of uniformly paracompact and uniformly Lindelöf spaces were studied in [1], [2], [6].

A uniform space (X,U) is called pre-Lindelöf ($\aleph_0$-bounded) if it has a base consisting of countable covers [5]. A uniform space (X,U) is called uniformly paracompact if it is pre-Lindelöf and uniformly paracompact [1].

The following theorem characterizes Lindelöf spaces with the help of uniform structures.

Theorem 1. A Tychonoff space X is Lindelöf if and only if, for every pre-Lindelöf uniform structure U, the uniform space (X,U) is uniformly Lindelöf.

Proof. Necessity. Let (X,τ)be a Lindelöf space.Let us show that the uniform space (X,U) is uniformly Lindelöf. Let α be an arbitrary open cover of the space (X,U). For every point $x \in X$, there are $A \in \alpha$ and $\beta \in U$such $A \in \alpha$that $\beta(x) \subset A$.It is clear that for the covering $\beta \in U$ there exists $\gamma \in U$ such that $\gamma * \succ \beta$ .Let $\lambda = \{\beta(x) : x \in X\}$. Since the space X is Lindelöf, the open cover of λ contains a countable subcover of $\lambda_0 = \{\beta_n(x) : n \in N\}$. We form a sequence of uniform covers $\{\gamma_n : n \in N\}$. Then for each point $x \in X$ there is a number $n \in N$ such that $x \in \gamma_n(x)$.It is clear that $\gamma_n(x) \subset \beta_n(x) \subset A$.Thus, the uniform paracompactness of the space (X,U) is proved. (X,U) is uniformly Lindelöf since it is pre-Lindelöf and uniformly paracompact.

Sufficiency. Let (X,U) be obtained such a pre-Lindelöf uniform space that $\tau_U = \tau$. Let α is an arbitrary open cover of (X,τ_U). For an open cover α of a space

(X,U), there exists a normal sequence of countable covers $\{\beta_n : n \in N\} \subset U$ that satisfies the condition (BP). Then there exists a separable pseudometric d on X such that $\beta_{n+1}(x) \subset \{y : d(x,y) < \frac{1}{2^{n+1}}\} \subset \beta_n(x)$, for any $x \in X$ and for each $n \in N$. Consequently, the interior $\langle \alpha^{\angle} \rangle = \{\langle A \rangle : A \in \alpha^{\angle}\}$ of α is an open cover of a separable pseudometric space (X,d). Since any separably pseudometrizable space is Lindelöf, there exists a countable open cover λ of the space (X,τ_d) inscribed in the cover α . It follows from the inclusion $\tau_d \subset \tau_U$ that the cover of λ is an open cover of the space (X,τ_U).

Therefore, (X,τ_U) is uniformly Lindelöf.

Consequence 1. A Tychonoff space X is Lindelöf if and only if, for every precompact uniform structure U, the uniform space (X,U) is uniformly Lindelöf.

Theorem 2. A Tychonoff space X is Lindelöf if and only if, for a universal uniform structure U_X, the uniform space (X,U_X) is uniformly Lindelöf.

Proof. Necessity. Let the Tychonoff space (X,τ) is Lindelöf. Let us show that the uniform space (X,U_X) is uniformly Lindelöf. Let α be an arbitrary open covering of the space (X,U).Then for each point $x \in X$ there are $A \in \alpha$ and $\beta \in U$ such that $\beta(x) \subset A$. For $\beta \in U$ there is an $\gamma \in U$ such that $\gamma * \succ \beta$. Let $\lambda = \{\beta(x) : x \in X\}$. Since the Tychonoff space X is Lindelöf, the open cover λ contains a countable subcover $\lambda_0 = \{\beta_n(x) : n \in N\}$ .We form a sequence of uniform coverings $\{\gamma_n : n \in N\}$. Then for each point $x \in X$ there is such a number $n \in N$ that $x \in \gamma_n(x)$. It is clear that $\gamma_n(x) \subset \beta_n(x) \subset A$. This implies that (X,U) is uniformly paracompact.Since (X,τ) is Lindelöf, then (X,U_X) is pre-Lindelöf. Therefore, (X,U_X) is uniformly Lindelöf.

Sufficiency. Let (X,U_X) is a uniformly Lindelöf space. Let α be an arbitrary open covering of the space (X,τ_{U_X}). Then $\alpha \in U_X$. By virtue of the uniform Lindelöfspace (X,U), the covering α contains a countable subcover $\alpha_0 \in U_X$. Consequently, α contains a countable subcover of α_0. Hence, the space (X,τ) is uniformly Lindelöf, where $\tau = \tau_{U_X}$.

Theorem 3. A Tychonoff space X is compact if and only if, for every pre-Lindelöf uniform structure U, the uniform space (X,U) is sequentially complete and uniformly Lindelöf.

The proof follows from Lemma 1 and Theorems 1.

Consequence 2. A Tychonoff space X is compact if and only if, for every precompact uniform structure U, the uniform space (X,U) is sequentially complete and uniformly Lindelöf.

Theorem 4. A Tychonoff space X is compact if and only if, for the universal uniform structure U, the uniform space (X,U) is sequentially complete and uniformly Lindelöf.

Consequence 3. A Tychonoff space X is compact if and only if for every uniform structure U the uniform space (X,U) is complete.

Consequence 4. A Tychonoff space X is compact if and only if, for a universal uniform structure U, the uniform space (X,U) is complete.

Recall that a topological space X is countably paracompact if and only if each of its finitely additive countable open covers can be a refinement of the locally finite open cover [4].

The following theorem establishes a characterization of countably paracompact spaces with the help of universal uniform structures.

Theorem 5. A Tychonoff space X is countably paracompact if and only if the uniform space (X,U_X), where U_X is a universal uniform structure, is countably uniformly paracompact.

Proof. Necessity. Let X be countably paracompact. The set of all open coverings forms the basis of universal uniformity. It is easy to see that the uniform space (X,U_X) is countably uniformly paracompact.

Sufficiency. If the uniform space (X,U_X) is countably uniformly paracompact. Then by Proposition 1 [8, p. 320], the Tychonoff space X is countably paracompact.

Theorem 6. For a Tychonoff space X the following statements are equivalent:

1. X is countably paracompactness;

2. The uniform space (X,U_X), where U_X is a universal uniform structure, is countably uniformly paracompact.

3. There is a uniform structure U on the space X such that the uniform space (X,U) is countably uniformly paracompact.

The proof with minor modifications is similar to the proof of Theorem 5.

A uniform space (X,U) is called uniformly locally Lindelöf if it contains a uniform covering consisting of compact subsets [3].

Theorem 7. For a Tychonoff space X the following statements are equivalent:

1. X is locally Lindelöf and paracompactness;

2. The universal uniform structure U_X of the space X contains a covering consisting of Lindelöf subsets;

3. There is a uniform structure U on the space X containing a covering consisting of Lindelöf subsets.

Proof. $1. \Rightarrow 2.$ Let the space X be locally Lindelöf and paracompact. Then for each point $x \in X$ there exists a neighborhood of O_x such that the closure of $[O_x]$ is Lindelöf. Let $\alpha = \{[O_x] : x \in X\}$. It is clear that the coverage $\{O_x : x \in X\}$ belongs to the uniform structure U_X. Since the open cover of $\{O_x : x \in X\}$ is a refinement in the cover of α, then from $\alpha \in U_X$.

$2. \Rightarrow 3.$ Obviously.

$3. \Rightarrow 1.$ Let the uniform structure U contains a covering α consisting of Lindelöf subsets. Since the interior $\langle \alpha \rangle$ of the covering α is a uniform covering, i.e. belongs to the uniform structure U, then the space X is locally Lindelöf. The uniform space (X,U) is uniformly locally Lindelöf, since (X,U) contains a covering consisting of subsets whose closure is Lindelöf, and any uniformly Lindelöf space is uniformly paracompact. Thus, the local Lindelöf and paracompactness of the space X is proved.

Consequence 5. A Tychonoff space X is locally compact and paracompact if and only if the universal uniform structure U_X contains a covering consisting of compact subsets [3].

Consequence 6. For a Tychonoff space X the following statements are equivalent:

1. X is locally compact and paracompact;

2. The universal uniform structure U_X of the space X contains a covering consisting of compact subsets;

3. There is a uniform structure U on the space X containing a cover consisting of compact subsets.

Consequence 7. For a Tychonoff space X the following statements are equivalent:

1. X is Lindelöf;

2. The universal uniform structure U_X of the space X contains a countable cover consisting of Lindelöf subsets;

3. There is a uniform structure U on the space X containing a countable cover consisting of Lindelöf subsets.

Consequence 8. For a Tychonoff space X the following statements are equivalent:

1. X is compactness;

2. The universal uniform structure U_X of the space X contains a finite covering consisting of compact subsets;

3. There is a uniform structure U on the space X containing a finite cover consisting of compact subsets.

A uniform space (X,U) is called uniformly locally Lindelöf if it contains a uniform covering consisting of subsets whose closure is Lindelöf [5].

Theorem 8. For precompact uniform spaces (X,U) the following conditions are equivalent:

1. (X,U) is uniformly locally Lindelöf;

2. (X,τ_U) - Lindelöf.

Proof. $1.\Rightarrow 2.$ Let (X,U) is a uniform locally Lindelöfspace and α is an arbitrary finitely additive open cover of the space (X,τ_U). Then, by virtue of being precompact and uniformly locally Lindelöf, there exists a finite uniform covering of β consisting of subsets whose closure is Lindelöf. It is clear that $\beta \succ \alpha^{\angle}$, where $\alpha^{\angle} = \{\cup \alpha_0 : |\alpha_0| \leq \aleph_0\}$. It is easy to see that the covering α contains a countable subcover. Then from the fact that the space (X,τ) is Lindelöf if and only if every finitely additive open cover can be a refinement with a countable open cover, it follows that the space (X,τ_U) is Lindelöf.

$2. \Rightarrow 1.$ Conversely, let the topological space (X,τ_U) is Lindelöf. Then the space (X,U) is uniformly paracompact, i.e. uniformly Lindelöf.

Therefore, the space (X,U) is uniformly locally Lindelöf.

REFERENCES

1. Borubaev A.A. Uniform topology - Bishkek: Ilim, 2013. - 347 p.
2. Kanetov B.E. Some classes of uniform spaces and uniformly continuous mappings. - Bishkek, 2013. - 160 p.
3. Kelly J.L. General topology. – M.: Nauka, 1981. – 432 p.
4. Engelking R. General topology. – M.: Mir, 1986. – 752 p.
5. Borubaev A.A. Uniform topology and its applications. – Bishkek: Ilim, 2021.
6. Isbell J. Uniform space. - Providence, 1964. - 175 p.
7. Kanetov B., Kanetova D. Characterization of some types of compactness and a construction of index compactness ≤ τ extensions by means of uniform structures // AIP Conference Proceedings. “International Conference on Analysis and Applied Mathematics, ICAAM 2018”. – Melville. – New York. - 2018. - Vol. 1997. - P. 020023-1-020023-5.
8. Marconi, U. On the uniform paracompactness // Rendiconti del Seminario Matematico della Universita di Padova. - 1984. - T.72. - P. 319-328.

MSC 54E15

EXTENSION TO MAPPINGS UNIFORMLY PARACOMPACT AND STRONGLY UNIFORMLY PARACOMPACT SPACES

Borubaev A.A. [1], Kanetov B.E. [2], Zhumaliev T.Zh. [3], Mamatnazarova T.A. [4]
[1]*Institute of Mathematics at the NAS of Kyrgyz Republic*
[2,4]*Kyrgyz National University named after J. Balasagyn*
[3]*Kyrgyz National Agrarian University named after K.I. Skryabin*

In this article, maps are extended to uniformly paracompact and strongly uniformly paracompact spaces. In particular, it is established that under such mappings the uniformly paracompact and strongly uniformly paracompact properties are preserved towards the inverse image.
Keywords: Uniformly continuous mapping, finitely additive open cover, uniformly locally finite cover, uniform paracompact space, uniform R-paracompact space, complete mapping.

Бул макалада бир калыптуу паракомпактуу жана күчтүү бир калыптуу паракомпактуу мейкиндиктер чагылдырууларга жайылтылган. Мындай чагылдырууларда бир калыптуу паракомпактуу жана күчтүү бир калыптуу паракомпактуу мейкиндиктердин касиеттери кайра элес жагына сакталат.
Урунттуу сөздөр: Бир калыптуу үзгүлтүксүз чагылдыруулар, чектүү аддитивдүү ачык жабдуу, бир калыптуу локалдык чектүү жабдуу, бир калыптуу паракомпакт, бир калыптуу R-паракомпакт, толук чагылдыруу.

В настоящей статье на отображения распространяется равномерно паракомпактные и сильно равномерно паракомпактные пространства. В частности, устанавливаются, что при таких отображениях равномерно паракомпактные и сильно равномерно паракомпактные свойства сохраняются в сторону прообраза.
Ключевые слова: Равномерно непрерывное отображение, конечно аддитивное открытое покрытие, равномерно локально конечным покрытие, равномерный паракомпакт, равномерный R-паракомпакт, полное отображение.

Let $f:(X,U)\to(Y,V)$ - uniformly continuous mapping of a uniform space (X,U) to a uniform space (Y,V).

Definition 1. Uniformly continuous mapping $f:(X,U)\to(Y,V)$ uniform space (X,U) to a uniform space (Y,V) is called uniformly paracompact, if for any finite additive open cover α uniform space (X,U) there exists a finitely additive open cover β uniform space (Y,V) and a uniformly locally finite open cover γ such that $f^{-1}\beta\wedge\gamma\succ\alpha$.

Proposition 1. If a $f:(X,U)\to(Y,V)$ uniformly continuous uniform space

(X,U) uniformly R - paracompact, then the mapping f uniformly paracompact.

Proof. Let α be an arbitrary finitely additive open cover of the uniform space (X,U). Due to the uniform R-paracompactness (X,U) we have $\alpha \in U$. There exists a uniformly locally finite open cover $\gamma \in U$, what $\gamma \succ \alpha$. Choose any open cover β space (Y,V). Then $f^{-1}\beta^{\angle} \wedge \gamma \succ \alpha$, hence the mapping f is a uniformly paracompact.

Lemma 1. If a α and β - uniformly locally finite covers of the space (X,U), then $\alpha \wedge \beta$ is also a uniformly locally finite cover of the space (X,U).

Proof. Let α, β be a uniformly local finite covers (X,U). Then there are such uniform covers $\lambda,\eta \in U$, that for any $L \in \lambda$, $E \in \eta$ we have $L \subset \bigcup_{i=1}^{n} A_i$ and $E \subset \bigcup_{j=1}^{k} B_j$, where $A_i \in \alpha$, $B_j \in \beta$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,k$, consequently, $L \cap E \subset (\bigcup_{i=1}^{n} A_i) \cap (\bigcup_{j=1}^{k} B_k) \subset \bigcup_{i=1}^{n} \bigcup_{j=1}^{k} (A_i \cap B_j)$. It's clear that $\lambda \wedge \eta$ - even cover and $L \cap E \in \lambda \wedge \eta$. So each $L \cap M \in \lambda \wedge \eta$ intersects only with a finite number of cover elements $\alpha \wedge \beta$, means, $\alpha \wedge \beta$ is a uniformly locally finite cover of the space (X,U).

Lemma 2. Let $f:(X,U) \to (Y,V)$ - uniformly continuous mapping. If a β - uniformly local finite cover of uniform space (Y,V), then $f^{-1}\beta$ is a uniformly local finite cover of the uniform space (X,U).

Proof. Let β - uniformly local finite cover (Y,V). Let us show that the cover $f^{-1}\beta$ is a uniformly locally finite cover of the space (X,U), because the β uniformly locally finite, then there exists a uniform cover $\lambda \in V$, that each element of which intersects only a finite number of elements of the cover β, that is for everybody $L \in \lambda$ there are $B_1, B_2, \ldots, B_n$ from β, what $L \subset \bigcup_{i=1}^{n} B_i$, consequently, $f^{-1}L \subset f^{-1}(\bigcup_{i=1}^{n} B_i) = \bigcup_{i=1}^{n} f^{-1}(B_i)$, where $f^{-1}(B_i) \in \beta$, $i = 1,2,\ldots,n$. It's obvious that $f^{-1}\lambda \in U$, then $f^{-1}\lambda$ - the desired cover, hence the cover $f^{-1}\beta$ is a uniformly locally finite cover of the space (X,U).

Lemma 3. If a α and β - uniformly star finite covers of space (X,U), then $\alpha \wedge \beta$ is also a uniformly star finite cover of the space (X,U).

The **proof** follows from Lemma 1 and from the fact that the intersection of any two star finite covers is a star finite cover.

Lemma 4. Let $f:(X,U)\to(Y,V)$ - uniformly continuous mapping. If a β - uniformly star finite cover of the uniform space (Y,V), then $f^{-1}\beta$ is a uniformly star finite cover of the uniform space (X,U).

The **proof** follows from Lemma 2 and from the fact that the inverse image of any star finite cover is a star finite cover.

Proposition 2. If a $f:(X,U)\to(Y,V)$ - uniformly paracompact mapping and uniform space (Y,V) is compact, then the uniform space (X,U) uniformly paracompact.

Proof. $f:(X,U)\to(Y,V)$ - uniformly paracompact mapping and α - arbitrary finitely additive open cover of the uniform space (X,U), then there is a finitely additive open cover β uniform space (Y,V) and a uniformly locally finite open cover γ such that $f^{-1}\beta\wedge\gamma\succ\alpha$. Without loss of generality, we will assume that β final open coating. Let's put $\lambda=f^{-1}\beta\wedge\gamma$, then according to Lemma 1. the cover λ is a uniformly locally finite open cover of the space (X,U), means, space (X,U) is uniformly R-paracompact.

Proposition 3. If a $f:(X,U)\to(Y,V)$ - a uniformly paracompact uniformly continuous mapping, and $Y=\{y\}$, then the uniform space (X,U) uniformly R-paracompact.

Proof. $f:(X,U)\to(Y,V)$ - uniformly paracompact mapping and α - arbitrary finitely additive open cover of the uniform space (X,U), then there are finitely additive open coverings β uniform space (Y,V) and a uniformly locally finite open cover γ, what $f^{-1}\beta\wedge\gamma\succ\alpha$, notice, that $f^{-1}\beta\wedge\gamma=\gamma$, Consequently, (X,U) is uniformly R-paracompact.

Proposition 4. If a $f:(X,U)\to(Y,V)$ - uniformly paracompact mapping and $M\subset X$ - closed subset, then its restriction $f|_M:(M,U_M)\to(Y,V)$ is also a uniformly paracompact mapping.

Proof. α_M - arbitrary finitely additive open space cover (M, U_M), and λ - finite additive open space family (X,U) such that $\lambda \wedge \{M\} = \alpha_M$, it is clear that the family $\alpha = \{\lambda, X \setminus M\}$ is a finitely additive open cover of the space (X,U), then there is a finitely additive open cover β uniform space (Y,V) and a uniformly locally finite open cover γ such that $f^{-1}\beta \wedge \gamma \succ \alpha$, it is easy to see that $(f|_M)^{-1}\beta \wedge \gamma_M \succ \alpha_M$, hence the mapping $f|_M$ uniformly paracompact.

Theorem 1. Let $f:(X,U) \to (Y,V)$ - uniformly paracompact mapping of uniform space (X,U) onto a uniformly R-paracompact space (Y,V). Then (X,U) is also uniformly R-paracompact.

Proof. f - a uniformly paracompact mapping, and (Y,V) - uniformly R-paracompact. Consider α - arbitrary finitely additive open cover of the uniform space (X,U), then there are finitely additive open coverings β uniform space (Y,V) and a uniformly locally finite open cover γ, what $f^{-1}\beta \wedge \gamma \succ \alpha$. In a finitely additive open cover β uniform space (Y,V) we inscribe uniformly locally a finite open cover λ space (Y,V). By Lemma 2. the cover $f^{-1}\beta$, and by Lemma 1. the cover $f^{-1}\lambda \wedge \gamma$ are uniformly locally finite open covers, hence the uniform space (X,U) uniformly R-paracompact.

Proposition 5. The composition of two uniformly paracompact mappings is a uniformly paracompact mapping.

Proof. $f:(X,U) \to (Y,V)$ and $g:(Y,V) \to (Z,W)$ - uniformly paracompact mappings and α - any finitely additive open covering of uniform spacesa (X,U), then there is a finitely additive open cover β space (Y,V) and a uniformly locally finite open cover γ, what $f^{-1}\beta \wedge \gamma \succ \alpha$. In turn, for open coverage β space (Y,V) there exists a finitely additive open cover δ space (Z,W) and a uniformly locally finite open cover λ, what $g^{-1}\delta \wedge \lambda \succ \beta$, notice, that $(g \circ f)^{-1}\delta \wedge (f^{-1}\lambda \wedge \gamma) \succ f^{-1}\beta \wedge \gamma \succ \alpha$, that is $(g \circ f)^{-1}\delta \wedge \eta \succ \alpha$ and $\eta \in U$, where $\eta = f^{-1}\lambda \wedge \gamma$. According to Lemma 2, the cover $f^{-1}\lambda$ is a uniformly locally finite cover, and by Lemma 1. the cover $\eta = f^{-1}\lambda \wedge \gamma$ is uniformly locally finite, hence the composition $(g \circ f):(X,U) \to (Z,W)$ uniformly paracompact.

Theorem 2. Every uniformly paracompact mapping $f:(X,U) \to (Y,V)$ uniform space (X,U) to a uniform space (Y,V) is a complete mapping.

Proof. Consider the uniformly paracompact mapping $f:(X,U) \to (Y,V)$. We choose such a Cauchy filter F in uniform space (X,U), what fF converges to some point (Y,V), let's pretend that F does not converge at any point x space (X,U), then for each point $x \in X$ there is a neighborhood O_x and element Φ_x from F such that $\Phi_x \cap O_x = \varnothing$. Let $\alpha = \{O_x : x \in X\}$, then, due to the uniform paracompactness of the mapping f there exists a finitely additive open cover β space (Y,V) and a uniformly locally finite open cover γ that, $f^{-1}\beta \wedge \gamma \succ \alpha^{\angle}$, because the fF converges to $y \in Y$, then for any $B \in \beta$ such that $B \ni y$ $B \in fF$, hence it follows that $f^{-1}B \in f^{-1}\beta \cap F$, note that there is $\Gamma \in \gamma$, what $\Gamma \in F$, then $f^{-1}B \cap \Gamma \neq \varnothing$, because the $f^{-1}\beta \wedge \gamma \succ \alpha^{\angle}$, then there is $\bigcup_{i=1}^{n} O_{x_i} \in \alpha^{\angle}$ such that $f^{-1}B \cap \Gamma \subset \bigcup_{i=1}^{n} O_{x_i}$, so, $\bigcup_{i=1}^{n} O_{x_i} \in F$ and $\bigcap_{i=1}^{n} \Phi_{x_i} \in F$, that is $(\bigcap_{i=1}^{n} \Phi_{x_i}) \cap (\bigcup_{i=1}^{n} O_{x_i}) \neq \varnothing$, then there is a number $i_0 \le n$, what $\Phi_{x_{i_0}} \cap O_{x_{i_0}} \neq \varnothing$, so we got a contradiction, therefore, f - complete.

Definition 2. Mapping $f:(X,U) \to (Y,V)$ uniform space (X,U) to a uniform space (Y,V) is called strongly uniformly paracompact if for any finite additive open cover α uniform space (X,U) there exists a finitely additive open cover β uniform space (Y,V) and uniformly star finite open cover γ such that $f^{-1}\beta \wedge \gamma \succ \alpha$.

Proposition 6. Every strongly uniformly paracompact mapping is uniformly paracompact.

Proof. Given a uniformly continuous mapping $f:(X,U) \to (Y,V)$ uniform space (X,U) to a uniform space (Y,V) is strongly uniformly paracompact and α - arbitrary finitely additive open cover of the uniform space (X,U), then there is a finitely additive open cover β uniform space (Y,V) and uniformly star finite open cover γ

such that $f^{-1}\beta \wedge \gamma \succ \alpha$, since every uniformly star finite open cover is a uniformly locally finite open cover, we conclude that the mapping f uniformly paracompact.

Proposition 7. If a $f:(X,U)\to(Y,V)$ uniformly continuous mapping and uniform space (X,U) strongly evenly R - paracompact, then the mapping f strongly uniformly paracompact.

Proof. Let α be an arbitrary finitely additive open cover of the uniform space (X,U), due to the strongly uniform R - paracompact space (X,U) we have $\alpha \in U$, there exists a uniformly star finite open cover $\gamma \in U$, what $\gamma \succ \alpha$. Let β - arbitrary open space cover (Y,V). Let's put $\beta^{\angle} = \{\cup\beta_0 : \beta_0 \subset \beta - \text{final}\}$, than $\beta^{\angle} = \{\cup\beta_0 : \beta_0 \subset \beta - \text{final}\}$ has a finitely additive open cover, hence $f^{-1}\beta^{\angle} \wedge \gamma \succ \alpha$, so the display f is strongly uniformly paracompact.

Proposition 8. If a $f:(X,U)\to(Y,V)$ - strongly uniformly paracompact uniformly continuous mapping and uniform space (Y,V) is compact, then the uniform space (X,U) uniformly paracompact.

Proof. Given $f:(X,U)\to(Y,V)$ - a strongly uniform paracompact mapping, and α an arbitrary finitely additive open cover of the uniform space (X,U), then there is a finitely additive open cover β uniform space (Y,V) and uniformly star finite open cover γ such that $f^{-1}\beta \wedge \gamma \succ \alpha$. In what follows, we will assume that β a finite open cover, set $\lambda = f^{-1}\beta \wedge \gamma$, then according to Lemma 3. the cover λ is a uniformly star finite open cover of the space (X,U), means space (X,U) is strongly uniformly R - paracompact.

Proposition 9. If a $f:(X,U)\to(Y,V)$ a uniformly paracompact uniformly mapping, and $Y=\{y\}$, then the uniform space (X,U) strongly uniformly R - paracompact.

Proof. Let $f:(X,U)\to(Y,V)$ be an strongly uniform paracompact mapping, and α - arbitrary finitely additive open cover of the uniform space (X,U), then there are finitely additive open covers β uniform space (Y,V) and uniformly star finite open cover γ, what $f^{-1}\beta \wedge \gamma \succ \alpha$. It's obvious that $f^{-1}\beta \wedge \gamma = \gamma$, hence the space (X,U) is strongly uniform R-paracompact.

Proposition 10. If $f:(X,U) \to (Y,V)$ an strongly uniform paracompact mapping, and $M \subset X$ - closed subset, then its restriction $f|_M : (M,U_M) \to (Y,V)$ is also a strongly uniformly paracompact mapping.

Proof. Let α_M be a some finitely additive open cover (M,U_M), let λ - finite additive open family (X,U) such that $\lambda \wedge \{M\} = \alpha_M$, it is clear that the family $\alpha = \{\lambda, X \setminus M\}$ is a finitely additive open cover of the space (X,U), then there is a finitely additive open cover β uniform space (Y,V) and uniformly star finite open cover γ such that $f^{-1}\beta \wedge \gamma \succ \alpha$. It's obvious that $(f|_M)^{-1}\beta \wedge \gamma_M \succ \alpha_M$, hence the mapping $f|_M$ strongly uniformly paracompact.

Theorem 3. Let $f:(X,U) \to (Y,V)$ be a strongly uniformly paracompact mapping of a uniform space (X,U) is strongly uniformly R- paracompact space (Y,V), then (X,U) is also strongly uniform R-paracompact.

Proof. Let f - a strongly uniform paracompact mapping, and (Y,V) - strongly uniform R-paracompact, let α - arbitrary finitely additive open cover of the uniform space (X,U), then there are finitely additive open covers β uniform space (Y,V) and uniformly star finite open cover γ, what $f^{-1}\beta \wedge \gamma \succ \alpha$. In a finitely additive open cover β uniform space (Y,V) refines a uniformly star finite open cover λ space (Y,V). According to Lemma 4, the cover $f^{-1}\beta$, while Lemma 3 covers $f^{-1}\lambda \wedge \gamma$ are uniformly star finite open covers, hence the uniform space (X,U) strongly uniformly R- paracompact.

Corollary 1. Let $f:(X,U) \to (Y,V)$ be a strongly uniformly paracompact mapping of a uniform space (X,U) onto a uniformly R-paracompact space (Y,V). Then (X,U) is also uniformly R-paracompact.

Proof. Let f be a strongly uniformly paracompact mapping, (Y,V) - uniformly R-paracompact space and α - arbitrary finitely additive open cover of the uniform space (X,U), then there are finitely additive open covers β uniform space (Y,V) and uniformly star finite open cover γ, what $f^{-1}\beta \wedge \gamma \succ \alpha$. In a finitely additive open cover β uniform space (Y,V) we refines uniformly locally a finite open cover λ space (Y,V). According to Lemma 2, the cover $f^{-1}\beta$ is a uniformly local finite cover, it is easy to see that the coverage $f^{-1}\beta \wedge \gamma$ is a uniformly locally finite open cover, as the intersection of uniformly locally finite and uniformly star finite covers, hence the uniform space (X,U) uniformly R-paracompact.

Proposition 11. The composition of two strongly uniformly paracompact mappings is a strongly uniformly paracompact mapping.

Proof. Given strongly uniform paracompact mappings $f:(X,U)\to(Y,V)$ and $g:(Y,V)\to(Z,W)$, and α - any finitely additive open covering of the uniform space (X,U), then there is a finitely additive open cover β space (Y,V) and uniformly star finite open cover γ, what $f^{-1}\beta \wedge \gamma \succ \alpha$. In turn, for open cover β space (Y,V) there exists a finitely additive open cover δ space (Z,W) and uniformly star finite open cover λ, what $g^{-1}\delta \wedge \lambda \succ \beta$, notice, that $(g\circ f)^{-1}\delta \wedge (f^{-1}\lambda \wedge \gamma) \succ f^{-1}\beta \wedge \gamma \succ \alpha$, that is $(g\circ f)^{-1}\delta \wedge \eta \succ \alpha$ and $\eta \in U$, where $\eta = f^{-1}\lambda \wedge \gamma$. According to the lemma, the cover $f^{-1}\lambda$ uniformly star finite cover, and by the lemma the cover $\eta = f^{-1}\lambda \wedge \gamma$ is uniformly star finite, hence $(g\circ f):(X,U)\to(Z,W)$ strongly uniformly paracompact.

Theorem 4. Every strongly uniformly paracompact mapping $f:(X,U)\to(Y,V)$ of the uniform space (X,U) to a uniform space (Y,V) is a complete.

The **proof** follows from Theorem 2 and Proposition 6.

REFERENCES

1. Borubaev A.A. Uniform topology. – Bishkek: Ilim, 2013. – 347 p.
2. Kanetov B.E. Some classes of uniform spaces and uniformly continuous mappings. – Bishkek, 2013. – 160 p.
3. Pasynkov B.A. On the extension to mappings of some concepts and statements concerning spaces // Mappings and functors. – M., 1984. – P. 72-102.
4. Engelking R. General topology. – M.: Mir, 1986. – 752 p.

MSC 54E15, 54E35

ON SOME EXTENSIONS OF UNIFORM AND TOPOLOGYCAL SAPCES

Borubaev A.A. [1], Namazova G.O.[2], Chamashev M.K.[3], Bekbolsunova A.B.[4]
[1,2]Institute of Mathematics of the NAS of Kyrgyz Republic,
[3]Osh State University,
[4]Kyrgyz State Technical University after I. Razzakov

In this article extensions of real-complete Tychonoff and uniform spaces are considered, as well as locally compact paracompact and locally compact Lindelöff extensions of Tychonoff and uniform spaces.
Keywords: Uniform real complete extension, locally compact paracompact extension, locally compact Lindelöff extension.

Бул макалада чыныгы толук тихоновдук жана бир калыптагы мейкиндиктерди кеңейтүүлөр, ошондой эле тихоновдук жана бир калыптагы мейкиндиктердин жергиликтүү компакттуу паракомпакттуу жана жергиликтүү компакттуу линделөфтүк кеңейтүүлөр каралат.
Урунттуу сөздөр: Бир калыптагы чыныгы толук кеңейтүү, жергиликтүү компакттуу паракомпакт, жергиликтүү компакттуу линделөфтүк кеңейтүү.

В этой статье рассматриваются расширения вещественно полных тихоновских и равномерных пространств, а также локально компактно паракомпактные и локально компактно линделёфовые расширения тихоновских и равномерных пространств.
Ключевые слова: Равномерно вещественно полное расширение, локально компактный паракомпакт, локально компактное линделёфово расширение.

The real complete spaces introduced by Edwin Hewitt. [1] The properties of real complete or in other terminology of complete Hewitt spaces are presented in the book. The maximal real complete Tychonoff spaces extensions called the Hewitt extension (the Hewitt extension). The first Tychonoff spaces constructed by E. Hewitt ([1]). Uniform analogues analysis of other important classes topological spaces and formation all extensions of such Tychonoff spaces classes considered ([3]). Real complete extensions are considered in [4].

Definition 1. A uniform space (X, U) is called a uniformly functional space, and the uniformity U is functional if the uniformity U is generated by some family of functions $C_U(X)$, i.e. U is generated by a family of coverings of the form $(f^{-1}\alpha: f \epsilon C_U(X), \alpha \epsilon E_R)$, where f: X → R, and E_R-natural uniformity of the number line R.

Proof. Let $C_U(X)$ - set of all uniformly continuous functions, where $f\colon (X,U) \to (R,E_R)$. We denote by U_F the uniformity generated by coverings of the form $\{f^{-1}\alpha\colon f\epsilon C_U(x), \alpha\epsilon E_R\}$. Then U_F is the desired uniformity of X.

Definition 2. A uniform space (X, U) is called uniformly real complete if it is uniformly functionally and complete.

Theorem 1. Let (X, U) be a uniformly function space. Then its completion ($\widetilde{X}$, $\widetilde{U}$) is uniformly real complete, and its topological space (X,τ_U) will be real complete spaces.

Proof. Let $C_U(X)$ be the set of all uniformly continuous functions $f\colon (X,U) \to (R,E_R)$. By R^f denotes a copy of the number space R for each $f \in C_U(X)$ and by $\Delta f = \Delta\{f\colon f \in C_U(X)\}$ the diagonal product of mappings $\{f\colon f \in C_U(X)\}$. Then the mapping $\Delta f\colon (X,U) \to \prod\{(R^f, E_R^f)\}$ is uniformly continuous. Since the space (X, U) is uniformly functional and the set of maps CU (X) generates the uniformity of U, then CU (X) also generates the Tychonoff topological space (X, τ_U). Then the mapping $\Delta f\colon (X,U) \to \prod\{\left(R^f, E_R^f\right)\}$ is a homeomorphic embedding ([2]). Since U is the weakest uniformity on X for which the whole mapping $f \in C_U(X)$ is uniformly continuous, the diagonal mapping $\Delta f\colon (X,U) \to \prod\{(R^f, E_R^f\}$ is also a uniformly homeomorphic embedding.

Then we consider the image Δf(X,U) as a uniform subspace of the uniform space $\prod\{(R^f, E_R^f),\ \ f \in C_U(X)\}$. By $\tilde{X}$ denotes the closure Δf (X) in the products $\prod\{\left(R^f, E_R^f\right)\}$. Then $\tilde{X}$ as a closed subspace of space and $\prod\{(R^f, f \in C_U(X)\}$ of substances is a real complete space [2]. Denote by $\widetilde{U}$ the uniformity on $\tilde{X}$ induced by the uniformity $\prod\{(E^f, f \in C_U(X)\}$. The uniform space $\left(\widetilde{X},\ \widetilde{U}\right)$ is the completion of the uniform space (X, U) and it is a uniformly real complete space.

Theorem 1 is proved.

Let (X, U) be an arbitrary uniform space. Then, by Proposition 1, there exists maximal functional uniformity U_F contained in U. By Theorem 1, the completions $\left(\widetilde{X},\ \ \widetilde{U}\right)$ of the uniform space (X, U) are uniformly real complete, and its topological space $(\widetilde{X},\ \widetilde{U})$is a real complete space. We denote it by $V_U X$ and call it the Hewitt extension of the uniform space (X, U).

If U is the maximal uniformity of the Tychonoff space X, then $V_U X$ coincides with the classical Hewitt extension VX of the Tychonoff space X.

Let X be an arbitrary real complete space. By C (X) we denote the set of all continuous functions f: X → R, which generates the maximum functional uniformity of U_F. We show that the uniformity of U_F is complete. By the external characteristic, the real-complete space X is a closed subspace of the product $\prod\{(R^f, f \in C(X)\}$ of the set of copies R^f of the real line R ([2]). We denote by U the uniformity on X induced by the product $\prod\{(E_R^f, f \in C(X)\}$ the set of natural uniformities E_R^f of the real line R^f. . The uniform space (X, U) is complete as a closed subspace of the complete of the uniform space $\prod\{(R^f, E_R^f): f \in C(X)\}$. The uniformity U is generated by the restriction family by the projection $pr_f \prod\{R^f: f \in C(X)\} \rightarrow R^f$. Since $pr_f \in C(X)$, for each $f \in C(X)$, then $U \subseteq U_F$.

Hense, U_F is the complete functional uniformity on X. Then the topology of the space X is also determined by this maximal functional uniformity U_F.

Definition 3. A uniform space (X, U) is called a pre-maximal functionally uniform space if its completion $(\widetilde{X},\ \widetilde{U})$ is uniformly real complete and uniformity U is an maximal functional uniformity.

Let X be an arbitrary Tychonoff space. Now we construct real complete extensions of the Tychonoff space by means of uniform structures.

We denote by V (X) the set of all pre-maximal uniformities of the Tychonoff space X. The sets V (X) are partially ordered by inclusion. We denote by H (X) the set (identifying the equivalent extension) of all real complete extensions of the Tychonoff space X. The set H (X) is also partially ordered in a natural way [2], [3]. On every real complete extension HX of the Tychonoff space X, there exists a unique complete maximal functional uniformity ϑ. It induces on X the pre-maximal functional uniformity $\vartheta \in$V (X). Each uniformity corresponds to a unique real complete extension (H_U, X) obtained as a completion of the uniform space (X, U). It is easy to see that this correspondence between the partially ordered sets H (X) and V (X) preserves a partial order.

So, we have obtained the following theorem

Theorem 2. Partially ordered sets V (X) and H (X) are isomorphic.

Lemma. Every uniformly real complete space is uniformly homeomorphic to a closed subspace of the product of some set of copies of a real line with natural uniformity.

Proof. Let (X, U) be an arbitrary uniformly real complete space. By his definition, it is complete, and U is the maximum functional uniformity. Let $C_U(X)$ be the set of all uniformly continuous functions $f:(X,U) \to (R,E_R)$ and $\Delta F = \{f: f \in f C_U(X)\}$ be the diagonal product $\prod\{R^F,\ E_R^f\}: f \in C_U(X)\}$, where (R^f, E_R^F) is a copy of the uniform space (R, E_R) for each $f \in C_U(X)$.

It is easy to verify that the map $\Delta F: (X,U) \to \prod\{(R^F, E_R^F)\}$ is a uniformly homeomorphic embedding. The fact that ΔF is a homeomorphic embedding is indicated in [2]. The uniformly homeomorphic embedding of the map ΔF follows from the fact that uniformity is generated by the family of functions $C_U(X)$. Since the uniform space (X, U) is complete, its image ΔF (X, U) is also complete, and the complete subspace of any uniform space is closed.

The lemma is proved.

Theorem 3. For each uniform space (X, U) there is exactly one (up to a uniform homeomorphism) uniformly real-complete space $(\vartheta_U X, \vartheta_U)$ with the following properties:

(1) There is a uniformly homeomorphic enclosure $i: (X, U_F) \to (\vartheta_U X, \vartheta_U)$, for which $(\vartheta_U X, \vartheta_U)$ is the completion of the uniform space (X, U_F), where U_F is the maximum functional uniformity contained in U.

(2) For any continuous function $f: (X,U) \to (R, E_R)$, there is a uniformly continuous function $\tilde{f}: (\vartheta_U X, \vartheta_U) \to (R, E_R)$ such that $\tilde{f}\, i = f$.

The spaces $(\vartheta_U X, \vartheta_U)$ also satisfy the condition:

(3) For each uniformly continuous mapping f: (X, U) → (γ, M) of the uniform space (X, U) into an arbitrary uniformly real complete space (γ, M), there is a uniform mapping $\tilde{f}: (\vartheta_U X, \vartheta_U) \to (\gamma, \mathrm{M})$ such that $\tilde{f}\ i = f$.

Proof. Let (X, U) be an arbitrary uniform space and U_F the maximal functional uniformity of those contained in U (exists by Proposition 1). By $(\vartheta_U, X, \vartheta_U)$ we denote the completion of the uniform space (X, U_F). Then the uniform space $(\vartheta_U, X, \vartheta_U)$ is uniformly real complete, i.e. condition (1) is satisfied.

Let $f: (X, U) \to (R, E_R)$ be an arbitrary uniformly continuous function. Then the mapping $f: (X, U_F) \to (R, E_R)$ will also be uniformly continuous. By $\tilde{f}$ we denote the continuation of the mapping f onto the completions $(\vartheta_U, X, \vartheta_U)$ of the space (X, U_F) and $i: (X, U_F) \to (\vartheta_U, X, \vartheta_U)$ is a natural uniformly homeomorphic embedding. Then $\tilde{f}\ = \tilde{f}\ i$ condition (2) is satisfied.

It follows from conditions (1), (2) and the lemma that a uniformly real-complete space $(\vartheta_U, X, \vartheta_U)$ satisfies condition (3).

Let $(\vartheta_U', X, \vartheta_U')$ be some uniformly real complete space for which conditions (1) and (2) are satisfied. Then $(\vartheta_U', X, \vartheta_U'$, also satisfies condition (3), which implies that $\left(\vartheta_U', X, \vartheta_U'\right)$ is uniformly homeomorphic $(\vartheta_U, X, \vartheta_U)$.

Theorem 3 is proved

A uniformly real complete space $(\vartheta_U, X, \vartheta_U)$ is called the Hewitt completion of the uniform space (X, U). Generally, it differs from the completion $(\tilde{X}, \tilde{U})$ of the uniform space (X,U).

Example. Let R be the space of real numbers. By $E_R -$ we denote the natural uniformity of the space R, and E_p the maximal functional uniformity on R, and also by Ep the maximal precompact uniformity on R contained in E_R. Then $E_p \neq E_R$ and $E_R \neq E_F$. The first inequality follows from the fact that Ep is incomplete uniformity, and E_R is complete uniformity. The second inequality follows from the fact that the function $f(x) = x^2$ is continuous on R, but is not uniformly continuous on (R, E_R). Therefore, there exists a uniform covering $\alpha \in E_R$ such that $f\alpha \notin E_R$, but by construction of E_F, the covering $f^{-1}\alpha \in E_F$. Therefore, $E_R \neq E_F$. The uniform spaces (R,E_F) and (R,E_R) are uniformly real compact spaces.

Now we consider locally compact paracompact, locally compact Lindelof extensions of Tychonoff and uniform spaces.

Definition 4. Let $(X,\mathcal{U})$ be a uniform space. The uniformity $\mathcal{U}$ is called:

1) preparacompact if every cover γ of the set X such that $\gamma \cap \mathcal{F} \neq \varnothing$ for any $\mathcal{F} \in \varphi(\mathcal{U})$ belongs to $\mathcal{U}$;

2) strongly preparacompact if $\mathcal{U}$ is a preparacompact and has a base consisting of a star-finite coverings;

3) preLindeloff if $\mathcal{U}$ is a preparacompact and has a base consisting of countable coverings.

We denote by $\mathcal{U}_D(X)$ (respectively $\mathcal{U}_P(X), \mathcal{U}_S(X), \mathcal{U}_L(X)$) the set of all preuniversal (respectively preparacompact, strongly preparacompact, preLindeloff) uniformities of the Tychonoff space X. The sets $\mathcal{U}_D(X), \mathcal{U}_P(X), \mathcal{U}_S(X)$ are partially ordered by inclusion.

Theorem 4. For any Tychonoff space X the following partially ordered sets

1) $(D(X),\leq)$ and $(\mathcal{U}_D(X),\subset)$;

2) $(P(X),\leq)$ and $(\mathcal{U}_P(X),\subset)$;

3) $(S(X),\leq)$ and $(\mathcal{U}_S(X),\subset)$;

4) $(L(X),\leq)$ and $(\mathcal{U}_L(X),\subset)$.

are isomorphic.

Proposition 3. A Tychonoff space X is locally compact and paracompact (respectively Lindelof) if and only if it contains a universal uniformity $\mathcal{U}^*$ contains a cover (countable cover respectively) consisting of compact subsets.

Proof. Let the space X be locally compact and paracompact (respectively Lindelof). Then for any point $x \in X$ there is a neighborhood O_x such that $[O_x]$ is compact. Then, by the paracompactness of X the cover $\{[O_x]: x \in X\}$ belongs to universal uniformity $\mathcal{U}^*$. If X is Lindelof, then the cover $\{O_x : x \in X\}$ contains a countable subcover $\{O_{x_i} : i \in N\}$. Then the cover $\{\langle A \rangle : A \in \alpha\}$ belongs to the universal uniformity $\mathcal{U}^*$. Conversely, suppose that the universal uniformity $\mathcal{U}^*$ of the space X contains a cover (countable cover) α consisting of compact subsets. Then the cover $\{\langle A \rangle : A \in \alpha\}$ also belongs to the uniformity $\mathcal{U}^*$. This implies that X is locally compact. Uniform space $(X, \mathcal{U}^*)$ is complete and $ic(\mathcal{U}) = 1$. Since the well-known theorem of Stone states that every metrizable space is paracompact and the preimage of a paracompact space under a perfect map is paracompact. If there is a countable cover of X consisting of compact subsets, then its any open covering contains a countable subcovering, i.e. X is Lindeloff.

Proposition is proved.

Remark 1. Let $(X, \mathcal{U})$-be a uniform space, and $(X, \mathcal{U})$- be its completion. If $(A, \mathcal{U}_A)$ is a precompact subspace of the space $(X, \mathcal{U})$, then the subspace $\left([A]_X, \mathcal{U}_{[A]_X}\right)$ of the space $(X, \mathcal{U})$ is compact.

Theorem 5. There is an isomorphism between the partially ordered the set of all locally compact paracompact (locally compact Lindelöff) extensions of the given Tychonoff space X and partially ordered set of all preuniversal uniformities of the space X containing a uniform cover (respectively countable uniform cover) consisting of precompact subsets.

A partially ordered set $(D(X), \leq)$ has the greatest element. This element is the extension $t_{\mathcal{U}^*} \geq X$ corresponding to the universal uniformity $\mathcal{U}^*$ of the space X. The rest partially ordered sets $(P(X), \leq)$, $(P(X), \leq)$ and $(L(X), \leq)$ generally speaking, do not have greatest elements.

Theorem 6. For Tychonoff space X the following conditions are equivalent:

(1) Partially ordered set $(P(X), \leq)$ has a greatest element.

(2) Universal uniformity U^* of the space X is preparacompact.

If U_X is a universal (the maximal) uniform of a Tychonoff space X, then a maximal locally compact paracompact (locally compact Lindeloff, respectively) extension of a uniform space (X, U_X) is a maximal locally compact paracompact (maximal locally compact Lindeloff, respectively) extension of the Tychonoff space X.

From the above results, one can get the following theorem.

Theorem 7. Among all locally compact paracompact (locally compact Lindeloff, respectively) extensions of a Tychonoff space X there is a maximal extension.

Let μX be a maximal Dieudonne complete extension of the space X and pX (spX, lX) a maximal locally compact paracompact (a maximal locally compact strongly paracompact, a maximal locally compact Lindeloff respectively) extension of the space X. Then we get the following inclusions $\mu X \subseteq pX \subseteq spX \subseteq lX \subseteq \beta X$.

If νX is a maximal real compact Hewitt extension of a space X, then the following inclusions $\mu X \subseteq \nu X \subseteq lX \subseteq \beta X$. hold.

Remark 2. Locally compact paracompact space is strongly paracompact. The locally compact strongly paracompact extensions coincide with locally compact paracompact extensions

REFERENCES

1. Edwin Hewitt. Rings of real-valued continuous functions I. Transactions of the American Mathematical Society 64(1948), 45-99.
2. Borubaev A.A. Ravnomernaya topologiya. Bishkek. Izdatelstvo “Ilim” 2013.
3. Aparina L.V. Hewitovskie rasshireniya blizostnych I ravnomernych prostranstv // Sibirskii mat. jurnal.—1974.-T.15.-№4.-P.707-729.
4. Engeiking R. General topology. Moskow.Mir, 1986. P.752
5. Borubaev A.A. On completions of uniform spaces and extensions of topological spaces. Bolg.Math.Spis., 1989, V.15.1. P. 63-73. (in Russian)

MSC 54E15, 54D20

ON PRECOMPACT UNIFORM STRUCTURES

Chekeev A.A. [1], **Kanetov B.E.** [2], **Baidzhuranova A.M.** [3], **Zhusupbekova N.** [4]
[1,2,4] *Kyrgyz National University named after J. Balasagyn*
[3] *Institute of Mathematics of NAS of Kyrgyz Republic*

In this paper, studied precompact spaces. In particular, a criterion for the precompactness of uniform structures is established, and countably precompact structures are extended to the mapping.

Keywords: Uniform space, uniformly continuous mapping, precompact, countably precompact, pre-Lindelöf.

Бул илимий макалада санактуу предкомпакттуу мейкиндиктер изилденилет. Предкомпакттуу бир калыптуу структуралардын критерийи тургузулат, санактуу предкомпакттуу структуралар чагылдырууларга жайылтылат.

Урунттуу сөздөр: Бир калыптуу мейкиндик, бир калыптуу үзгүлтүксүз чагылдыруу, предкомпакт, санактуу предкомпакт, пред-Линделёф.

В настоящей статье вводятся и исследуются счетно предкомпактные пространства. В частности, устанавливается критерий предкомпактности равномерных структур, на отображение распространяется счетно предкомпактные структуры.

Ключевые слова: Равномерное пространство, равномерно непрерывное отображение, предкомпакт, счетно предкомпакт, пред-Линделёф.

A uniform space (X,U) is called precompact if the uniformity U has a base consisting of finite covers.

A uniform space (X,U) is called pre-Lindelöf if the uniformity U has a base consisting of countable coverings.

A uniform space (X,U) is called countably precompact if every countable uniform cover can be refinement with a finite uniform cover.

Proposition 1. Every precompact uniform space is countably precompact.

Proof. Let (X,U) be a precompact uniform space and $\alpha \in U$ is an arbitrary countable uniform cover. Then there is a finite uniform cover $\beta \in U$ such that $\beta \succ \alpha$. Thus, (X,U) is a countably precompact uniform space.

Corollary 1. Every compact uniform space is countably precompact.

Proposition 2. Every subspace of a countably precompact uniform space is countably precompact.

Proof. Let (X,U) be a countably precompact uniform space and (M,U_M) is a subspace of it. Let $\alpha_M \in U_M$ be an arbitrary countable uniform cover. Then there is a countable uniform cover $\alpha \in U$ such, that $\alpha \wedge \{M\} = \alpha_M$. Let $\beta \in U$ be a finite uniform cover such, that $\beta \succ \alpha$. It is easy to see that $\beta_M \succ \alpha_M$ and $\beta_M \in U_M$ are final coverage. Therefore, the uniform space (X,U) is countably precompact.

Theorem 1. A Tychonoff space X is countably compact if and only if the uniform space (X,U_X) is countably precompact, where U_X is a universal uniform structure.

Proof. Necessity. Let the Tychonoff space X be countably compact and let $\alpha \in U_X$ be an arbitrary open countable cover. Then there is a finite open cover β of the space X such, that $\beta \succ \alpha$. Clearly $\beta \in U_X$. Therefore, the space (X,U_X) is countably precompact.

Sufficiency. Let a uniform space (X,U_X) be countably precompact, where U_X is a universal uniform structure. Let prove that the Tychonoff space X is countably compact. Let α be an arbitrary countable open covering. Then $\alpha \in U_X$ and, since the space (X,U_X) is countably precompact, there exists a finite open cover of $\beta \in U_X$ such that $\beta \succ \alpha$. Consequently, the Tychonoff space X is countably compact.

Theorem 2. A Tychonoff space X is Lindelöf if and only if the uniform space (X,U_X) is pre-Lindelöf, where U_X is a universal uniform structure.

Proof. Necessity. Let a Tychonoff space X be a Lindelöf space and let $\alpha \in U_X$ be an arbitrary open uniform cover. Then there is a finite open cover β of the space X such, that $\beta \succ \alpha$. Obviously $\beta \in U_X$. Therefore, the space (X,U_X) is pre-Lindelöf.

Sufficiency. Let the uniform space (X,U_X) be pre-Lindelöf, where U_X is the universal uniform structure. Let prove that the Tychonoff space X is Lindelöf. Let α be an arbitrary open covering. Then $\alpha \in U_X$ and, since the uniform space (X,U_X) is pre-Lindelöf, there is a countable open cover $\beta \in U_X$ such, that $\beta \succ \alpha$. Therefore, the Tychonoff space X is Lindelöf.

Theorem 3. A Tychonoff space X is compact if and only if the uniform space

(X, U_X) is countably precompact and pre-Lindelöf, where U_X is a universal uniform structure.

The proof follows from Theorems 1 and 2.

Theorem 4. A uniform space (X,U) is precompact if and only if the space (X,U) is pre-Lindelöf and countably precompact.

Proof. Necessity. Let (X,U) be a precompact uniform space. It is clear that (X,U) is a pre-Lindelöf space. Let $\alpha \in U$ be an arbitrary countable uniform cover. Then there exists a finite uniform cover $\beta \in U$ which is refinement in the uniform cover α. Therefore, (X,U) is a countably precompact space.

Sufficiency. Let $\alpha \in U$ be an arbitrary uniform cover. Then, since the space (X,U) is pre-Lindelöf, there exists a countable uniform cover $\beta \in U$ such that $\beta \succ \alpha$. In turn, for the cover $\beta \in U$, since the space (X,U) is countably precompact, there exists a finite uniform cover $\gamma \in U$ refinement in β. It is clear that $\gamma \succ \alpha$. Hence (X,U) is precompact.

Corollary 2. Let (X,U) be a complete uniform space. A uniform space (X,U) is compact if and only if the space (X,U) is pre-Lindelöf and countably precompact.

Let $f:(X,U) \to (Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) into a uniform space (Y,V).

A mapping f is called countably precompact if for any countable uniform cover $\alpha \in U$ there exists a finite uniform cover $\gamma \in U$ and a uniform cover $\beta \in V$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$.

A mapping f is called pre-Lindelöf if for any uniform cover $\alpha \in U$ there exists a countable uniform cover $\gamma \in U$ and a uniform cover $\beta \in V$ such, that $f^{-1}\beta \wedge \gamma \succ \alpha$.

A mapping f is said to be precompact if for any uniform cover $\alpha \in U$ there exist a finite uniform cover $\gamma \in U$ and a uniform cover $\beta \in V$ such, that $f^{-1}\beta \wedge \gamma \succ \alpha$.

Theorem 5. Let $f:(X,U) \to (Y,V)$ be a uniformly continuous mapping from a uniform space (X,U) to a uniform space (Y,V). A mapping f is precompact if and only if it is pre-Lindelöf and countably precompact.

Proof. Necessity. Let the mapping $f:(X,U)\to(Y,V)$ is uniform space (X,U) to uniform space (Y,V) be precompact. Then it is easy to see that it is pre-Lindelöf and countably precompact.

Sufficiency. Let the mapping f be pre-Lindelöf and countably precompact, and let $\alpha\in U$ be an arbitrary uniform cover. Then, since the mapping f is pre-Lindelöf, there exist a countable uniform cover $\gamma\in U$ and a uniform cover $\beta\in V$ such, that $f^{-1}\beta\wedge\gamma\succ\alpha$. Further, for a cover $\gamma\in U$, there exist a finite uniform cover $\gamma_0\in U$ and a uniform cover $\beta_0\in V$ such that $f^{-1}\beta_0\wedge\gamma_0\succ\gamma$. Then $(f^{-1}\beta_0\wedge f^{-1}\beta)\wedge\gamma_0\succ f^{-1}\beta\wedge\gamma\succ\alpha$. We put $f^{-1}\beta_0\wedge f^{-1}\beta=f^{-1}(\beta_0\wedge\beta)$ and $\beta_0\wedge\beta=\lambda$. It is clear that $\beta_0\wedge\beta\in U$ and $f^{-1}\lambda\wedge\gamma_0\succ\alpha$. Therefore, f is a precompact mapping.

REFERENCES

1. Borubaev A.A. Uniformly spaces and uniformly continuous mappings, Frunze, Ilim, 1990.
2. Borubaev A.A. Uniformly spaces, Bishkek, KGU, 1987.
3. Borubaev A.A. Uniform topology, Bishkek, Ilim, 2013.
4. Kanetov B.E. Some classes of Uniform Spaces and Uniformly Continuous Mappings, Bishkek, 2013.

MSC 54E15

ON STRONG UNIFORMLY τ-FINALLY PARACOMPACT SPACES

Kanetov B.E.[1], Altybaev N.[2], Anarbek kyzy J.[3]

[1,3]*Kyrgyz National University named after J. Balasagun*

[2]*Institute of Mathematics of NAS of KR*

As we know the strong τ-finally paracompactness play an important role in the General Topology. Therefore, the finding of uniform analogues of strong τ-finally paracompactness is an important and interesting problem in the theory of uniform spaces.

In the paper, strongly uniformly τ-finally paracompact spaces and their connection with other uniform properties of compactness type are studied, and characterizes of these classes are established with the help of finitely additive open covers, compact Hausdorff extensions, and ω-mappings.

Keywords: strong uniform τ -finally paracompactness, strong τ -finally paracompactness, τ -finally paracompactness.

Күчтүү τ -финалдуу паракомпактуулук жалпы топологияда негизги рольду ойнойт. Күчтүү τ -финалдуу паракомпактуулуктун бир калыптуу аналогдорун табуу бир калыптуулуктар теориясынын негизги жана кызыктуу маселелеринен болуп саналат.

Бул илимий макалада күчтүү бир калыптуу τ -финалдуу паракомпактуу мейкиндиктер жана алардын башка компактуу типтүү бир калыптуу касиеттер менен болгон байланыштары изилденент жана чектүү аддитивдүү ачык жабдуулар, Хаусдорфтук компактуу кеңейүүлөр жана ω -чагылдыруулар аркылуу мүнөздөмөлөрү берилет.

Урунттуу сөздөр: күчтүү бир калыптуу τ -финалдуу паракомпактуулук, күчтүү τ -финалдуу паракомпактуулук, τ -финалдуу паракомпактуулук.

Как известно, сильно τ -финально паракомпактные пространства играют важную роль в общей топологии. Нахождение равномерных аналогов сильно τ -финально паракомпактности является важной и интересной задачей теории равномерных пространств.

В настоящей статье изучается сильно равномерно τ -финально паракомпактные пространства и их связь с другими равномерными свойствами типа компактности, устанавливается характеристики при помощи конечно аддитивных открытых покрытий, Хаусдорфовых компактных расширений и ω -отображений.

Ключевые слова: сильно равномерно τ -финальная паракомпактность, сильно τ -финальная паракомпактность, τ -финальная паракомпактность.

1. Introduction

Throughout this paper all uniform spaces are assumed to be Hausdorff, mappings are uniformly continuous.

For coverings α and β of the set X, the symbol $\alpha \succ \beta$ means that the covering α is a refinement of the covering β, i.e. for any $A \in \alpha$ there exists $B \in \beta$ such that $A \subset B$ and, for coverings α and β of a set X, we have: $\alpha \wedge \beta = \{A \cap B : A \in \alpha, B \in \beta\}$. The covering α finitely additive if $\alpha^{\angle} = \alpha$, $\alpha^{\angle} = \{\cup \alpha_0 : \alpha_0 \subset \alpha$ is finite$\}$. $\alpha(x) = \cup St(\alpha, x)$, $St(\alpha, x) = \{A \in \alpha : A \ni x\}$, $x \in X$, $\alpha(H) = \cup St(\alpha, H)$, $St(\alpha, H) = \{A \in \alpha : A \cap H \neq \varnothing\}$, $H \subset X$.

A covering α of the uniform space (X, U) is called uniformly locally finite if there exists a uniform covering $\beta \in U$ such that every $B \in \beta$ meets α only for a finite number of elements of α [5]; a uniform space (X, U) called uniformly τ -finally paracompact, if every open covering has an open uniformly locally finite refinement cardinality $\leq \tau$ [2]; a uniform space (X, U) is called uniformly τ -locally compact if the uniformity of U contains a uniform covering cardinality $\leq \tau$ consisting is compact sets [3]; a uniformly continuous mapping $f:(X,U)\to(Y,V)$ of uniform space (X,U) onto a uniform space (Y,V) is called a precompact, if for each $\alpha \in U$ there exist a uniform covering $\beta \in V$ and finite uniform covering $\gamma \in U$, such that $f^{-1}\beta \wedge \gamma \succ \alpha$ [2]; a uniformly continuous mapping $f:(X,U)\to(Y,V)$ of uniform space (X,U) onto a uniform space (Y,V) is called a uniformly perfect, if it is both precompact and perfect [2]. For the uniformity U by τ_U we denote the topology generated by the uniformity and symbol U_X means the universal uniformity.

2. Strong uniformly τ-finally paracompactness

Definition 1. A uniform space (X,U) is called strongly uniformly τ-finally paracompactness if each of its open covers can be a refinement with a uniformly star-finite open cover of cardinality $\leq \tau$.

Moreover, a covering α of a uniform space (X,U) is called uniformly star-finite if it is star-finite and uniformly locally finite.

Theorem 1. If a uniform space (X,U) is strongly uniformly τ-finally paracompact, then the topological space (X,τ_U) is strongly τ-finally paracompact. Conversely, if the Tychonoff space (X,τ) is strongly τ-finally paracompact, then the uniform space (X,U_X), where U_X is universal uniformity, is strongly uniformly τ-finally paracompact.

Proof. Let α be an arbitrary open covering of the space (X,τ_U). Then there exists a uniformly star-finite open covering β of cardinality $\leq \tau$ a refinement in it. Since every uniformly star-finite open cover is a star-finite open cover, then the cover β is a star-finite cover of cardinality $\leq \tau$. Thus, the space (X,τ_U) is strongly τ-finally paracompact.

Conversely, let the space (X,τ) be strongly τ-finally paracompact. Then the set of all open covers forms a base of universal uniformity U_X of the space (X,τ). It is easy to see that the uniform space (X,U_X) is strongly uniformly τ-finally paracompact.

The following theorem characterizes strongly uniformly τ-finally paracompactness in the spirit of Tamano [6].

Theorem 2. Let (X,U) be a uniform space, bX is an arbitrary compact extension. A uniform space (X,U) is strongly uniformly τ-finally paracompact if and only if for any compact set $K \subset bX \setminus X$ there exists a uniformly star-finite open cover α of cardinality $\leq \tau$ such that $[A]_{bX} \cap K = \varnothing$ for any $A \in \alpha$.

Proof. Necessity. Let (X,U) is strongly uniformly τ-finally paracompact and $K \subset bX \setminus X$ is an arbitrary compactum. Then for each point $x \in X$ there exists a open in bX neighborhood O_x such that $[O_x]_{bX} \cap K = \varnothing$. Put $\beta = \{O_x \cap X : x \in X\}$. It is clear that β is an open covering of the space (X,U). We refinement a uniformly star-finite open cover β of cardinality γ into the cover $\leq \tau$. Then $[\Gamma]_{bX} \cap K = \varnothing$ for any $\Gamma \in \gamma$.

Sufficiency. Let α be obtained an open cover of the space (X,U). Then there is an open family β in bX such that $\beta \wedge \{X\} = \alpha$. Put $K = bX \setminus \cup\beta$. Then for a compact set K there exists a uniformly star-finite open cover γ of cardinality $\leq \tau$ such that $[\Gamma]_{bX} \cap K = \varnothing$ for any $\Gamma \in \gamma$. Due to the compactness of the set $[\Gamma]_{bX}$, there are $B_1, B_2, ..., B_n \in \beta$ such that $[\Gamma]_{bX} \subset \bigcup_{i=1}^{n} B_i$. Then $\Gamma \subset \bigcup_{i=1}^{n} A_i$, where $\bigcup_{i=1}^{n} A_i \in \alpha$.

Consequently, the uniform space (X,U) is a strongly uniformly τ-finally paracompact space.

The following theorem is an intrinsic characterizes for strongly uniformly τ-finally paracompactness spaces.

Theorem 3. For a uniform space (X,U) the following statements are equivalent:

1) (X,U) is strongly uniformly τ - finally paracompact;

2) (X,U) is uniformlyly τ -finally paracompact and the topological space (X,τ_A) is strongly τ -finally paracompact.

Proof. $1) \Rightarrow 2)$ obviously.

$2) \Rightarrow 1)$. Let α be an arbitrary open cover of the uniform space (X,U) . We refinement in it a star-finite open covering β of cardinality $\leq \tau$. In turn, we refinement into the cover β a uniformly locally finite open cover γ of cardinalit y $\leq \tau$. It is easy to see that β is a uniformly finite open covering of cardinality $\leq \tau$. Thus, the uniform space (X,U) is strongly uniformly τ-finally paracompact.

Theorem 4. For a locally compact space (X,U) the following conditions are equivalent:

1) (X,U) is uniformly τ - locally compact;

2) (X,U) is strongly uniformly τ - finally paracompact.

Proof. $1) \Rightarrow 2)$ obviously.

$2) \Rightarrow 1)$. From the local compactness of (X,U) it follows that for every point $x \in X$ there is an open neighborhood O_x such that $[O_x]$ is compact. The family $\alpha = \{O_x : x \in X\}$ forms an open covering of the space (X,U). In α refinement a uniformly star-finite open covering β of cardinality $\leq \tau$. Each $B \in \beta$ is contained in a certain set $\bigcup_{i=1}^{n} O_{x_i}$, whence, due to the monotonicaly of the closure, we have that $[B] \subset [\bigcup_{i=1}^{n} O_{x_i}]$, and therefore $[B]$ is compact. So $[\beta] = \{[B] : B \in \beta\}$ is a uniform covering consisting of compact subsets. Hence (X,U) is uniformly τ -locally compactness.

Theorem 5. A uniform space (X,U) is strongly uniformly τ-finally paracompactness if and only if any finitely additive open cover of (X,U) can be refinement with a uniformly star-finite open cover of cardinality $\leq \tau$.

Proof. Necessity. Let (X,U) be a strongly uniformly τ -finally paracompactness space and α is an arbitrary finitely additive open cover. In it we refinement a uniformly star-finite open covering β of cardinality $\leq \tau$ of thes pace (X,U). Consequently, the space (X,U) is strongly uniformly τ -finally paracompact.

Sufficiency. Let in any finitely additive open cover of the space (X,U) one can refinement a uniformly star- finite open cover of cardinality $\leq \tau$. Let us show that (X,U) is a strongly uniformly τ -finally paracompact space. Let α be an arbitrary open covering. By virtue of the strongly uniformly τ-finally paracompact of (X,U), a finitely additive open cover $\alpha^{\angle}$ can be refinement with a uniformly star-finite open cover β of cardinality $\leq \tau$ of the space (X,U). Consequently, the space (X,U) is strongly uniformly τ -finally paracompact.

An infinite discrete uniform space (X,U_D) , where U_D is a discrete uniformity of cardinality $\tau \geq \aleph_0$ is a strongly uniformly τ-finally paracompact but not compact uniform space.

The property of strongly uniformly τ-finally paracompactness is preserved in passing to closed subspaces and to any disjoint sum of uniform spaces.

Proposition 2. Any uniformly τ-locally compact space is strongly uniformly τ-finally paracompact.

Proof. Let a uniform space (X,U) be a uniformly τ -locally compact space. Then the topological space (X,τ_U) is locally compact and τ -finally paracompactness, i.e. strongly τ-finally paracompact. According to Theorem 3, the uniform space (X,U) is a strongly uniformly τ-finally paracompact space.

Proposition 2 is a uniform analog of A.V. Arkhangelskii [1].

Proposition 3. Any uniformly τ -finally paracompact space (X,U) whose topological space is τ -locally compact is strongly uniformly τ-finally paracompact.

Proof. Let (X,U) be a uniformly τ-finally paracompact space and let its topological space (X,τ_U) be locally compact. Then the space (X,τ_U) is τ -finally paracompact. Consequently, the topological space (X,τ_U) is strongly τ-finally paracompact. Then, by Theorem 3, the uniform space (X,U) is strongly uniformly τ -finally paracompact.

Lemma 1. Any (uniformly) perfect mapping $f:(X,U)\to(Y,V)$ of a uniform space (X,U) onto a uniform space (Y,V) is a ω -mapping for any finitely additive open cover ω of the space (X,U) .

Proof. Let ω be an arbitrary finitely additive open cover of the space (X,U). It is easy to see that $\alpha=\{f^{-1}y : y\in Y\}$ is refinement in ω . For each $f^{-1}y\in\alpha$, we choose $W_y\in\omega$ such that $f^{-1}y\subset W_y$. Then, since the mapping f is closed, there exists a neighborhood $O_y \ni y$ such that $f^{-1}O_y\subset W_y$.

Lemma 2. Let $f:(X,U)\to(Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) onto a uniform space (Y,V). If β is a uniformly star-finite open cover of cardinality $\le\tau$ of the space (Y,V), then $f^{-1}\beta$ is a uniformly star- finite open cover of cardinality $\le\tau$ of the space (X,U).

Proof. Let β be a uniformly star- finite open covering of cardinality $\le\tau$ of the space (Y,V). Then, by virtue of the uniform continuity of the mapping f, the covering $f^{-1}\beta$ is open in the space (X,U). Let $\gamma\in V$ be a uniform cover such that Γ has only a finite number of elements β, i.e. $\Gamma\subset\bigcup_{i=1}^{n}B_i$. It follows from here that $f^{-1}\beta(f^{-1}\Gamma)\subset\bigcup_{i=1}^{n}f^{-1}B_i$, where , $f^{-1}B_i\in f^{-1}\beta$, $f^{-1}\Gamma\in f^{-1}\gamma$, $f^{-1}\gamma\in U$. It is easy to see that $f^{-1}\beta$ is a star- finite covering. Hence, $f^{-1}\beta$ is a uniformly star- finite open covering of cardinality $\le\tau$ of the space (X,U).

The following theorem is a uniform analog of V.I. Ponomarev on the characterization of strongly paracompact and finally compact spaces using ω-mappings.

Theorem 6. Let $f:(X,U)\to(Y,V)$ be ω-mapping of the uniform space (X,U) into the uniform space (Y,V). If a uniform space (Y,V) is strongly uniformly τ-finally paracompact, then the space (X,U) is also strongly uniformly τ-finally paracompact.

Proof. Let ω be an arbitrary finitely additive open cover of the space (X,U). Then for each point $y\in Y$ there is a neighborhood $O_y\ni y$ such that $O_y\subset W$ for some $W\in\omega$. Let $\beta=\{O_y : y\in Y\}$. Then there exists a uniformly star-finite open cover γ of cardinality $\le\tau$ of the space (Y,V) such that $\gamma\succ\beta$. It follows from Lemma 2 that $f^{-1}\gamma$ is a uniformly star-finite open cover of cardinality $\le\tau$ of the space (X,U). It is clear that $f^{-1}\gamma\succ\omega$. Consequently, (X,U) is strongly uniformly τ -finally paracompact.

Corollary 1. Under perfect (uniformly perfect) mappings, strong uniformly τ-finally paracompactness is preserved towards the inverse image.

The proof follows from Lemmas 1 and 2.

REFERENCES

1. Arkhangelskii A.V. On the coincidence of dimensions $indG$ and $\dim G$ // Dokl. Academy of Sciences of the USSR. - 1960. - T.132, No. 5. - S. 980-981.
2. Borubaev A.A. Uniform spaces and uniformly continuous mappings. - Frunze: Ilim, 1990. - 172 p.
3. Kanetov B.E. Some classes of uniform spaces and uniformly continuous mappings. - Bishkek: KNU im. Zh. Balasagyna, 2013. - 160 p.
4. Ponomarev V.I. On paracompact and finally compact spaces // Dokl. Academy of Sciences of the USSR. - 1961. - T.141, No. 3. - P.56-563.
5. Rice M.D. A note on uniform paracompactness // Proc. amer. Math. soc. - 1977. - Vol. 62. - No. 2. - P. 359-362.
6. Tamano H. On paracompactness // Pacific. J Math. - 1960. - Vol. 10. - P. 1043-1047.

MSC 54E15

ABOUT REMAINDERS OF UNIFORMLY CONTINUOUS MAPPINGS

Kanetov B.E. [1]**, Esenkanova N.** [2]
[1,2]*Kyrgyz National University named after J. Balasagun*

A uniform space can be considered as a special case of a uniformly continuous mapping, identifying the uniform space under consideration with its uniformly continuous mapping to a point. Therefore, the idea arises of extending to uniformly continuous mappings the concepts and statements available for uniform spaces. The method of mappings is of universal importance for the classification of uniform spaces. The mutual classification of uniform spaces and uniformly continuous mappings is an important trend in modern uniform topology. In this article, we study some properties of remainders of uniformly continuous mappings.

Key words: uniformly continuous mapping, remainders, Samuel compactification, locally perfect mapping.

Каралып жаткан бир калыптуу мейкиндикти бир калыптуу үзгүлтүксүз чагылдыруу менен туюндуруп бир калыптуу мейкиндикти бир калыптуу үзгүлтүксүз чагылдыруунун айрым учуру катары кароого мүмкүн болот. Ошондуктан бир калыптуу мейкиндикке тиешелүү болгон түшүнүктөрдү жана натыйжаларды бир калыптуу үзгүлтүксүз чагылдырууларга жайылтуу идеясы келип чыгат. Чагылдыруу ыкмасы бир калыптуу мейкиндиктерди квалификациялоодо универсалдуу мааниге ээ. Бир калыптуу мейкиндиктерди жана бир калыптуу үзгүлтүксүз чагылдырууларды өз ара квалификациялоо заманбап топологиянын маанилүү багыты болуп саналат. Бул макалада бир калыптуу үзгүлтүксүз чагылдыруулардын өсүндүлөрүнүн айрым касиеттери изилденет.

Урунттуу сөздөр: бир калыптуу үзгүлтүксүз чагылдыруулар, өсүндүлөр, Самуэлдик компактификация, локалдуу жеткилең чагылдыруу.

Равномерное пространство можно рассматривать как частный случай равномерно непрерывного отображения, отождествляя рассматриваемое равномерное пространство с равномерно непрерывным отображением его в точку. Поэтому возникает идея распространения на равномерно непрерывные отображения понятий и утверждений, имеющихся для равномерных пространств. Метод отображений имеет универсальное значение для классификации равномерных пространств. Взаимная классификация равномерных пространств и равномерно непрерывных отображений составляет важное направление современной равномерной топологии. В настоящей статье изучаются некоторые свойства наростов равномерно непрерывных отображений.
Ключевые слова: равномерно непрерывное отображение, наросты, Самуэловская компактификация, локально совершенное отображение.

One of the important concepts of the theory of uniform spaces are the concepts of completeness and completion of uniform spaces. These concepts were introduced and studied by A. Weil [7]. It is known [6] that for every uniformity U on a Tychonoff space X there exists a maximal precompact uniformity U_0 contained in the uniformity U. The completion (sX, sU_0) of the uniform space (X, U_0) is called the Samuel compactification of the space X with respect to this precompact uniformity U. It was established [5] that the Samuel compactification of the Tychonoff space X with respect to the universal uniformity on X coincides with the Stone-Ceh compactification of the space X.

The idea of extending some concepts of uniform spaces to uniformly continuous mappings led to the construction of an extensive theory. Thus, the concepts of completeness and completion have been transferred from uniform spaces to uniformly continuous mappings [1]. In particular, the Samuel compactification of uniform spaces is carried over to mappings, i.e. the concept of Samuel compactification of uniformly continuous mappings is introduced and studied.

Let $f:(X,U)\to(Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) into a uniform space (Y,V).

Recall [2] that a uniformly continuous mapping $\tilde{s}f:(\tilde{s}X,\tilde{s}U)\to(Y,V)$ of a uniform space $(\tilde{s}X,\tilde{s}U)$ into a uniform space (Y,V) is called the Samuel compactification of a mapping f if the following conditions are satisfied:

1. (X, U) is an everywhere dense subspace of a uniform space $(\hat{s}X, \hat{s}U)$.

2. $f = \hat{s}f\ |_X$.

3. The mapping $\hat{s}f$ is uniformly perfect.

It is known [2] that any uniformly continuous mapping $f : (X, U) \to (Y, V)$ of a uniform space (X, U) into a uniform space (Y,V) has a unique Samuel compactification.

The notion of a locally perfect mapping was introduced and studied in [4]. In [3], the notion of a uniformly locally perfect mapping was introduced and studied.

Theorem 1. A uniformly continuous mapping $f:(X,U)\to(Y,V)$ from a uniform space (X,U) to a uniform space (Y, V) is perfect if and only if X is a closed subset of the space $(\hat{s}X, \hat{s}U)$.

Proof. Necessity. Let $f:(X,U)\to(Y,V)$ be a perfect mapping from a uniform space (X,U) to a uniform space (Y,V) and let $\hat{s}f:(\hat{s}X,\hat{s}U)\to(Y,V)$ be a Samuel compactification. Then $f=\hat{s}f$. Therefore, X is a closed subset of the space $(\hat{s}X,\hat{s}U)$.

Sufficiency. Let $f:(X,U)\to(Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) into a uniform space (Y,V) and $\hat{s}f:(\hat{s}X,\hat{s}U)\to(Y,V)$ be its Samuel compactification. Let X be closed in $(\hat{s}X,\hat{s}U)$. Then the mapping $f=\hat{s}f\ |_X$ is a perfect mapping, as the restriction of the mapping $f:(X,U)\to(Y,V)$ to the space (X,U).

Corollary 1. A uniformly continuous mapping $f : (X, U) \to (Y, V)$ from a uniform space (X,U) to a uniform space (Y,V) is uniformly perfect if and only if X is a closed subset of the space $(\hat{s}X,\hat{s}U)$.

Corollary 2. A uniform space (X,U) is compact if and only if X is a closed subset of the space (sX,sU).

Let $f : (X, U) \to (Y, V)$ be a uniformly continuous mapping and (X_0, U_0) is subspace of a uniform space (X, U). The restriction of $f_0 = f\ |_{X_0}:(X_0,U_0)\to(Y,V)$ is called a submapping of $f:(X,U)\to(Y,V)$.

f_0 is said to be a closed submapping if X_0 is closed in (X,U).

Theorem 2. If $f:(X,U)\to(Y,V)$ is uniformly continuous and the submapping $f_0 = f|_{X_0}: (X_0, U_0) \to (Y,V)$ is a uniformly perfect mapping, then f_0 is a closed submapping.

In this sense, uniformly perfect mappings in the class of uniformly continuous mappings have the property of absolute closure, like compact spaces in the class of separable uniform spaces. Recall [2] that a uniform space (X,U) is called (strongly) uniformly locally compact if there exists a (locally finite) uniform cover $\alpha \in U$ consisting of compact subsets.

Theorem 3. Any uniformly continuous mapping $f:(X,U)\to(Y,V)$ from a locally compact uniform space (X,U) into a uniform space (Y,V) is a locally perfect mapping.

Proof. Let (X,U) be a locally compact space and $f:(X,U)\to(Y,V)$ a uniformly continuous mapping of a uniform space (X,U) into a uniform space (Y,V). Let x be an arbitrary point of the uniform space (X,U). Then, due to the local compactness of the space (X,U), there exists an open neighborhood O of the point $x \in X$ such that $[O]$ is compact. The restriction $f|_{[O]}$ of the mapping f to the compact set $[O]$ is perfect. It is clear that $f[O]$ is closed in (Y,V). Therefore, f is a perfect mapping.

Corollary 3. Any uniformly continuous mapping $f:(X,U)\to(Y,V)$ of a uniformly locally compact uniform space (X,U) into a uniform space (Y,V) is a locally perfect mapping.

Corollary 4. Any uniformly continuous mapping $f:(X,U)\to(Y,V)$ of a trongly uniformly locally compact uniform space (X,U) into a uniform space (Y,V) is locally perfect.

Theorem 4. A uniformly continuous mapping $f:(X,U)\to(Y,V)$ from a uniform space (X,U) to a uniform space (Y,V) is locally perfect if and only if X is an open subset of the space $(\hat{s}X, \hat{s}U)$.

Proof. Necessity. Let $f:(X,U)\to(Y,V)$ be a locally perfect mapping of a uniform space (X,U) into a uniform space (Y,V) and $\hat{s}f:(\hat{s}X,\hat{s}U)\to(Y,V)$ is the Samuel compactification of a mapping f. Then for any point $x\in X$ there exists an open set O and a closed subspace (H,U_H) of a uniform space (X,U) such that $x\in O\subset H$, the mapping $f|_H$ is perfect, and fH is closed. Since H is closed in $(\hat{s}X,\hat{s}U)$, then $[O]_{\hat{s}X}\subset[H]_{\hat{s}X}=H\subset X$. Let $\hat{s}O=\hat{s}X\setminus[X\setminus O]_{\hat{s}X}$. Then $\hat{s}O=\hat{s}X\setminus[X\setminus O]_{\hat{s}X}$. The space (X,U) is everywhere dense in $(\hat{s}X,\hat{s}U)$, since $\hat{s}O\setminus[O]_{\hat{s}X}=\varnothing$. Thus, $O=\hat{s}O$. Therefore, X is open in $(\hat{s}X,\hat{s}U)$.

Sufficiency. Let X be open in $(\hat{s}X,\hat{s}U)$ and $x\in X$ is an arbitrary point. Then there is an open neighborhood O in $(\hat{s}X,\hat{s}U)$ such that $[O]_{\hat{s}X}\subset X$. Put $H=[O]_{\hat{s}X}$. Then the restriction $\hat{s}f|_H$ of the uniformly continuous mapping $\hat{s}f$: $(\hat{s}X,\hat{s}U)\to(Y,V)$ to the closed subspace (H,U_H) is perfect. It is easy to see that $\hat{s}f|_H=f|_H$ and fH are closed.

Corollary 5. A uniformly continuous mapping $f:(X,U)\to(Y,V)$ from a uniform space (X,U) to a uniform space (Y,V) is uniformly locally perfect if and only if X is an open subset of the space $(\hat{s}X,\hat{s}U)$.

Corollary 6. A uniform space (X,U) is locally compact if and only if X is an open subset of the space (sX,sU).

Corollary 7. If $f:(X,U)\to(Y,V)$ is a uniformly perfect mapping and A is an open set in (X,U), then the restriction of $f|_A$ is locally perfect.

Let $\hat{s}f:(\hat{s}X,\hat{s}U)\to(Y,V)$ be the Samuel compactification of a uniformly continuous mapping $f:(X,U)\to(Y,V)$. The restriction $\hat{f}|_{\hat{X}\setminus X}:(\hat{s}X_{i-1}\setminus X_{i-1},\hat{s}U_{\hat{s}X_{i-1}\setminus X_{i-1}})\to(Y,V)$ of a mapping $\hat{s}f$ on a uniform space $(\hat{s}X_{i-1}\setminus X_{i-1},\hat{s}U_{\hat{s}X_{i-1}\setminus X_{i-1}})$ is called the remainder of the Samuel compactification of a uniformly continuous mapping $f:(X,U)\to(Y,V)$.

Theorem 5. The remainder $\hat{s}f|_{\hat{s}X\setminus X}:(\hat{s}X\setminus X,\hat{s}U_{\hat{s}X\setminus X})\to(Y,V)$ of a uniformly continuous mapping $f:(X,U)\to(Y,V)$ is perfect if and only if the uniformly continuous mapping $f:(X,U)\to(Y,V)$ is locally perfect.

Proof. Let the remainder $\hat{s}f|_{\hat{s}X\setminus X}:(\hat{s}X\setminus X,\hat{s}U_{\hat{s}X\setminus X})\to(Y,V)$ of a uniformly continuous mapping f be perfect. Then $\hat{s}X\setminus X$ will be closed in the space $(\hat{s}X,\hat{s}U)$, i.e. X is open in $(\hat{s}X,\hat{s}U)$. Thus, according to Theorem 4, the mapping f is locally perfect.

Conversely, let $f:(X,U)\to(Y,V)$ be locally perfect. Then, according to Theorem 4, the set X is open in $(\hat{s}X,\hat{s}U)$, i.e., the set $\hat{s}X\setminus X$ is closed in $(\hat{s}X,\hat{s}U)$. Therefore, the mapping $\hat{s}f|_{\hat{s}X\setminus X}:(\hat{s}X\setminus X,\hat{s}U_{\hat{s}X\setminus X})\to(Y,V)$ is perfect.

Corollary 8. The remainder $\hat{s}f|_{\hat{s}X\setminus X}:(\hat{s}X\setminus X,\hat{s}U_{\hat{s}X\setminus X})\to(Y,V)$ of a uniformly continuous mapping $f:(X,U)\to(Y,V)$ is uniformly perfect if and only if the uniformly continuous mapping $f:(X,U)\to(Y,V)$ is uniformly locally perfect.

Corollary 9. The remainder $(sX\setminus X,sU_{sX\setminus X})$ of a uniform space (X,U) is compact if and only if the uniform space (X,U) is locally compact.

REFERENCES

1. Borubaev A.A. Uniform spaces and uniformly continuous mappings. - Frunze: Ilim, 1990. - 172 p.
2. Kanetov B.E. Some classes of uniform spaces and uniformly continuous mappings. - Bishkek, 2013. - 160 p.
3. Kasymova T.D. On complete and uniformly perfect mappings and their inverse spectra: Cand. for the competition scientist degree cand. Phys.-Math. Sciences. - Bishkek, 1998. - 97 p.
4. N. Krolevets, On locally perfect mappings // Dokl. Academy of Sciences of the USSR. - 1967. - T. 175. - S. 1008-1011.
5. Engelking R. General topology. - M.: Mir, 1986. - 752 p.
6. Samuel P. Ultrafilters and compactifications of uniform spaces // Trans. Amer. Math. Soc. - 1948. - Vol. 64. - P. 100-132.
7. Weil A. Sur les espaces a structure uniforme et sur la topologie generale, Act. sci. Ind. no. 551. - Paris, 1937.

MSC 54E15

ON THE UNIFORMLY ANALOG OF COUNTABLY PARACOMPACT SPACES

Kanetov B.E. [1]**, Urgaziev A.B.** [2]
[1,2]*Kyrgyz National University named after J. Balasagun*

Countable uniform paracompactness is one of the most important uniform analogues of compactness, which was studied by A. Hohti [4] and U. Marconi [5]. At present paper countably uniformly paracompact space is studied. A uniform space is called countably uniformly paracompact, if every countably open covering has an open uniformly locally finite refinement. The topology of countably uniformly paracompact is countably paracompact. If we have countably paracompact space, then uniform space with the universally uniformity is countably uniformly paracompact. Any closed subspace of a countably uniformly paracompact space is countably uniformly paracompact. It is proved that a uniform space is countably uniformly paracompact if and only if every finitely additive countably open covering has an open uniformly locally finite refinement. It is established that for perfect mappings the countably uniform paracompactness is preserved in the direction of the image and in the direction of the preimage. If a uniform space is countably uniformly paracompact then every finitely open covering has an open uniformly locally finite refinement. The sum of finitely family countably uniformly paracompact spaces is countably uniformly paracompact. The product a countably uniformly paracompact space onto a compact space is a countably uniformly paracompact. The space of real numbers (with natural uniformity) is a countably uniformly paracompact space. A discrete uniformly space is a countably uniformly paracompact space.

Keywords: uniform space, countably uniformly paracompact space, finitely additive countable covering, countable open covering, uniform covering, uniformly continuous mapping.

Санактуу бир калыптуу паракомпакттуулук компакттуулуктун негизги бир калыптуу аналогдорунун бири болуп саналат. Аны А. Хохти [4] жана У. Маркони [5] изилдешкен. Сунуш кылынган илимий макалада санактуу бир калыптуу паракомпакттуу мейкиндиктер изилденет. Бир калыптуу мейкиндик санактуу бир калыптуу паракомпакттуу деп аталат, эгерде ар бир ачык жабдууга бир калыптуу локалдуу чектүү ачык жабдууну ичтен сызууга мүмкүн болсо. Санактуу бир калыптуу паракомпакттуу мейкиндиктин топологиясы санактуу паракомпакттуу болот. Эгер санактуу паракомпакттуу мейкиндик берилсе, анда бир калыптуу мейкиндик универсалдуу бир калыптуулугу менен санактуу бир калыптуу паракомпакттуу болот. Санактуу бир калыптуу паракомпакттуу мейкиндиктин каалагандай туюк камтылган мейкиндиги санактуу бир калыптуу паракомпакттуу болот. Бир калыптуу мейкиндик санактуу бир калыптуу паракомпакттуу мейкиндик болот, качан гана анын ар бир чектүү аддитивдүү санактуу ачык жабдуусуна бир калыптуу локалдуу чектүү ачык жабдууну ичтен сызууга мүмкүн болсо. Санактуу бир калыптуу паракомпакттуулук бир калыптуу жеткилең чагылдырууда образ жагына да, прообраз жагына да сакталышы тургузулган. Эгер бир калыптуу мейкиндик санактуу бир калыптуу паракомпакттуу болсо, анда анын ар бир чектүү ачык жабдууга бир калыптуу локалдуу чектүү ачык жабдууну ичтен сызууга мүмкүн болот. Чектүү сандагы санактуу бир калыптуу паракомпакттуу мейкиндиктердин суммасы санактуу бир калыптуу паракомпакттуу болот. Санактуу бир калыптуу паракомпакттуу мейкиндиктин компакттуу бир калыптуу мейкиндикке болгон көбөйтүндүсү санактуу бир калыптуу паракомпакттуу болот. Анык сандардын мейкиндиги (табигый бир калыптуулугу менен) санактуу бир калыптуу паракомпакттуу болот. Дискреттик бар калыптуу мейкиндик санактуу бир калыптуу паракомпакттуу болот.

Урунттуу сөздөр: бир калыптуу мейкиндик, санактуу бир калыптуу паракомпакттуу мейкиндик, чектүү аддитивдүү санактуу жабдуу, санактуу ачык жабдуу, бир калыптуу жабдуу, бир калыптуу үзгүлтүксүз чагылдыруу.

Счетная равномерная паракомпактность является одним из важнейших равномерных аналогов компактности, которая исследовалась А. Хохти [4] и У. Маркони [5]. В предлагаемой статье исследуется счетно равномерно паракомпактное пространство. Равномерное пространство называется счетно равномерно паракомпактным, если в каждое его счетно открытое покрытие можно вписать равномерно локально конечное открытое покрытие. Топология счетно равномерно паракомпактного пространства является счетно паракомпактным. Если задано счетно паракомпактное пространство, то равномерное пространство с универсальной равномерностью является счетно равномерно паракомпактным. Замкнутое подпространство счетно равномерно паракомпактного пространства является счетно равномерно паракомпактным. Доказано, что равномерное пространство является счетно равномерно паракомпактным тогда и только тогда, когда в каждое его конечно аддитивное счетное открытое покрытие можно вписать равномерно локально конечное открытое покрытие. Установлено, что при равномерно совершенных отображениях счетно равномерно паракомпактность сохраняется как в сторону образа, так и в сторону прообраза. Если равномерное пространство является счетно равномерно паракомпактным, то в каждое его конечное открытое покрытие можно вписать равномерно локально конечное открытое покрытие пространства. Сумма конечного семейства счетно равномерно паракомпактных пространств счетно равномерно паракомпактна. Произведение счетно равномерно паракомпактного пространства на компактное равномерное пространство является счетно равномерно паракомпактным. Пространство вещественных чисел (с естественной равномерностью) является счетно равномерно паракомпактным. Дискретное равномерное пространство является счетно равномерно паракомпактным.
Ключевые слова: равномерное пространство, счетно равномерно паракомпактное пространство, конечно аддитивное счетное покрытие, счетное открытое покрытие, равномерное покрытие, равномерно непрерывное отображение.

Definition 1. A covering α of the uniform space (X,U) is called uniformly locally finite, if there exists a uniform covering $\beta \in U$ such that every $B \in \beta$ meets α only for a finite number of elements of α.

Definition 2. A uniform space (X,U) is called countably uniformly paracompact, if every countably open covering has an open uniformly locally finite refinement.

Proposition 1. If a uniform space (X,U) is countably uniformly paracompact then the topological space (X,τ_U) is countably paracompact. Conversely, if (X,τ) is countably paracompact space then the uniform space (X,U_X) is countably uniformly paracompact, where U_X is a universally uniformities of the space (X,τ).

Proof. Let (X,U) be countably uniformly paracompact space and α be an arbitrary countably open covering of the space (X,τ_U). Since the space (X,U) is countably uniformly paracompact, there exists a uniformly locally finite open covering β of the space (X,U) which is a refinement of covering α. Every uniformly locally finite covering is locally finite. Therefore covering β is locally finite open covering. Thus, locally finite open covering β refined in the countably open covering α. Consequently, the space (X,τ_U) is countably paracompact.

Conversely, let (X,τ) be countably paracompact space and α be an arbitrary countably open covering of the space (X,U_X). Since the space (X,τ) is countably paracompact, there exists a locally finite open covering β which is a refinement of covering α. For every point $x\in X$ there exists a neighborhood O_x that intersects only with a finite number of elements of covering β. Denote $\gamma=\{O_x : x\in X\}$. Since the system of all open coverings forms a base of universal uniformity U_X of the space (X,τ), $\gamma\in U_X$. Consequently, β is uniformly locally finite open covering.

Lemma 1. Let α and β be a covering of the space (X,U) and $M\subset X$ be any subset. If the covering α is refined in the covering β then the covering α_M is also refined in the covering β_M, where $\alpha_M=\alpha\wedge\{M\},\beta_M=\beta\wedge\{M\}$.

Proof. Let $A_M\in\alpha_M$, where $A_M=A\cap M$. The covering α is a refinement of the covering β, then for $A\in\alpha$ there exists $B\in\beta$ such that $A\subset B$. It follows from this that $A\cap M\subset B\cap M$, where $B\cap M\in\beta_M$. Consequently, the covering α_M is refined in the covering β_M.

Proposition 2. Any closed subspace of a countably uniformly paracompact space is countably uniformly paracompact.

Proof. Let (M,U_M) be closed subspace of a countably uniformly paracompact space (X,U). Let α_M be an arbitrary countably open covering of the space (M,U_M). Denote $\alpha=\{\alpha_M, X\setminus M\}$. Obviously the covering α is countably open covering of the space (X,U). Since the space (X,U) is countably uniformly paracompact, there exists a uniformly locally finite open covering β of the space (X,U) which is a refinement of covering α. Let $\beta_M = \beta \wedge \{M\}$. Since β is uniformly locally finite open covering of the space (X,U), there exists a uniform covering $\gamma \in U$ such that every $\Gamma \in \gamma$ meets β only for a finite number of elements of β. Let $\gamma_M = \gamma \wedge \{M\}$. For every $\Gamma \in \gamma$ there exists $B_1, B_2, ..., B_k \in \beta$ such that $\Gamma \subset \bigcup_{i=1}^{k} B_i$. Then $\Gamma \cap M \subset \left(\bigcup_{i=1}^{k} B_i \right) \cap M = \bigcup_{i=1}^{k} (B_i \cap M)$. Thus, every element of covering $\gamma_M \in U_M$ meets β_M only for a finite number of elements of β_M. Therefore β_M is uniformly locally finite open covering of the space (M, U_M). Since the covering β is refined in the covering α, then by lemma 1 the covering β_M is refined in the covering α_M. Consequently, (M, U_M) is countably uniformly paracompact.

Theorem 1. A uniform space (X,U) is countably uniformly paracompact if and only if every finitely additive countably open covering has an open uniformly locally finite refinement.

Proof. Necessity. Let (X,U) be a countably uniformly paracompact space and α be an arbitrary finitely additive countably open covering of the space (X,U). Since α is countably open covering, then covering α has an open uniformly locally finite refinement.

Sufficiency. Let α be an arbitrary countably open covering of the space (X,U). Denote $\alpha^{\angle} = \{\cup \alpha_0 : \alpha_0 \subset \alpha$ is finite$\}$. Since the set of all finite subsets of a countable set is countable, then $\alpha^{\angle}$ is finitely additive countably open covering. Accordingly to the condition of the theorem, there exist an open uniformly locally finite covering β refined in the covering $\alpha^{\angle}$. For every $B \in \beta$ there exists $\bigcup_{i=1}^{k} A_i \in \alpha^{\angle}, A_i \in \alpha$ such that $B \subset \bigcup_{i=1}^{k} A_i$. Denote $\lambda = \cup\{\lambda_B : B \in \beta\}$, $\lambda_B = \{B \cap A_i : i = 1, 2, ..., k\}$. Then λ is an uniformly locally finite open covering refined in the countably open covering α. Consequently, (X,U) is countably uniformly paracompact.

Lemma 2. A uniform space (X,U) is countably uniformly paracompact if and only if $\alpha^{\angle}\in U$ for any countably open covering α of the space (X,U).

Proof. Necessity. Let (X,U) be a countably uniformly paracompact space and α be an arbitrary countably open covering of the space (X,U). Then there exist an open uniformly locally finite covering β of the space (X,U) refined in the covering α. By definition of uniform local finiteness of covering β there exists a uniform covering $\gamma\in U$ such that every $\Gamma\in\gamma$ meets β only for a finite number of elements of β. Then each element $\Gamma\in\gamma$ is contained in some set $B_{\Gamma}\in\beta^{\angle}$, where $B_{\Gamma}=\bigcup_{i=1}^{k}B_i$. Consequently, the covering γ is a refinement of the covering $\beta^{\angle}$. It follows from this that the covering γ is a refinement of the covering $\alpha^{\angle}$. By axiom (P1) of the definition of a uniform space $\alpha^{\angle}\in U$.

Sufficiency. Let $\alpha^{\angle}\in U$ for any countably open covering α of the space (X,U). By proposition 3 [5] the covering α has an open uniformly locally finite refinement. Then the uniform space (X,U) is countably uniformly paracompact.

Lemma 3. Let $f:(X,U)\to(Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) onto a uniform space (Y,V). If β is open covering of space (Y,V) then $(f^{-1}\beta)^{\angle}=f^{-1}\beta^{\angle}$.

Proof. Let $\bigcup_{i=1}^{n}f^{-1}B_i\in(f^{-1}\beta)^{\angle}$. Then $\bigcup_{i=1}^{n}f^{-1}B_i=f^{-1}\bigcup_{i=1}^{n}B_i$. Obviously that $f^{-1}\bigcup_{i=1}^{n}B_i\in f^{-1}\beta^{\angle}$. Consequently, $(f^{-1}\beta)^{\angle}=f^{-1}\beta^{\angle}$.

Lemma 4. Let $f:(X,U)\to(Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) onto a uniform space (Y,V). If α is finitely additive open covering of space (X,U) and $\beta=\{f^{\#}A:a\in\alpha\}$, where $f^{\#}A=Y\setminus f(X\setminus A)$, then the covering $f^{-1}\beta^{\angle}$ is refined in the covering α.

Proof. Let $f^{-1}f^{\#}A\in f^{-1}\beta$. Then $f^{-1}(Y\setminus f(X\setminus A))=X\setminus f^{-1}f(X\setminus A)$. Since $X\setminus A\subset f^{-1}f(X\setminus A)$, then $X\setminus f^{-1}f(X\setminus A)\subset X\setminus(X\setminus A)=A$. Consequently, the covering $f^{-1}\beta$ is refined in the covering α. It follows from this that the covering $f^{-1}\beta^{\angle}$ is refined in the covering α.

Theorem 2. Let $f:(X,U)\to(Y,V)$ be a uniformly perfect mapping of a uniform space (X,U) onto a uniform space (Y,V). Then the countably uniform paracompactness is preserved in the direction of the image and in the direction of the preimage.

Proof. . Let (X,U) be a countably uniformly paracompact space and β be an arbitrary finitely additive countably open covering of the space (Y,V). Then $f^{-1}\beta$ is finitely additive countably open covering of the space (X,U) and by lemma 2 $f^{-1}\beta\in U$. Since the mapping f is precompact, there exist such a covering $\lambda\in V$ and finite covering $\gamma\in U$ that $f^{-1}\lambda\wedge\gamma\succ f^{-1}\beta$. It is easy to see that $(f^{-1}\lambda\wedge\gamma)^{\angle}\succ(f^{-1}\beta)^{\angle}$. Since $(f^{-1}\lambda\wedge\gamma)^{\angle}=(f^{-1}\lambda)^{\angle}$ and $(f^{-1}\beta)^{\angle}=f^{-1}\beta^{\angle}$, then $(f^{-1}\lambda)^{\angle}\succ f^{-1}\beta^{\angle}$. Consequently, the covering λ is a refinement of the covering β. Then $\beta\in V$ and by lemma 2 the space (Y,V) is countably uniformly paracompact.

Conversely, let (Y,V) be a countably uniformly paracompact space and α be an arbitrary finitely additive countably open covering of the space (X,U). It is easy to see that the family $\{f^{-1}y:y\in Y\}$ of all compact sets $f^{-1}y$ is a refinement of the covering α. Since the mapping f is closed, $\beta=\{f^{\#}A:A\in\alpha\}$ is countably open covering of the space (Y,V), where $f^{\#}A=Y\setminus f(X\setminus A)$. Then by virtue of lemma 2 $\beta^{\angle}\in V$. By lemma 4 the covering $f^{-1}\beta^{\angle}$ is refined in the covering α. Since $f^{-1}\beta^{\angle}\in U$, then $\alpha\in U$. Consequently, by virtue of lemma 2 (X,U) is countably uniformly paracompact.

Proposition 3. If a uniform space (X,U) is countably uniformly paracompact then every finitely open covering has an open uniformly locally finite refinement.

Proof. Let (X,U) be countably uniformly paracompact space and α be an arbitrary finitely open covering of the space (X,U). Let λ be an arbitrary countably open covering of the space (X,U). It is easy to see that $\alpha \wedge \lambda$ is countably open covering of the space (X,U). Consequently, there exist an open uniformly locally finite covering β of the space (X,U) refined in the covering $\alpha \wedge \lambda$. Since the covering $\alpha \wedge \lambda$ is a refinement of the covering α, the covering β is a refinement of the finitely open covering α.

Proposition 4. The sum of two countably uniformly paracompact spaces is countably uniformly paracompact.

Proof. Let (X_1,U_1) and (X_2,U_2) are countably uniformly paracompact spaces, $(X_1 \coprod X_2, U_1 \coprod U_2)$ is the sum of countably uniformly spaces. Consider an arbitrary countably open covering α of the space $(X_1 \coprod X_2, U_1 \coprod U_2)$. It is easy to see that $\beta = \{X_i \cap A : A \in \alpha, i = 1,2\}$ is again a countably open covering of the space $(X_1 \coprod X_2, U_1 \coprod U_2)$ is a refinement α. Let $\beta_1 = \{X_1 \cap A : A \in \alpha\}$ and $\beta_2 = \{X_2 \cap A : A \in \alpha\}$. Then at least one of these coverings is countably open. Let β_1 be countably open covering of the space (X_1,U_1), then β_2 is finitely open covering of the space (X_2,U_2). Since (X_1,U_1) and (X_2,U_2) are countably uniformly paracompact spaces, there exists an open uniformly locally finite covering γ_1 of the space (X_1,U_1) refined in the covering β_1 and an open uniformly locally finite covering γ_2 of the space (X_2,U_2) refined in the covering β_2. It is clear that a family γ that is a union of families γ_1 and γ_2 is a uniformly locally finite open covering of the space $(X_1 \coprod X_2, U_1 \coprod U_2)$. And the covering γ is a refinement of the covering α.

Corollary 1. The sum of finitely family countably uniformly paracompact spaces is countably uniformly paracompact.

Proposition 5. The product $(X,U) \times (Y,V)$ a countably uniformly paracompact space (X,U) onto a compact space (Y,V) is a countably uniformly paracompact.

Proof. Let (X,U) be a countably uniformly paracompact space and (Y,V) be a compact uniform space. It is known that if (X,U) is an arbitrary uniform space and (Y,V) is a compact uniform space then the projection $\pi_X:(X,U)\times(Y,V)\to(X,U)$ is uniformly perfect. Then by theorem 2 the product $(X,U)\times(Y,V)$ the countably uniformly paracompact space (X,U) onto the compact space (Y,V) is a countably uniformly paracompact.

Proposition 6. The space of real numbers (with natural uniformity) is a countably uniformly paracompact space.

Proof. Let (R,U_R) be a uniform space, R be a real line and U_R be a natural uniformity on the R. Let's show that the space (R,U_R) is countably uniformly paracompact. Let α be an arbitrary countably open covering of the space (R,U_R). It is enough to show that $\alpha^{\angle}\in U_R$. Let $\beta=\{(n-1;n+1):n\in Z\}$ be some element of the uniformity U_R. It is easy to see that β is uniformly locally finite open covering. Note that $[n-1:n+1]$ is compact. Therefore, there exists the finite family $\{A_1,A_2,...,A_n\}\subset\alpha$ such that $[n-1;n+1]\subset\bigcup_{i=1}^{n}A_i$, i.e. $(n-1;n+1)\subset\bigcup_{i=1}^{n}A_i$. It is clear that $\bigcup_{i=1}^{n}A_i=A^{\angle}\in\alpha^{\angle}$. Hence, the covering β is a refinement of the covering $\alpha^{\angle}$. According to the axiom (P1) of uniformity, $\alpha^{\angle}\in U_R$. Consequently, (R,U_R) is countably uniformly paracompact.

Proposition 7. A discrete uniformly space is a countably uniformly paracompact space.

Proof. Let (X,U) be a discrete uniformly space and α be an arbitrary countably open covering of the space (X,U). The family $\mathcal{B}=\{\{\{x\}:x\in X\}\}$ forms a base of discrete uniformity U. Then the covering $\beta=\{\{x\}:x\in X\}$ is a refinement of the countably open covering α. It is clear that α is uniformly locally finite open covering of the space (X,U). Consequently, (X,U) is countably uniformly paracompact.

REFERENCES

1. Archangielski, A.V. Fundamentals of General topology in tasks and exercises [Text] / A.V. Archangielski, V.I. Ponomarev. – Moscow: Nauka, 1974. – 424 p. (In Russian)
2. Borubaev, A.A. Uniform Topology [Text] / A.A. Borubaev. – Bishkek: Ilim, 2013. – 347 p. (In Russian)
3. Engelking, R. General topology [Text] / R. Engelking. – Moscow: Mir, 1986. – 752 p. (In Russian)
4. Hohti, A. On uniform paracompactness [Text] / A. Hohti // Ann. Acad. Sc. Fenn., ser. A, Math. Diss., – 1981. – No. 36.
5. Marconi, U. on the uniform paracompactness [Text] / U. Marconi // Rendiconti del Seminario Matematico della Universita di Padova. – 1984. – T.72. – P. 319-328.

MSC 54E15

SOME PROPERTIES OF REMAINDERS OF TOPOLOGICAL GROUPS

Kanetov B.E.[1], Abdykaimov I.Z.[2]

[1,2]Kyrgyz National University named after J. Balasagyn

In this article some questions about remainder of the uniformed space are considered. Uniformed spaces have their own theoretical sphere of interesting and important mathematical problems. The reason for this is that there are many interconnection between the theory of uniformed spaces and the general topology. All completely regular spaces can be characterized as topological space, inducted by some uniform structure. Some special constructions of compactifications of topological spaces can be determined using the completion of uniformed spaces, having precompact uniform structure. Such as there is big meaning of the uniform structures for topological groups: possibility of the completion of the topological group depends on uniformed structure, which is considered to be agreed with topology. Under what necessary and sufficient conditions does its remainders imposed on uniform spaces have some uniform property? For uniform spaces this question has answer, responding for it. This work is trying to determine such conditions for some natural uniformed structures on topological groups.

Keywords: uniform structure, topological group, free Cauchy filter, co-cover.

Бул макалада бир калыптуу мейкиндиктердин өсүндүлөрү жөнүндөгү бир нече суроолор каралат. Бир калыптуу мейкиндиктер өзүдүк теориялык областка, кызыктуу жана маанилүү көгөйлөргө ээ. Себеби, бир калыптуу мейкиндиктердин теориясы жана топологиянын арасында көп байланышуулук бар. Толук регулярдуу мейкиндиктердин бардыгын кандайдыр бир калыптуу структура аркылуу берилген топологиялык мейкендик катары мүнөөздөөгө болот. Топологиялык мейкиндиктердин компактификациясын предкомпактуу бир калыптуу мейкиндиктердин толуктануусу катары аныктоого болот. Бир калыптуу структуралар топологиялык группалар үчүн маанилүү, мисалы, топологиялык группалардын толуктануусу бир калыптуу структуралар аркылуу аныкталат. Кандай зарыл жана жетиштүү шарттарда бир калыптуу мейкиндиктердин өсүндүлөрү айрым бир бир калыптуу касиетке ээ болот ? деген суроо жаралат. Бир калыптуу мейкиндиктер үчүн мындай суроо чечилген. Бул макалада топологиялык группалар үчүн бул маселе чечилет.

Урунттуу сөздөр: бир калыптуу структура, топологиялык группа, эркин Кошинин фильтри, ко-жабдуу.

В этой статье рассматриваются некоторые вопросы о наростах равномерного пространства. Равномерные пространства имеют свою собственную теоретическую область, интересных и важных математических проблем. Причина этого заключается в том, что существует много взаимосвязей между теорией равномерных пространств и общей топологией. Все вполне регулярные пространства могут быть охарактеризованы как топологические пространства, индуцированные некоторой равномерной структурой. Некоторые специальные конструкции компактификаций топологических пространств могут быть определены с помощью пополнения равномерных пространств, имеющих предкомпактную равномерность. Большое значение имеют равномерные структуры для топологических групп, например, пополнения топологической группы определяется через равномерной структуры. Возникает вопрос: при каких необходимых и достаточных условиях налагаемые на равномерные пространства его нарост обладает некоторым равномерным свойством? Для равномерных пространств этот вопрос имеет решение. В данной работе делается попытка определить такие условия для некоторых естественных равномерностей на топологических группах.

Ключевые слова: равномерная структура, топологическая группа, свободный фильтр Коши, ко-покрытие.

Let us see at the some uniformed space *(X, U)*, its completion $(\tilde{X},\tilde{U})$ and its remainder *(Y, V)*: $Y=\tilde{X}\setminus X$, $V=\tilde{U}\Big|_{\tilde{X}\setminus X}$.

Definition 1. (X, U) is called τ-bounded, if for any uniform covering α of the space (X,U) exists some its uniform covering β such, that $\beta\succ\alpha$ and β has cardinal τ .

Definition 2. (X, U) is called precompact, if for any uniform covering α of the space (X,U) exists some its uniform cover β such, that $\beta\succ\alpha$ and β is finite.

Definition 3. (X, U) is called I-lindelof, if for any uniform cover α of the space (X,U) exists some its uniform cover β such, that $\beta\succ\alpha$ and β has countably infinite cardinal.

Definition 4. (G,*, τ) is called topological group if (G,*) is group, (G, τ) is topological space and such operation as $f(x,y)=x*y^{-1}$ determines mapping, which is continuous in the (G, τ).

Definition 5. If (G,*, τ) is the topological group, then U_r is the right uniform structure of the (G,*, τ), if it consists of covers α_H such that $\alpha_H = \{H*x : x \in G\}$ for all H, which are elements of the basis of the identity element. Analogically to this: U_l is the left uniform structure of the (G,*, τ), if it consists of covers such that $\alpha_H = \{H*x : x \in G\}$ for all H, which are elements of the basis of the identity element.

Definition 6. Let us talk that the topological group (G,*, τ) is precompact, if uniformed space (G, U_r) is precompact as uniformed space, where U_l is the left uniform structure of the (G,*, τ).

Definition 7. Let us talk that the topological group (G,*, τ) is τ-bounded, if uniformed space (G, U_r) is τ-bounded as uniformed space, where U_l is the left uniform structure of the (G,*, τ).

Definition 8. Let α be a family of subsets of the topological group (G,*, τ). Then we will call α a co-cover of the (G,*, τ), if for every free Cauchy filter $\tilde{\Im}$ of the (G,*, τ) exists $F \in \tilde{\Im}$ which is common with, i.e.: $F \in \alpha$.

Theorem 1. (G,*, τ) has precompact remainder if and only if in every uniform cover α of the space (G,*, τ) there exists its subcover α_0, which is a finite co-cover.

Proof. Let (G,*, τ) has a precompact remainder and $\tilde{G}$ is its completion. Then for any uniformed cover $\alpha = \{x*H : x \in G\}$ of a U_l we can determine $\tilde{\alpha} = \{\tilde{x}*\tilde{H} : \tilde{x} \in \tilde{G}\}$, where $\tilde{H} = \tilde{G} \setminus [G \setminus H]_{\tilde{G}}$, *i:* $G \to \tilde{G}$ - is trivial injection: $\tilde{x} = i(x)$. From this we can see the track of the $\tilde{\alpha}$ at the remainder $\tilde{G} \setminus G$: $\hat{\alpha} = \tilde{\alpha} \cap (\tilde{G} \setminus G) \in \hat{U}_l$, where $\hat{U}_l$ is got as $\hat{U}_l = \tilde{U}_l \cap (\tilde{G} \setminus G)$. $(\tilde{G} \setminus G, \hat{U}_l)$ is precompact, that's why $\hat{\alpha}$ has a subcover $\hat{\alpha}_0 \in \hat{U}_l$, which is finite. Then there exists such $\tilde{\alpha}_0 = \{x*\tilde{H}_0 : x \in \tilde{X}\} \in \tilde{U}_l$, that its track on a (G,*, τ) is $\hat{\alpha}_0$, and $\tilde{\alpha}_0$ is finite with number of elements $n = card(\tilde{X})$, where $\tilde{X} \subset G$ what is implication of building of such covers as: $\tilde{\alpha}$ and $\tilde{\alpha}_0$, and of the including of $\tilde{\alpha}_0$ by $\tilde{\alpha}$. It is so because of the fact that completion of topological group by left uniform structure is topological group (in the case of its existing). And now let us look at the family of sets $\alpha_0 = \{x*H_0 : x \in i^{-1}(\tilde{X}) = X\} = \{x_i * H_0 : 1 \le i \le n\}$, which is got as a track of $\tilde{\alpha}_0$ on the *(G, U_l)*. Of course, α_0 is subset of α. Let us show that α_0 is co-covering on the *(G, U_l)*. Let us see at the free Cauchy filter $\Im$ in the space *(X,U_l)*. Then it is track of some Cauchy filter $\tilde{\Im}$ in the space $(\tilde{G},\tilde{U}_l)$, and $\tilde{\Im}$ has a limit on the $(\tilde{X},\tilde{U}_l)$, because $(\tilde{G},\tilde{U}_l)$ is complete. But because of $\Im$ is free, the condensation point $\hat{x}$ of $\tilde{\Im}$ lies in $(\tilde{G},\tilde{U}_l)$. Let us consider that $\tilde{B}(\hat{x})$ is filter of neighbourhoods of the point $\hat{x}$. Then let us think that $\Im_0 = \tilde{B}(\hat{x}) \cap X$. Then because of being by $\tilde{\Im}$ the Cauchy filter we notice that $\Im_0 \subset \Im$. But $\hat{\alpha}_0$ is cover, so there exists such $\hat{A}_0 \in \hat{\alpha}_0$, that $\hat{x} \in \hat{A}_0$, it means that exists some $1 \le i \le n$ such that $\hat{x} \in x_i * \tilde{H}_0$. It is clear that $x_i * \tilde{H}_0 = \tilde{A}_0 \in \tilde{B}(\hat{x})$, where $\tilde{A}_0$ is some neighbourhood, inducting the $\hat{A}_0$ as its track in the (G,*, τ). So the track of the $\tilde{A}_0$ in the (G,*, τ), which we will call as $A_0 = x_i * H_0 \in \Im_0$. It means that $A_0 \in \Im_0$ so as $A_0 \in \alpha_0$. From this and because of the definition of co-cover we can get that α_0 is co-cover in the (G,*, τ) as the family of sets, which has minimally one common element with every free Cauchy filter from *(G,U_l)*.

Let us make a proof for the inversed direction. Let us think that the condition, declared in the condition of the theorem, is realized. Let us prove that $(\tilde{G},\tilde{U}_l)$ is precompact uniformed space. Let $\hat{\alpha} \in \hat{U}$. Then it is got as the track of some uniform covering $\tilde{\alpha} = \{\tilde{x} * \tilde{H} : \tilde{x} \in \tilde{G}\}$ on the (G,*, τ): $\hat{\alpha} = \tilde{\alpha} \cap (\tilde{G} \setminus G)$. Then there exists some $\alpha = \{x * H : x \in G\}$ as a track of the $\tilde{\alpha}$ on the G. From the considered in the theorem we can get that it means that there exists $\alpha_0 = \{x_i * H_0 : 1 \le i \le n\}$, which is finite subset of α and is a co-cover. It means that the track of the continuation $\tilde{\alpha}_0 = \{x * \tilde{H}_0 : x \in \tilde{X} = i(X)\} = \{i(x_i) * \tilde{H}_0 : 1 \le i \le n\}$ of the α_0 on the $(\tilde{G},\tilde{U}_l)$, which we will call as $\hat{\alpha}_0$, will be a subset of the track of the continuation of the α on the $(\tilde{G},\tilde{U}_l)$, which is $\hat{\alpha}$ by the building of the α_0. But by the building: $\hat{\alpha}_0 \in \hat{U}_l$. So for any $\hat{\alpha}_0 \in \hat{U}_l$ by this way we are able to see that exists some its subset $\hat{\alpha}_0 \in \hat{U}_l$, having the elements, which's set has natural number of points. It means that $(\tilde{G}\backslash G,\hat{U}_l)$ has the property of precompactness. So the theorem is proved.

Notification 1. This result can be got with the right uniform structure on the topological group. Formally, proving of such result will be respectively similar to that, which is shown higher for the left uniform structure.

Theorem 2. (G,*, τ) has τ-bounded remainder if and only if in every uniform cover α of the space (G,*, τ) there exists its subcover α_0, which is a co-cover and has a cardinal τ.

Proof. Let (G,*, τ) has a τ-bounded remainder and $\tilde{G}$ is its completion. . Then for any uniformed cover $\alpha=\{x*H:x\in G\}$ of a U_l we can determine $\tilde{\alpha}=\{\tilde{x}*\tilde{H}:\tilde{x}\in\tilde{G}\}$, where $\tilde{H}=\tilde{G}\backslash[G\backslash H]_{\tilde{G}}$, *i:* $G\to\tilde{G}$ - is trivial injection: $\tilde{x}=i(x)$. From this we can see the track of the $\tilde{\alpha}$ at the remainder $\tilde{G}\backslash G$: $\hat{\alpha}=\tilde{\alpha}\cap(\tilde{G}\backslash G)\in\hat{U}_l$, where $\hat{U}_l$ is got as $\hat{U}_l=\tilde{U}_l\cap(\tilde{G}\backslash G)$. $(\tilde{G}\backslash G,\hat{U}_l)$ is τ-bounded, that's why $\hat{\alpha}$ has a subcover $\hat{\alpha}_0\in\hat{U}_l$, which has an cardinal τ. Then there exists such $\tilde{\alpha}_0=\{x*\tilde{H}_0:x\in\tilde{X}\}\in\tilde{U}_l$, that its track on a (G,*, τ) is $\hat{\alpha}_0$, and the cardinal of the $\tilde{\alpha}_0$ is $\tau=card(\tilde{X})$, where $\tilde{X}\subset G$ what is implication of building of such covers as: $\tilde{\alpha}$ and $\tilde{\alpha}_0$, and of the including of $\tilde{\alpha}_0$ by $\tilde{\alpha}$. It is so because of the fact that completion of topological group by left uniform structure is topological group (in the case of its existing). And now let us look at the sets-family $\alpha_0=\{x*H_0:x\in i^{-1}(\tilde{X})=X\}=\{x_i*H_0:1\le i\le n\}$, which is got as a track of $\tilde{\alpha}_0$ on the *(G, U_l)*. Of course, α_0 is subset of α. Let us show that α_0 is co-covering on the *(G, U_l)*. Let us see at the free Cauchy filter $\Im$ in the space *(X,U_l)*. Then it is track of some Cauchy filter $\tilde{\Im}$ in the space $(\tilde{G},\tilde{U}_l)$, and $\tilde{\Im}$ has a limit on the $(\tilde{X},\tilde{U}_l)$, because $(\tilde{G},\tilde{U}_l)$ is complete. But because of $\Im$ is free, the condensation point $\hat{x}$ of $\tilde{\Im}$ lies in $(\tilde{G},\tilde{U}_l)$. Let us consider that $\tilde{B}(\hat{x})$ is filter of neighbourhoods of the point $\hat{x}$. Then let us think that $\Im_0=\tilde{B}(\hat{x})\cap X$. Then because of being by $\tilde{\Im}$ the Cauchy filter we notice that $\Im_0\subset\Im$. But $\hat{\alpha}_0$ is cover, so there exists such $\hat{A}_0 \in \hat{\alpha}_0$, that $\hat{x} \in \hat{A}_0$, it means that exists some $1 \le i \le n$ such that $\hat{x} \in x_i * \tilde{H}_0$. It is clear that $x_i * \tilde{H}_0 = \tilde{A}_0 \in \tilde{B}(\hat{x})$, where $\tilde{A}_0$ is some neighbourhood, inducting the $\hat{A}_0$ as its track in the (G,*, τ). So the track of the $\tilde{A}_0$ in the (G,*, τ), which we will call as $A_0 = x_i * H_0 \in \Im_0$. It means that $A_0 \in \Im_0$ so as $A_0 \in \alpha_0$. From this and because of the definition of co-cover we can get that α_0 is co-cover in the (G,*, τ) as the family of sets, which has minimally one common element with every free Cauchy filter from *(G,U_l)*.

Let us make a proof for the inversed direction. Let us think that the condition, declared in the condition of the theorem, is realized. Let us prove that $(\tilde{G},\tilde{U}_l)$ is a τ-bounded uniformed space. Let $\hat{\alpha} \in \hat{U}$. Then it is got as the track of some uniform covering $\tilde{\alpha} = \{\tilde{x} * \tilde{H} : \tilde{x} \in \tilde{G}\}$ on the (G,*, τ): $\hat{\alpha} = \tilde{\alpha} \cap (\tilde{G} \setminus G)$. Then there exists some $\alpha = \{x * H : x \in G\}$ as a track of the $\tilde{\alpha}$ on the G. From the considered in the theorem we can get that it means that there exists $\alpha_0 = \{x_i * H_0 : 1 \le i \le n\}$, which is subset of α, and is a co-cover, and $X \subset G$ has the cardinal τ. It means that the track of the continuation $\tilde{\alpha}_0 = \{x * \tilde{H}_0 : x \in \tilde{X} = i(X)\} = \{i(x_i) * \tilde{H}_0 : 1 \le i \le n\}$ of the α_0 on the $(\tilde{G},\tilde{U}_l)$, which we will call as $\hat{\alpha}_0$, will be a subset of the track of the continuation of the α on the $(\tilde{G},\tilde{U}_l)$, which is $\hat{\alpha}$ by the building of the α_0. But by the building: $\hat{\alpha}_0 \in \hat{U}_l$. So for any $\hat{\alpha}_0 \in \hat{U}_l$ by this way we are able to see that exists some its subset $\hat{\alpha}_0 \in \hat{U}_l$, having the cardinal τ. It means that $(\tilde{G} \setminus G, \hat{U}_l)$ has the property of τ-boundedness. So the theorem is proved.

Notification 2. Analogically to this case we are able to get result with the right uniform space on the topological group. Formally, proving of such result will be respectively similar to that, which is shown higher.

Corollary 1. (G,*, τ) is I-lindelof topological group if and only if in every uniform cover α of the space (G,*, τ) there exists its countably infinite subcover α_0.

Proof. Let us make an accent at the fact of that property of being I-lindelof for the topological group (G,*, τ) means that this topological group (G,*, τ) is τ-bounded, where $\tau = \aleph_0$. That's why, using the result, which is got in the previous theorem, we can understand that being by the topological group (G,*, τ) I-lindelof is equivalent for (G,*, τ) to have such countably infinite subcover α_0 for any uniform cover α of the space *(G,U_l)* that α_0 is co-cover of the (G,*, τ). This sentence gives the proof of the proposition, given at the implication.

REFERENCES

1. Borubaev A.A. Uniformed topology and its applications. – Bishkek, "Ilim", 2021, 334 p.
2. Kanetov, B.E. Some classes of uniform spaces and uniformly continuous mappings. - Bishkek, 2013. - 160 p.
3. Uspensky, V.V. Characterization of compactness in terms of uniform structure in the space of functions // Successes mat. Sciences. - 1982. - T. 37. - issue. 4. – P.183–184.

MSC 34K20, 45J05

PARTIAL CUTTING METHOD AND EVALUATION OF THE SOLUTION OF A LINEAR VOLTERROAN INTEGRAL EQUATION OF THE SECOND KIND ON A HALF-AXIS

Iskandarov S.[1], Iskandarova G.S.[2]

[1,2]*Institute of Mathematics of the NAS of Kyrgyz Republic*

Sufficient conditions are established for estimating, boundedness, exponential law absolute integrability on the half-axis, tending to zero (including the exponential and exponential law) with an unlimited growth of the argument of the solution of a linear Volterra integral equation of the second order without assuming that the free term of this equation has these properties. The partial cutting method is being developed in combination with other methods. An illustrative example is being constructed.

Keywords: linear integral equation of the Volterra type of the second kind, estimate of the solution, boundedness, exponential absolute integrability, tending to zero, exponential law, exponential law.

Экинчи түрдөгү Вольтерра тибиндеги сызыктуу интегралдык теңдеменин чыгарылышынын жарым окто бааланышы, чектелгендеги, даражалуу абсолюттук интегралданышы, аргумент чексиз чоңойгондо нөлгө, экспоненциалдык жана даражалуу закондору боюнча, умтулушунун жетиштүү шарттары, бул теңдеменин бош мүчөсү аталган касиеттерге ээ болбой калуу учурунда, табылат. Жекече кесүү методу башка методдор менен айкалышта өнүктүрүлөт. Иллюстративдик мисал тургузулат.
Урунттуу сөздөр: экинчи түрдөгү Вольтерра тибиндеги сызыктуу теңдеме, чыгарылыштын бааланышы, чектелгендик, даражалуу абсолюттук интегралданыш, нөлгө умтулгандык, экспоненциялдык закон, даражалуу закон.

Устанавливаются достаточные условия для оценки, ограниченности, степенной абсолютной интегрируемости на полуоси, стремления к нулю (по экспоненциальному и степенному закону) при неограниченном росте аргумента решения линейного волтеррова интегрального уравнения второго рода без предположения, что указанными свойствами обладает свободный член этого уравнения. Развивается метод частичного срезывания в сочетании с другими методами. Строится иллюстративный пример.
Ключевые слова: линейное интегральное уравнения типа Вольтерра второго рода, оценка решения, ограниченность, степенная абсолютная интегрируемость, стремление к нулю, экспоненциальный закон, степенной закон.

All appearing functions and their derivatives are continuous and the relations hold for $t \geq t_0$; $t \geq \tau \geq t_0$; $J = [t_0, \infty)$; IE - integral equation; IDE - integro-differential equation.

Problem. Establish sufficient conditions for boundedness, exponential absolute integrability on a half-interval $J = [t_0, \infty)$, tending to zero, including exponential and exponential law, for $t_0 \to \infty$ solutions of a linear IE of the second kind of the Volterra type of the form:

$$x(t) + \int_{t_0}^{t} K(t,\tau)x(\tau)d\tau = f(t), t \geq t_0, \quad (1)$$

without the assumption that the free member $f(t)$ of this IE has the indicated asymptotic properties.

Note that such a statement of the problem was first posed in the work of the first author [1] to establish $x(t) \in L^2(J, R)$ and further considered in numerous works of this author. In [2], the problem posed above was solved by developing the method of weight and cut functions [3, p. 41], and in this work, to solve the problem, first the IE (1) multiplied by some weight function $\varphi(t) > 0$, [4], then the partial shearing method [5] is developed in combination with other well-known methods.

Let us proceed to obtain the main result.

First, according to [4], both parts of IE (1) are multiplied by some weight function $\varphi(t) > 0$, then both parts of the resulting relation are differentiable with respect to t. Then we get the following IDE of the form

$$(\varphi(t)x(t)' + \varphi(t)K(t,\tau)x(t) + \int_{t_0}^{t}\big(\varphi(t)K(t,\tau)\big)'_t x(\tau)d\tau =$$

$$= (\varphi(t)f(t))', t \geq t_0. \qquad (2)$$

Note that the solution of IDE (2) with the initial condition

$$x(t_0) = f(t_0) \qquad (3)$$

coincides with the solution of IE (1).

The method of partial cutting [5] is applicable to the study of IDE (2). Let [3,5]:

$$(\varphi(t)K(t,\tau))'_t = \sum_{i=1}^{n} K_i(t,\tau), \qquad (K)$$

$$(\varphi(t)f(t))' = \sum_{i=1}^{n} f_i(t), \qquad (f)$$

$\psi_i(t)$ $(i = 1..n)$ - some cutting functions,

$$P_i(t) \equiv \varphi(t)K_i(t,t)\big(\psi_i(t)\big)^{-2}, Q_i(t,\tau) \equiv \varphi(t)K_i(t,\tau)\big(\psi_i(\tau)\big)^{-1},$$

$$E_i(t) \equiv \varphi(t)f_i(t)\big(\psi_i(t)\big)^{-1}, P_i(t) = A_i(t) + B_i(t) \ (i = 1..n), \qquad \text{(P)}$$

$c_i(t)$ $(i = 1..n)$ - some functions.

For an arbitrarily fixed solution $x(t)$ IDE (2) multiply by $\varphi(t)x(t)$ [3, p. 46-47], integrate within t_0до t, including by parts, while applying the lemma [5], lemma 1.2 [3, c.44-45], enter conditions $(K), (f)$, functions $\psi_i(t), P_i(t), Q_i(t,\tau), E_i(t)$, condition (P), functions $c_i(t)$ $(i = 1..n)$. Then we get the following identity:

$$\varphi^2(t)\big(x(t)\big)^2 + 2\int_{t_0}^{t} \varphi^2(s)K(s,s)(x(s))^2 ds + \sum_{i=1}^{n}\{A_i\,(t)(X_i(t,t_0)^2 +$$

$$+B_i(t)X_i(t,t_0)^2 - 2E_i(t)X_i(t,t_0) + c_i(t) - \int_{t_0}^{t}[B_i'(s)(X_i(s,t_0))^2 -$$

$$-2E_i'(s)X_i(s,t_0) + c_i'(s)]ds \equiv c_* + \sum_{i=1}^{n}\int_{t_0}^{t}[A_i'(s)\big(X_i(s,t_0)\big)^2 +$$

$$+\int_{t_0}^{s}[Q_{i\tau}'(s,\tau)X_i(\tau,t_0)x(s)d\tau]ds, \qquad (4)$$

where $X_i(t,t_0) \equiv \int_{t_0}^{t} \psi_i(\eta)x(\eta)d\eta \ (i = 1..n),\ c_* = \varphi^2(t_0)\big(x(t_0)\big)^2 + \sum_{i=1}^{n} c_i(t_0)$.

Passing from identity (4) to the integral inequality, applying Lemma 1 on the integral inequality [6], and similarly to the theorem 1.1 [3, p.48-50], theorem [5] proves

Theorem. Let 1) $\varphi(t) > 0$, conditions are met $(K),(f),(P)$; 2) $K(t;t) \geq 0$; 3) $A_i(t) > 0, B_i \geq 0, B_i'(t) \leq 0$, существуют функции $A_i^*(t) \in L^1(J,R_+)$ functions exist. $c_i(t)\ (i = 1..n)$ such as $A_i'(t) \leq A_i^*(t)A_i(t)$, $(E_i^{(k)}(t))^2 \leq B_i^{(k)}(t)c_i^{(k)}(t)$ $(i = 1..n; k = 0,1)$; 4) $\int_{t_0}^{t} |Q_{i\tau}'(t,\tau)| A_i(\tau)^{-\frac{1}{2}} (\varphi(t))^{-1} d\tau \in L^1(J,R_+)\ (i = 1..n)$. Then for any solution $x(t)$ ИДУ(2), therefore, for the solution $x(t)$ ИУ(1) the estimate is true:

$$x(t) = \big(\varphi(t)\big)^{-1} O(1) \tag{5}$$

and the ratio is correct

$$\varphi^2(t)K(t,t)\big(x(t)\big)^2 \in L^1(J,R_+). \tag{6}$$

From estimate (5), similarly to the corollaries 3.1-3.4 [3, c.117] the following propositions follow.

Corrolary 1. If all the conditions of the theorem are satisfied and $(\varphi(t))^{-1} = O(1)$, then the solution IE(1) is restricted to J.

Corrolary 2. If all the conditions of the theorem are satisfied and $\varphi(t) \to \infty$ at $t \to \infty$, then the solution IE (1) $x(t) \to 0$ and $t \to \infty$.

Corrolary 3. If all the conditions of the theorem are satisfied and $(\varphi(t))^{-1} = e^{-\lambda t}O(1)\ (\lambda - const > 0)$, then the solution IE (1) $x(t) = e^{-\lambda t}O(1)\ (\lambda - const > 0)$, solution is $x(t)$ IE (1) tends to zero at $t \to \infty$ according to the exponential law.

Corrolary 4. If all the conditions of the theorem are satisfied and $t_0 = 0$, $(\varphi(t))^{-1} = (t+\delta)^{-\gamma}O(1)$ $(\delta,\gamma - const > 0)$ solutions of IE is (1) $\varphi(t) = (t+\delta)^{-\gamma}O(1)\ (\delta,\gamma - const > 0)$, solution is $x(t)$ IE (1) tends to zero and $t \to \infty$ according to the exponential law.

Corrolary 5. If all the conditions of the theorem are satisfied and $\big(\varphi(t)\big)^{-1} \in L^p(J,R_+\{0\})\ (p > 0)$, solution is IE (1) $x(t) \in L^p(J,R)(p > 0)$.

Here is an illustrative

Example. For IE:

$$x(t)+\int_0^t e^{-\sqrt{2t}}\left(\int_0^t e^{-\sqrt{2s}+5s+5\tau}\sqrt{16+(s-\tau)e^{-20s}}ds\right)x(\tau)d\tau=$$

$$=-e^{-\sqrt{2t}}\int_0^t e^{-\sqrt{2s}+5s}ds,\ \ t\geq 0 \qquad (1_*)$$

all conditions of the theorem and corollaries are satisfied 1,2,4,5 about $\varphi(t)\equiv e^{\sqrt{2t}}$, where $t_0=0, n=1,\ \psi_1(t)\equiv e^{5t},\ P_1(t)\equiv 4,\ A_1(t)\equiv 3,\ B_1(t)\equiv 1, E_1(t)\equiv -1$,

$$c_1(t)\equiv 1, Q_1(t,\tau)\equiv e^{5t}\sqrt{16+(t-\tau)e^{-20t}}, Q'_{1\tau}(t,\tau)\equiv -\frac{e^{-15t}}{2\sqrt{16+(t-\tau)e^{-20t}}}$$

Thus, we managed to find a class of IE of the form (1) for which the above problem is solvable.

Note that to study the properties of the solution IE (1_*) it is impossible to apply the results of work (2), since in this case $Q'_{1\tau}(t,\tau)<0$. This means that it is impossible to IE (1_*) use the method of cutting functions from [3].

Note also that using the results of [7], it is possible to extend the results of this work to an IE system of the form (1).

REFERENCES

1. Iskandarov S. O prinadlezhnosti prostranstvu $L^2[t_0,\infty))$ resheniya integral'nogo uravneniya Vol'terra // Tez.dokl. Vsesoyuz.konf. po asimptoticheskim metodam v teorii singulyarno-vozmushchennykh uravneniy. – Alma-Ata: Nauka, 1979. – Ch.1. – P.150-151. (in Russian)
2. Iskandarov S. Asimptotika resheniya lineynogo integral'nogo uravneniya tipa Vol'terra vtorogo roda // Ukrainian conf. “Modeling and research. systems sustainability", Kyiv, May 1993.: Abstracts of the conference-Kyiv: Society "Knowledge" of Ukraine, 1993. – Ch.1. – P.55. (in Russian)
3. Iskandarov S Metod vesovykh i srezyvayushchikh funktsiy i asimptoticheskiye svoystva resheniy integro-differentsial'nykh i integral'nykh uravneniy tipa Vol'terra.- Bishkek: Ilim, 2002.-P. 216. (in Russian)

4. Pakhyrov Z., Iskandarov S. Ob asimptoticheskom povedenii resheniy i ikh pervykh proizvodnykh slabo nelineynoy vol'terrovoy sistemy integro-differentsial'nykh uravneniy // Issled. po integro-differents. uravneniyam.- Bishkek: Ilim, 1991. –Vyp.23.-P.22-26. (in Russian)
5. Iskandarov S., Shabdanov D.N. Metod chastichnogo srezyvaniya i ogranichennost' resheniy neyavnogo vol'terrova integro-differentsial'nogo uravneniya pervogo poryadka // Issled. po integro-differents. uravneniyam. – Bishkek: Ilim, 2004. – Vyp.33.-P.67-71. (in Russian)
6. Ved' YU.A., Pakhyrov Z. Dostatochnyye priznaki ogranichennosti resheniy lineynykh integro-differentsial'nykh uravneniy // Issled. po integro-differents. uravneniyam v Kirgizii. – Frunze: Ilim, 1973. – Vyp.9. – P.68-103. (in Russian)
7. Iskandarov S. Matrichnyy metod chastichnogo srezyvaniya i stepennaya absolyutnaya integriruyemost' na poluosi resheniya sistemy lineynykh integral'nykh uravneniy vtorogo roda tipa Vol'terra // Vestnik Instituta matematiki NAN KR. – Bishkek, 2018. – №1. – P.55-62. (in Russian)

MSC 34K28, 34K26, 65L10

FINITE ELEMENT METHOD FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS WITH THE PARAMETER

Omuraliev A. [1], **Mederbek kyzy A.** [2]
[1,2]*Kyrgyz-Turkish Manas University*

The work is devoted to the construction of a numerical solution of a boundary value problem for an ordinary singular perturbed second-order differential equation. To apply the finite element method in singular perturbed problems, this problem is first regularized, then the finite element method is applied to the obtained regular problem.
Keywords: finite element method, singularly perturbed problem, basis functions.

Чектелген элементтер ыкмасын сингулярдуу козголгон маселелерде колдонуу үчүн алгач маселе регулярланат, алынган регулярдуу маселеге чектелген элементтер ыкмасы колдонулат. Регулярлоо Ломов ыкмасы менен аткарылат, ал ыкма боюнча кошумча өзгөрмө киргизилип маселе мейкиндикти бир өлчөмгө жогорулатуу менен аткарылат. Статьяда экинчи тартиптеги сингулярдуу козголгон кадимки дифференциалдык теңдеме үчүн четтик маселенин сандык чыгарылышы тургузулат.
Урунттуу сөздөр: чектелген элементтер методу, сингулярдуу козголгон маселе, базистик функциялар.

Работа посвящена построению численного решения краевой задачи для обыкновенного сингулярно возмущенного дифференциального уравнения второго порядка. Для применения метода конечных элементов в сингулярно возмущенных задачах сначала данная задача регуляризуется, далее полученной регулярной задаче применяется метод конечных элементов.

Ключевые слова: метод конечных элементов, сингулярно-возмущенная задача, базисные функции.

Numerical solution of a singular perturbed equation based on finite elements.

The paper is devoted to the numerical solution of a singular perturbed boundary value problem

$$L_\varepsilon u = \varepsilon^2 u''(x,\varepsilon) - a(x)u(x,\varepsilon) = f(x), \tag{1}$$

$$u(0,\varepsilon) = u^0, \quad u(1,\varepsilon) = u^1$$

Using the regularization method for singular perturbed problems [1], problem (1) is regularized. Next, the regularized problem is solved by the finite element method and the finite difference method.

The given functions $a(x)$, $f(x)$are differentiable a sufficient number of times.

Such a problem (1) has boundary layers along $x = 0$ and $x = 1$. We introduce two regularizing functions

$$\xi_l = \frac{\varphi_l(x)}{\varepsilon} = \frac{(-1)^{l-1}}{\varepsilon}\int_{l-1}^{x}\sqrt{a(s)}ds, \ \ l = 1,2 \tag{2}$$

and the extended function $\tilde{u}(x,\xi_1,\xi_2,\varepsilon)$ such that

$$\tilde{u}(x,\xi,\varepsilon)\Big|_{\xi=\frac{\varphi(x)}{\varepsilon}} \equiv u(x,\varepsilon) \tag{3}$$

We find from (3), on the basis of (2), the derivative $u''(x,\varepsilon)$

$$u'(x,\varepsilon) = \left(\partial_x\tilde{u} + \sum_{l=1}^{2}\frac{1}{\varepsilon}\varphi_l'(x)\partial_{\xi_l}\tilde{u}\right)_{\xi=\frac{\varphi(x)}{\varepsilon}},$$

$$u''(x,\varepsilon) = \partial_x^2\tilde{u} + \sum_{l=1}^{2}\left(\frac{\varphi_l'(x)}{\varepsilon}\right)^2\partial_{\xi_l}^2\tilde{u} + \sum_{l=1}^{2}\frac{1}{\varepsilon}\left(2\varphi_l'(x)\partial_{x\xi_l}^2\tilde{u} + \varphi_l''\omega(x)\partial_{x\xi_l}\tilde{u}\right)$$

then, instead of problem (1), we set the extended problem

$$\tilde{L}_\varepsilon \tilde{u} = \varepsilon^2 \partial_x^2 \tilde{u} + \sum_{l=1}^{2} \left[a(x) \partial_{\xi_l}^2 \tilde{u} + \varepsilon a(x) L_{\xi,l} \tilde{u} \right] - a(x) \tilde{u} = f(x) \tag{4}$$

$$\tilde{u}|_{x=0,\ \xi_1=0} = u^0,\ \tilde{u}|_{x=1,\ \xi_2=0} = u^1, \qquad L_{\xi,l} \equiv 2\varphi_l^{'}(x) \partial_{x\xi_l}^2 + \varphi_l^{"} \partial_{x\xi_l}$$

Note that there is an identity

$$\left(\tilde{L}\tilde{u}\right)_{\xi=\frac{\varphi(x)}{\varepsilon}} \equiv L_\varepsilon u \tag{5}$$

The solution of problem (4) will be defined as

$$\tilde{u}(x,\xi,\varepsilon) = V_1(x) + V_2(x) + W_1(x)\exp(-\xi_1) + W_2(x)\exp(-\xi_2), \tag{6}$$

with respect to $V_l(x), W_l(x),\ l=1,\ 2$ we will get the problem

$$-a(x)V_1(x) = f(x),\ L_\varepsilon V_2 = -\varepsilon^2 V_1^{"}(x),\ V_2|_{x=l-1} = 0, \tag{7}$$

$$\varepsilon W_l^{"}(x) - (-1)^{l-1}\left[2\varphi_l^{'}(x) W_l^{'}(x) + \varphi_l^{"}(x) W_l(x)\right] = 0\ ,\ l=1,\ 2. \tag{8}$$

Here V_2 is the remainder of the regular term. The equation with respect to $W_l(x)$ will be solved by the finite element method:

$$V_2(x) = \sum_{i=1}^{n-1} V_{2i}\omega_i(x), \tag{9}$$

$$W_i(x) = \sum_{i=1}^{n-1} W_{2,i}^l \omega_i(x) + \left(u^0 - V_1(0)\right)\omega_0(x) + \left(u^1 - V_1(1)\right)\omega_n(x), \tag{10}$$

where the trial functions $\omega_i(x)$ are defined as follows

$$\omega_i(x) = \begin{cases} \dfrac{(x - x_{i-1})}{h}, & x \in (x_{i-1}, x_i) \\ \dfrac{(x_{i+1} - x)}{h}, & x \in (x_i, x_{i+1}) \\ 0 \ \ in\, other\, cases, & \end{cases}$$

$$\omega_0(x) = \begin{cases} \dfrac{(x_i - x)}{h} & if\ x \in (x_0 = 0, x_1), \\ 0 & if\ \ x \notin (x_0, x_1), \end{cases}$$

$$\omega_n(x) = \begin{cases} \dfrac{(x - x_{n-1})}{h} & if\ \ x \in (x_{n-1}, x_n), \\ 0 & if\ x \notin (x_{n-1}, x_n), \end{cases}$$

$x_i = ih, h = \frac{1}{n}, \; i = 0,1,\ldots, \; n$

Substituting (9) and (10) respectively in (7) and (8), for the definition of V_{2i}, W_{2i}^l multiplying scalar by $\omega_i(x)$ we obtain the systems

$$\sum_{i=1}^{n-1} W_{2i}^l (\varepsilon^2 \omega_i^{"}(x) - (-1)^{l-1} \varepsilon \left(2\varphi_l^{'}(x)\omega_i^{'}(x) + \varphi_l^{"}(x)\omega_i(x)\right), \omega_k(x)) =$$

$$= -\left(u^0 - V_1(0)\right)\left(\omega_0(x), \omega_k(x)\right) - \left(u' - V_1(1)\right)(\omega_n(x), \omega_k(x)), \; k = 1,2,..,n-1 \quad (11)$$

where $\left[\omega_i, \omega_k\right] = -\varepsilon^2\left(\omega_i^{'}, \omega_k^{'}\right) - \left(a(x)\omega_i(x)\omega_k(x)\right)$ entering the designation

$$b_{ij}^l = \left(\varepsilon^2 \omega_i^{"}(x) - (-1)^{l-1}\left(2\varphi_l^{'}(x)\omega_i^{'}(x) + \varphi_l^{"}(x)\omega_i(x)\right), \omega_j(x)\right),$$

$$f_j = -\varepsilon^2\left(a^{-1}(x) f(x)\right)^{"}, \omega_j(x)),$$

$$g_j(x) = -\left(u^0 - V_1(0)\right)\left(\omega_0(x), \omega_j(x)\right) - \left(u' - V_1(1)\right)(\omega_n(x), \omega_j(x)),$$

$$a_{ij} = \left[\omega_i, \omega_j\right],$$

to determine V_{2i} and W_{2i}^l we obtain systems of algebraic equations

$$\sum_{i=1}^{n-1} a_{ij} V_{2i} = f_j$$

$$\sum_{i=1}^{n-1} b_{ij}^l W_{2i}^l = g_j, \quad j = 1,2,..,n-1, \; l = 1, \; 2$$

Let's describe these systems

$$a_{j-1,j} V_{2j-1} + a_{j,j} V_{2j} + a_{j+1,j} V_{2j+1} = f_j,$$

$$b_{j-1,j}^l W_{2j-1}^l + b_{j,j}^l W_{2j}^l + b_{j+1,j}^l W_{2j+1}^l = g_j, \; j = 1,2,\ldots,n-1 \quad (12)$$

here

$$a_{j-1,j} = -\varepsilon^2\left(\omega_{j-1}^{'}, \omega_j^{'}\right) - \left(a(x)\omega_{j-1}(x), \omega_j(x)\right) = +\varepsilon^2 \frac{1}{h} - a\left(\frac{x_{j-1} + x_j}{2}\right)\frac{h}{6}$$

$$a_{j.j} = -\varepsilon^2\left(\omega_j^{'}\omega_j^{'}\right) - \left(a(x)\omega_j, \omega_j\right) = -\frac{2\varepsilon^2}{h} - a\left(\frac{x_{j-1} + x_{j+1}}{2}\right)\frac{2h}{3}$$

$$a_{j+1,j} = -\varepsilon^2\left(\omega_{j+1}^{'}\omega_j^{'}\right) - \left(a(x)\omega_{j+1}, \omega_j\right) = \frac{\varepsilon^2}{h} - a\left(\frac{x_j + x_{j+1}}{2}\right)\frac{h}{6}$$

$$f_j = -\frac{\varepsilon^2 h}{2}\left[f\left(\frac{x_{j-1}+x_j}{2}\right) + f\left(\frac{x_j + x_{j+1}}{2}\right)\right], \quad f(x) = \left(a^{-1}(x) f(x)\right),$$

$$b_{j-1,j}^{l} = -\varepsilon^2\left(\omega_{j-1}^{'}, \omega_j^{'}\right) - 2\left(\varphi_l^{'}(x)\omega_{i-1}^{'}, \omega_j\right) -$$

$$-\left(\varphi_l^{''}(x)\omega_{j-1}, \omega_j = \varepsilon^2 \frac{1}{h} - 2\varphi_l^{'}\left(\frac{x_{j-1}+x_j}{2}\right)\left(-\frac{1}{2}\right) - \varphi_l^{''}\left(\frac{x_j + x_{j+1}}{2}\right)\frac{h}{2}\right)$$

$$b_{j,j}^{l} = -\frac{2\varepsilon^2}{h} - \frac{2}{h}\left[\varphi_l^{'}\left(\frac{x_{j-1}+x_j}{2}\right) + \varphi_l^{'}\left(\frac{x_j + x_{j+1}}{2}\right)\right] -$$

$$-\frac{h}{3}\left[\varphi_l^{''}\left(\frac{x_{j-1}+x_j}{2}\right) + \varphi_l^{''}\left(\frac{x_j + x_{j+1}}{2}\right)\right]$$

$$b_{j+1,j}^{l} = \varepsilon^2\left(-\frac{1}{h}\right) - 2\frac{1}{2}\varphi_l^{'}\left(\frac{x_j + x_{j+1}}{2}\right)\frac{h}{6}, \quad g_j = 0, j = 2, \ldots, n-2$$

$$g_1 = -\left(u^0 - V(0)\right)\left[\omega_0^{'}, \omega_1^{'}\varepsilon^2 - 2\left(\varphi_l^{'}\omega_0^{'}, \omega_1\right) - \left(\varphi_l^{''}\omega_0, \omega_1\right)\right] =$$

$$= -\left(u' - V(0)\right)\left[-\varepsilon^2 \frac{1}{2} + \varphi_l^{'}\left(\frac{x_0 + x_1}{2}\right) - \varphi_l^{''}\left(\frac{x_0 + x_1}{2}\right)\frac{h}{6}\right]$$

$$g_{n-1} = -\left(u' - V(1)\right)\left[-\varepsilon^2\left(\omega_n^{'}, \omega_{n-1}^{'}\right) - 2\left(\varphi_l^{'}\omega_n^{'}, \omega_{n-1}\right) - \left(\varphi_l^{''}\omega_n, \omega_{n-1}\right)\right] =$$

$$= -\left(u' - V(1)\right)\left[\frac{\varepsilon^2}{2} - 2\varphi_l^{'}\left(\frac{x_{n-1}+x_n}{2}\right)\left(-\frac{1}{n}\right) - \varphi_l^{''}\left(\frac{x_{n-1}+x_n}{2}\right)\frac{h}{6}\right]$$

Solving systems (12) definition V_{2i}, W_{2i}^{l}. By narrowing (6) we obtain an approximate solution of the problem (1).

Following Theorem 1.5 and [6] it is possible establish a pointwise estimate, that is, the validity of the following theorem.

Theorem. Let the second derivative $u''(x)$, be continuous, then there are estimates

$$|u - u''| < ch^2$$

Example. Using the MATLAB package we will solve the problem

$$\varepsilon^2 u'' - 4u = 1 + x^3 \,, \qquad (1)$$

$$u|_{x=0} = 2, \quad u|_{x=1} = 1$$

The exact solution of which has the form

x	Exact solution	Approximate Solution
0	2.0000	2.0000
0.05	-0.1697	-0.2678
0.1	-0.2473	-0.2500
0.15	-0.2505	-0.2508
0.2	-0.2517	-0.2519
0.25	-0.2536	-0.2538
0.3	-0.2563	-0.2566
0.35	-0.2602	-0.2606
0.4	-0.2655	-0.2658
0.45	-0.2722	-0.2726
0.5	-0.2806	-0.2811
0.55	-0.2909	-0.2914
0.6	-0.3032	-0.3038
0.65	-0.3178	-0.3184
0.7	-0.3348	-0.3355
0.75	-0.3545	-0.3552
0.8	-0.3769	-0.3777
0.85	-0.4025	-0.4033
0.9	-0.4329	-0.4332
0.95	-0.5165	-0.4640
1	-1.0000	-1.0000

REFERENCES

1. Марчук Г.И. Методы вычислительной математики. –М.: Наука, 1989
2. Флетчер К. Численные методы на основеметода Галёркина. –М.: Мир, 1988
3. Омуралиев А.С. “Регуляризация сингулярно возмущенной краевой задачи для обыкновенного диф.уравнения на основе конечных элементов”// Манас табигый илимдер журналы - 2003, №4, с.45-50.
4. Ломов С.А. Введение в общую теорию сингулярных возмущений. –М.: Наука, 1981
5. Марчук Г.И. Методы вычислительной математики. –М.: Наука, 1989
6. Г. Стренг Дж.Фикс Теория метода конечных элементов. – М.: Мир, 1977

MSC 34K20, 45J05

ON THE STRUCTURE OF SOLUTIONS OF THE INITIAL PROBLEM OF NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS IN PARTIAL DERIVATIVES OF THE FOURTH ORDER

Baizakov A.B. [1], Dzheenbaeva G.A. [2], Sharshenbekov M.M. [3]
[1,2,3]Institute of Mathematics of NAS of KR

The solvability of the Cauchy problem for partial integro-differential equations can be studied by the method of transforming solutions. The essence of this approach is the transformation of the original Cauchy problem into an integral equation equivalent to it, to which one can apply the topological method - the principle of compressed mappings.

In this paper, the solvability of the Cauchy problem for nonlinear integro-differential equations in partial derivatives of the fourth order is investigated and an integral representation of the obtained solutions is found.

Keywords: Sufficient condition for the solvability of the Cauchy problem for nonlinear systems of partial integro-differential equations, contraction mapping principle, Volterra integral equation of the second kind, space of continuous functions with their derivatives.

Жекече туундулуу интегро-дифференциалдык теңдемелер үчүн Коши маселесинин чыгарыла тургандыгын изилдөөдө чыгарылыштарды өзгөртүп түзүү ыкмасы менен жүргүзүүгө болот. Мындай ыкманын маңызы кысылган чагылдыруулар принциби деп аталган топологиялык ыкманы колдонууга мүмкүн болгон Кошинин баштапкы маселесин ага эквиваленттүү болгон интегралдык теңдемеге айландыруу болуп саналат.

Бул эмгекте төртүнчү тартиптеги жекече туундулуу сызыктуу эмес интегро-дифференциалдык теңдемелер үчүн Коши маселесинин чыгарыла тургандыгы изилденди жана алынган чыгарылыштардын интегралдык көрүнүшү табылды.

Урунттуу сөздөр: Жекече туундулуу сызыктуу эмес интегро-дифференциалдык теңдемелер системалары үчүн Коши маселесинин чыгарыла тургандыгынын жетиштүү шарты, кысылган чагылдыруулар принциби, экинчи түрдөгү Вольтерра интегралдык теңдемеси, туундулары менен үзгүлтүксүз функциялардын мейкиндиги.

Исследовать разрешимость задачи Коши для интегро-дифференциальных уравнений в частных производных можно провести методом преобразования решений. Сутью такого подхода является преобразование исходной задачи Коши в эквивалентное ей интегральное уравнение, к которой можно применить топологический метод – принцип сжатых отображений.

В данной работе исследована разрешимость задачи Коши нелинейных интегро-дифференциальных уравнений в частных производных четвертого порядка и найдена интегральное представление полученных решений.

Ключевые слова: достаточное условие разрешимости задачи Коши для нелинейных систем интегро-дифференциальных уравнений в частных производных, принцип сжатых отображений, интегральное уравнение Вольтерра второго рода, пространство функций непрерывных со своими производными**.**

Consider an integro-differential equation in partial derivatives of the fourth order of the form

$$u_{ttxx}+2\alpha u_{txx}+2\beta u_{ttx}+\alpha^2u_{xx}+4\alpha\beta u_{tx}+\beta^2u_{tt}+$$

$$+2\beta\alpha^2u_x+2\alpha\beta^2u_t+\beta^2\alpha^2u=F(t,x,u)+\int_0^t K\left(t,\tau,u\left(\tau,x\right)\right)d\tau \qquad (1)$$

where $\alpha,\ \beta\in R_+$, with initial condition

$$u(o,x)=\varphi(x),\ u_t(o,x)=\psi(x) \qquad (2)$$

The solution of problem (1), (2) will be sought in the form

$$u(t,x)=c(t,x)+\int_0^t\int_{-\infty}^x e^{-\alpha(t-\nu)-\beta(x-s)}(t-\nu)(x-s)Q(\nu,s)dsd\nu\,, \qquad (3)$$

where $c(t,x)$ is a known function such that

$$c(o,x)=\varphi(x),\ c_t(o,x)=\psi(x),$$

$Q(t,x)$ is an unknown function to be determined. To find the unknown function $Q(t,x)$, we will substitute (3) into (1). To this end, differentiating (3) with respect to t, we have

$$u_t=c_t-\alpha\left(u-c\right)+\int_0^t\int_{-\infty}^x e^{-\alpha(t-\nu)-\beta(x-s)}(x-s)Q(\nu,s)d\nu ds\,. \qquad (4)$$

Hence, differentiating with respect to t and replacing the resulting double integrals, taking into account (3), (4), we obtain

$$u_{tt}+\alpha u_t=c_{tt}+\alpha c_t-\alpha\left[u_t+\alpha\left(u-c\right)-c_t\right]+\int_{-\infty}^x e^{-\beta(x-s)}(x-s)Q(t,s)ds\,. \qquad (5)$$

Further, differentiating (5) with respect to x, we have

$$u_{ttx}+2\alpha u_{tx}+\alpha^2u_x\ =c_{ttx}+2\alpha c_{tx}+\alpha^2c_x-$$

$$-\beta\int_{-\infty}^x e^{-\beta(x-s)}(x-s)Q(t,s)ds+\int_{-\infty}^x e^{-\beta(x-s)}Q(t,s)ds\,. \qquad (6)$$

Equality (5) can be rewritten in the form

$$u_{tt}+2\alpha u_t+\alpha^2u-c_{tt}-2\alpha c_t-\alpha^2c=\int_{-\infty}^x e^{-\beta(x-s)}(x-s)Q(t,s)ds\,. \qquad (7)$$

In (6) replacing the first integral on the left - according to the formula (7) and differentiating with respect to x we get

$$u_{ttxx}+2\alpha u_{txx}+\alpha^2 u_{xx}+\beta\left[u_{ttx}+2\alpha u_{tx}+\alpha^2 u_x\right]=$$

$$=c_{ttxx}+2\alpha c_{txx}+\alpha^2 c_{xx}+\beta\left[c_{ttx}+2\alpha c_{tx}+\alpha^2 c_x\right]+\text{-}Q(t,s)-\beta\int_{-\infty}^{x} e^{-\beta(x-s)}Q(t,s)ds. \quad (8)$$

Considering (7) from (6) we have

$$u_{ttx}+2\alpha u_{tx}+\alpha^2 u_x+\beta\left[u_{tt}+2\alpha u_t+\alpha^2 u\right]=$$

$$=c_{ttx}+2\alpha c_{tx}+\alpha^2 c_x+\beta\left[c_{tt}+2\alpha c_t+\alpha^2 c\right]+\int_{-\infty}^{x} e^{-\beta(x-s)}Q(t,s)ds.$$

(9)

Multiplying equality (9) by β and adding the resulting expression with (8) term by term, we have

$$u_{ttxx}+2\alpha u_{txx}+\alpha^2 u_{xx}+2\beta\left[u_{ttx}+2\alpha u_{tx}+\alpha^2 u_x\right]+$$

$$+\beta^2\left[u_{tt}+2\alpha u_t+\alpha^2 u\right]=c_{ttxx}+2\alpha c_{txx}+\alpha^2 c_{xx}+$$

$$+2\beta\left[c_{ttx}+2\alpha c_{tx}+\alpha^2 c_x\right]+\beta^2\left[c_{tt}+2\alpha c_t+\alpha^2 c\right]+Q(t,x).$$

We write the last equality in the form

$$u_{ttxx}+2\alpha u_{txx}+2\beta u_{ttx}+\alpha^2 u_{xx}+4\alpha\beta u_{tx}+\beta^2 u_{tt}+$$

$$+2\beta\alpha^2 u_x+2\alpha\beta^2 u_t+\beta^2\alpha^2 u=N(t,c,\varepsilon)+Q(t,x), \quad (10)$$

where

$$N(t,c)\equiv c_{ttxx}+2\alpha c_{txx}+2\beta c_{ttx}+\alpha^2 c_{xx}+4\alpha\beta c_{tx}+\beta^2 c_{tt}+2\beta\alpha^2 c_x+2\alpha\beta^2 c_t+\alpha^2\beta^2 c.$$

Taking into account (3), (10) from (1) we have a nonlinear integral equation of the form

$$Q(t,x)=-N(t,c)+F(t,x,c(t,x)+$$

$$+\int_0^t\int_0^x e^{-\alpha(t-\nu)-\beta(x-s)}(t-\nu)(x-s)Q(\nu,s)dsd\nu\Bigg)+$$

$$+\int_0^t K\left(t,\tau,c(\tau,x)+\int_0^\tau\int_0^x e^{-\alpha(t-\nu)-\beta(x-s)}(\tau-\nu)(x-s)Q(\nu,s)dsd\nu\right)ds\equiv P(Q). \quad (11)$$

Nonlinear integral equation (11) will be solved by the contraction mapping method [1]. Let's pretend that:

$$N(t,c) \in \bar{C}(R_+ \times R), \tag{12}$$

$$F(t,x,u) \in \bar{C}(R_+ \times R \times R) \cap Lip\left(L|_u\right). \tag{13}$$

$$K(t,\tau,u) \in \bar{C}(R_+ \times R_+ \times R) \cap Lip\left(K_u\right) \tag{14}$$

We consider the right side of (11) as an operator $P[Q]$ acting on the function $Q(t,x)$. It's clear that

$$\|F(t,x,u) - N(t,c)\| \le N = const,$$

where

$$N = N_0 + N_1.$$

Taking into account (12)-(14) we estimate the difference

$$\|P[Q_1] - P[Q_2]\| \le \left\| F\left(t,x,c + \int_0^t \int_{-\infty}^x e^{-\alpha(t-\nu)-\beta(x-s)}(t-\nu)\,(x-s)Q_1(\nu,s)d\nu ds\right) - \right.$$

$$-F\left(t,x,c + \int_0^t \int_{-\infty}^x e^{-\alpha(t-\nu)-\beta(x-s)}(t-\nu)\,(x-s)Q_2(\nu,s)d\nu ds\right)\Bigg\| +$$

$$+\left\| \int_0^t K\left(t,\tau,c + \int_0^\tau \int_{-\infty}^x e^{-\alpha(t-\nu)-\beta(x-s)}(\tau-\nu)\times(x-s)Q_1(\nu,s)d\nu ds\right)\right) d\tau - $$

$$-\int_0^t K\left(t,\tau,c + \int_0^\tau \int_{-\infty}^x e^{-\alpha(t-\nu)-\beta(x-s)}(\tau-\nu)\times(x-s)Q_2(\nu,s)d\nu ds\right)\right) d\tau \Bigg\| \le$$

$$\le (L_u + K_u T)\left\{ \int_0^t e^{-\alpha(t-\nu)} \int_{-\infty}^x e^{-\beta(x-s)}(t-\nu)(x-s)\times \right.$$

$$\left. \times\|Q_1(\nu,s) - Q_2(\nu,s)\| d\nu ds \right\} \le (L_u + K_u T)\frac{T}{\alpha\beta^2}\|Q_1(t,x) - Q_2(t,x)\|.$$

At the same time, the integral was estimated

$$\left\| \int_0^t e^{-\alpha(t-\nu)}(t-\nu)d\nu \int_{-\infty}^x e^{-\beta(x-s)}(x-s)ds \right\| = \left\| \left[e^{-\alpha t}\int_0^t e^{\alpha\nu}(t-\nu)d\,\nu\right]\left[e^{-\beta x}\int_{-\infty}^x e^{\beta s}(x-s)d\,s\right] \right\| \le \frac{T}{\alpha}\frac{1}{\beta^2}.$$

Suppose that $\alpha,\ \beta$ are such that

$$(L_u + K_u T)\frac{T}{\alpha\beta^2} < 1. \tag{15}$$

Then, according to the contraction mapping principle, we conclude that the nonlinear integral equation (11) for all $(t,x)\in D$ has a unique continuous solution $Q(t,x)$.

Next, we prove the boundedness of the solutions of the Cauchy problem (1)-(2). From (3) we have the inequality

$$\|u(t,x)\| \le \|c(t,x)\| + \left\| \int_0^t e^{-\alpha(t-\nu)}(t-\nu)d\nu \int_{-\infty}^{x} e^{-\beta(x-s)}(x-s)\times Q(\nu,s)ds \right\| \le$$

$$\le c_0 + \frac{Tq}{\alpha\beta^2} = c_2 = const.$$

Thus, fair

Theorem. Let 1) the functions $c(t,x)$, $c(0,x)=\varphi(x)$, $c_t(0,x)=\psi(x)$, $c(t,x)\in \bar{C}^{(2,2)}(D)$; 2) conditions (12)-(15) are fulfilled. Then the nonlinear integro-differential equation in partial derivatives of the fourth order (1) with initial data (2) has a unique solution $u(t,x)\in \bar{C}^{(2,2)}(D)$.

REFERENCES

1. Imanaliev, M.I., Imanaliev, T.M., Kakishov, K. On Cauchy Problems for Sixth-Order Nonlinear Partial Differential Equations, Issled. by integro-differential. equations. - Bishkek: Ilim, 2007. - Issue. 36. - S. 19-28.
2. Imanaliev M.I., Baizakov A.B. On the solvability of the Cauchy problem for a class of non-linear partial differential equations // Poisk (scientific application of the international journal "Higher School of Kazakhstan"), ser. math.-tech. Sciences, Almaty, 2009. - P. 209-213.
3. Dzheenbaeva, G.A. On the initial problem of integro-differential equations in partial derivatives of the fourth order. Izvestiya VUZ of Kyrgyzstan. - Bishkek, 2020. - Issue No. 9. –S.9-14.

MSC 15B99, 15A83

MAGIC SQUARE: TERMS OF ARITHMETIC PROGRESSION - IDENTIFIERS

Baizakov A.B. [1]**, Sharshenbekov M.M.** [2]**, Aitbaev K.A.** [3]
[1,2,3]*Institute of Mathematics of NAS of KR*

This article illustrates the construction of M-matrices of the 12th order based on the available M-matrices of the third and fourth order. When constructing the M-matrix of the 12th order, the decomposition method is used, as well as the properties of the members of an arithmetic progression. Earlier in [1], it was noted that the creation of a database of low-order M-matrices is an important step in the decomposition method. It turns out that in this method the constants of squares forming an arithmetic progression of split block matrices act as identifiers.
Keywords: Decomposition method, arithmetic progression, square constants, matrix block.

Бул макалада үчүнчү жана төртүнчү тартиптеги мурдатан берилген М-матрицалардын базасында 12-тартиптеги М-матрицасын түзүү сүрөттөлөт. 12-тартиптеги М-матрицасын түзүүдө декомпозиция ыкмасы, ошондой эле арифметикалык прогрессиянын мүчөлөрүнүн касиеттери колдонулат. Буга чейин [1] иште төмөнкү тартиптеги М-матрицалардын маалымат базасын түзүү декомпозиция ыкмасынын маанилүү этабы экендиги белгиленген. Бул ыкмада бөлүнүүчү блок матрицалардын арифметикалык прогрессияны түзүүчү квадраттардын константалары идентификатор катары иш-аракет кылышат.
Урунттуу сөздөр: Декомпозиция ыкмасы, арифметикалык прогрессия, квадраттык константалар, матрицалардын блогу.

В данной статье иллюстрируется построение М-матриц 12го порядка на базе имеющихся М-матриц третьего и четвертого порядка. При построении М-матрицы 12го порядка используется метод декомпозиции, а также свойства членов арифметической прогрессии. Ранее в работе [1] было отмечено, что создание базы данных М-матриц низкого порядка – важный этап метода декомпозиции. Оказывается, что в этом методе константы квадратов образующие арифметическую прогрессию расщепленных блок матриц выступают как идентификаторы.
Ключевые слова: Метод декомпозиции, арифметическая прогрессия, константы квадратов, блок матрицы.

It was noted in [1] that the creation of a database of low-order *M*-matrices is an important step in the decomposition method. It turns out that in this method the constants of the squares of the split block matrices act as identifiers. Due to the properties of an arithmetic progression, the constants of squares of each split block matrix are calculated by the well-known formula

$$k_i = \frac{a_{i_1} + a_{i_n}}{2} \cdot m, \qquad (1)$$

where *m* is the order of the matrix of split groups, *i* - means the number of the group, a_{i_1}, a_{i_n} - the first and last member of the *m*-th group. Note that the set of values of square constants is modeled by a second-order difference equation of the form

$$u_{n+2} = 2u_{n+1} - u_n, \quad n \geq 1$$

with appropriate initial conditions. According to the Euler method, the general solution of Eq. (1) has the form

$$u(n) = c_1 + c_2 n$$

since the roots of the characteristic equation $\lambda^2 - 2\lambda + 1 = 0$: $\lambda_1 = \lambda_2 = 1$ coincide, where c_1, c_2 are arbitrary constants. The constants of squares forming an arithmetic progression act as identifiers.

In this article, we will illustrate the construction of a 12th order *M*-matrix based on the available *M*-matrices of the third and fourth order. Recall that the first chosen *M*-matrix of the fourth order is known as the "Dürer square", see figure 2.

6	7	2
1	5	9
8	3	4

Fig.1

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

13	2	12	7
16	3	9	6
1	14	8	11
4	15	5	10

Fig.2

We will construct a 12th order *M*-matrix using the 3*4 decomposition method. In our case, the construction is based on the properties of the members of an arithmetic progression. Now we divide the set of numbers from 1 to 144 into 9 groups. Each group consists of 16 numbers forming an arithmetic progression: 1st group - 1, 10, 19, ..., 127, 136. 2nd group - 2, 11, 20, ..., 128, 137, etc. ,VIIIth group - 8, 17, .26 ..., 134, 143, IXth group - 16.25, 34, ..., 135, 144.

When calculating the *n*th member of the arithmetic progression in each group, the formula $a_n = a_1 + (n-1)d$ was used, where $a_1 = 1,...,16; d = 9$.

Next, for each group, we calculate the constants of the squares of the block matrix of the order 4*4 according to the formula (1). For the first group we have

$$k_1 = \frac{a_{i_1} + a_{i_n}}{2} \cdot m = \frac{1+136}{2} \cdot 4 = 274. \text{ Further}$$

$$k_2 = \frac{a_{i_1} + a_{i_n}}{2} \cdot m = \frac{2+137}{2} \cdot 4 = 278,$$

$$k_3 = \frac{a_{i_1} + a_{i_n}}{2} \cdot m = \frac{3+138}{2} \cdot 4 = 282$$

…………………………………………

$$k_9 = \frac{a_{i_1} + a_{i_n}}{2} \cdot m = \frac{16+144}{2} \cdot 4 = 320.$$

Then the constants of the squares of the created groups form an arithmetic progression with the difference $d = 4$. We have the first 9 members of an arithmetic progression: $k_1 = 274$, $k_2 = 278$, $k_3 = 282$, ... , $k_9 = 320$ and we will place them in full accordance with the numbers in Fig. 1, i.e.

$$\begin{pmatrix} a_6 & a_7 & a_2 \\ a_1 & a_5 & a_9 \\ a_8 & a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 294 & 298 & 278 \\ 274 & 290 & 306 \\ 302 & 282 & 286 \end{pmatrix}$$

Fig.3

Note that the square constant of the last *M*-matrix is 870.

Further, we will consider the obtained 9 constants of squares as identifiers of an expanding order. The *M*-matrix of the 12th order is considered as a nested 3 * 4, or rather a block matrix. Each element of the *M*-matrix of the third order is considered as a block and expanded, keeping its location in Fig.3. So, for example, the constant 274 will consist of *M*-matrices of the fourth order of the form

136	19	10	109
37	82	91	64
73	46	55	100
28	127	118	1

By virtue of the constants of the square, each block can be painted in an arbitrary way. We have the right to take any other *M*-matrices of the fourth order from the database, for example

13	2	12	7
16	3	9	6
1	14	8	11
4	15	5	10

Fig.4

For the constants 278 and 282 we can take the *M*-matrix of the fourth order built on the basis of the last Fig. 4 contained in our database:

110	11	101	56
137	20	74	47
2	119	65	92
29	128	38	83

111	12	102	57
138	21	75	48
3	120	66	93
30	129	39	84

We will continue this procedure for all constants of the squares in Fig.3. As a result, instead of each constant of the square, we write the *M*-matrix of our group, we get *M* - a matrix of the 12th order. Here is a fragment of the arrangement instead of constants274, 278, 282 *M*-matrices of the 4th order of these groups.

								110	11	101	56
								137	20	74	47
								2	119	65	92
								29	128	38	83
136	19	10	109								
37	82	91	64								
73	46	55	100								
28	127	118	1								
				111	12	102	57				
				138	21	75	48				
				3	120	66	93				
				30	129	39	84				

Finally, we obtain one form of the *M*-matrix of the 12th order:

114	15	105	60	115	16	106	61	110	11	101	56
141	24	78	51	142	25	79	52	137	20	74	47
6	123	69	96	7	124	70	97	2	119	65	92
33	132	42	87	34	133	43	88	29	128	38	83
136	19	10	109	113	14	104	59	117	18	108	63
37	82	91	64	140	23	77	50	144	27	81	54
73	46	55	100	5	122	68	95	9	126	72	99
28	127	118	1	32	131	41	86	36	135	45	90
116	17	107	62	111	12	102	57	112	13	103	58
143	26	80	53	138	21	75	48	139	22	76	49
8	125	71	98	3	120	66	93	4	121	67	94
35	134	44	89	30	129	39	84	31	130	40	85

It is clear that

$$S = \frac{a_1 + a_{n^2}}{2} n = \frac{274 + 306}{2} \cdot 3 = 870.$$

This is the 12*12 magic square constant.

REFERENCES

1. Baizakov A.B., Mombekov A.D., Aitbaev K.A. On the diversity of square constants in subblocks during matrix decomposition // Reports of the National Academy of Sciences of the Kyrgyz Republic, Bishkek, 2017.-№2.- C19-24.
2. Baizakov A. B., Mombekov A. D., Sharshenbekov M. M. Creation of bases of data m-matrices short order - important stage of decomposition method // Theses of international scientific conference “Problems of modern mathematics and its applications”. Bishkek-2021. - 114p.

MSC 45A05, 45B 05

ON THE UNIQUENESS OF SOLUTIONS OF FREDHOLM LINEAR INTEGRAL EQUATIONS OF THE FIRST KIND ON THE SEMI-AXIS

Asanov A. [1]**, Kadenova Z.A.** [2]**, Bekeshova D.** [3]
[1]*Department of Mathematics, Kyrgyz-Turkish Manas University*
[2,3]*Institute of Mathematics of NAS of the Kyrgyz Republic*

In the present article the theorem about uniqueness of Fredholm linear integral equations of the first kind on the semi axis, with method of nonnegative quadratic forms and functional analysis methods.

Key words: Fredholm linear inteqral equations, first kind, the semiaxis, uniqueness of solutions.

Бул макалада терс эмес квадраттык формалар усулунун, функционалдык анализдин усулдарынын жардамы менен жарым октогу Фредгольмдун биринчи түрдөгү сызыктуу интегралдык тендемелеринин чечимдеринин жалгыздыгы далилденди.

Урунттуу сөздөр: Фредгольмдун сызыктуу интегралдык тендемелери, биринчи түрдөгү, жарым ок, чечимдеринин жалгыздыгы.

В настоящей статье доказана теорема о единственности линейных интегральных уравнений Фредгольма первого рода на полуоси с использованием метода неотрицательных квадратичных форм и методов функционального анализа.

Ключевые слова: Линейные интегральные уравнения Фредгольма первого рода, полу ось, единственность решений.

1. Introduction

Many problems of the theory of integral equations of the first kind were studied in [1-15]. But fundamental results for Fredholm integral equations of the first kind were obtained in [11-12], where regularizing operators in the sence of M.M. Lavrent'ev were constructed. Rezults on non-classical Volterra integral equations of the first kind can be found in [1]. In [2,6], problems of regularization, uniqueness and existence of solutions for Volterra integral and operator equations of the first kind are studied. In [13], for linear Volterra integral equations of the first and the third kind with smooth kernel, the existence of multiparameter family of solutions was proved. In [4,5], on the basis of theory of Volterra integral equations of the first kind, various inverse problems were studied. In [8], uniqueness theorems were proved and requlatizing operators in the sense of Lavrent'ev were constructed for systems of linear Fredholm integral equations of the third kind. In [10], problems of uniqueness and stability of solutions for linear integral equations of the first kind with two independent variables were investigated. In [3, 9], based on a new approach, the existence and uniquenes of solutions of Fredholm integral equations and the system linear Fredholm integral equations of the third kinds were studied. In [15], uniqueness theorems were proved for the linear Fredholm integral equations of the first kind in the axis.

In the prezent paper, on the basis of the method of integral transformation, uniqueness theorems for the new class of linear Fredholm integral equations of the first kind in the semiaxis were proved.

2. The linear Fredholm integral equations of the first kind

Consider the linear Fredholm integral equations of the first kind

$$Ku \equiv \int_{-\infty}^{a} K(t,s)u(s)ds = f(t), \quad t \in (-\infty, a] \tag{1}$$

where the u(t) is the desired function on $(-\infty, a]$, the given function f(t) is the continuous on $(-\infty, a]$,

$$\int_{-\infty}^{a}\int_{-\infty}^{a} |K(t,s)|^2 dsdt < \infty,$$

$$K(t,s) = \begin{cases} A(t,s), & -\infty < s \le t \le a, \\ B(t,s), & -\infty < t \le s \le a, \end{cases} \tag{2}$$

the given functions $A(t,s)$ and $B(s,t)$ are continuous on the domain

$$G = \{(t,s) : -\infty < s \le t \le a\}.$$

Let C$(-\infty, a]$ denote the space of all functions continuous on $(-\infty, a]$. Here C(G) denote the space of all functions continuous on G.

We introduce the notation

$$H(t,s) = A(t,s) + B(s,t), (t,s) \in G. \tag{3}$$

Assume that the following conditions are satisfied:

$(i) H(t,s), H'_t(t,s), H'_s(t,s), H''_{ts}(t,s) \in C(G), \alpha(t) = \lim_{s \to -\infty} H(t,s), t \in (-\infty, a], \alpha(a) \ge 0,$
$\alpha(t) \in C(-\infty, a], \alpha'(t) \le 0$ *for all* $t \in (-\infty, a], H''_{st}(t,s) \le 0$ *for all* $(t,s) \in G,$
$\alpha'(t) \in L_1(-\infty, a], \beta(s) = H'_s(a,s) \ge 0$ *for all* $s \in (-\infty, a], \beta(s) \in C(-\infty, a] \cap L_1(-\infty, a];$

$(ii)\ \underset{(t,s)\in G}{Sup}\left|H(t,s)\right| \le l < \infty,\ \underset{t\in(-\infty,a]}{Sup}\int_{-\infty}^{t}\left|H_s^{'}(t,s)\right| ds \le l_1 < \infty,$

$H_{ts}^{''}(t,s) \in L_1(G),\ \underset{t\in[s,a]}{Sup}\left|H_s^{'}(t,s)\right| \le \gamma(s) \in L_1(-\infty,a];$

(iii) At least one of the following three conditions holds:

1) $\alpha^{'}(t) < 0$ *for almost* $t \in (-\infty, a]$; 2) $\beta(s) > 0$ *for almost all* $s \in (-\infty, a]$;

3) $H_{ts}^{''}(t,s) < 0$ *for almost all* $(t,s) \in G$.

Theorem. Let conditions (i), (ii) and (iii) be satisfied. Then the solution of the integral equation (1) is unique in $L_1(-\infty, a]$.

Proof. Let $u(t) \in L_1(-\infty, a]$ be a solution of the integral equation (1). By virtue of (2), we can write of the integral equation (1) in the form

$$\int_{-\infty}^{t} A(t,s)u(s)ds + \int_{t}^{a} B(t,s)u(s)ds = f(t). \tag{4}$$

Multiplying both sides of the equation (4) by u(t) and integrating over the domain $(-\infty, a]$, we obtain.

$$\int_{-\infty}^{a}\int_{-\infty}^{t} A(t,s)u(s)u(t)dsdt + \int_{-\infty}^{a}\int_{t}^{a} B(t,s)u(s)u(t)dsdt = \int_{-\infty}^{a} f(t)u(t)dt. \tag{5}$$

Applying Dirichlet's formulas to (5) and taking into account (3), we have

$$\int_{-\infty}^{a}\int_{-\infty}^{t} H(t,s)u(s)ds\, u(t)dt = \int_{-\infty}^{a} f(t)u(t)dt. \tag{6}$$

We shall introduce the notation

$$z(t,s) = \int_{s}^{t} u(\nu)d\nu, (t,s) \in G. \tag{7}$$

Then from (7), we obtain

$$d_s z(t,s) = -u(s)ds, z(t,s)u(t)dt = \frac{1}{2}d_t(z^2(t,s)). \tag{8}$$

Let us transform the integral on the left hand of the identity (6). Taking into account (7), (8) and integrating by parts, we have

$$\int_{-\infty}^{a}\int_{-\infty}^{t} H(t,s)u(s)u(t)dsdt = \int_{-\infty}^{a}\alpha(t)z(t,-\infty)u(t)dt + \int_{-\infty}^{a}\int_{-\infty}^{t} H_s^{'}(t,s)z(t,s)u(t)dsdt.$$

Hence, applying Dirichlet's formula, we obtain

$$\int_{-\infty}^{a}\int_{-\infty}^{t} H(t,s)u(s)u(t)dsdt = \frac{1}{2}\int_{-\infty}^{a}\alpha(t)d_t(z^2(t,-\infty)) + \frac{1}{2}\int_{-\infty}^{a}\left[\int_{s}^{a} H_s^{'}(t,s)d_t(z^2(t,s))\right]ds =$$

$$= \frac{1}{2}\alpha(a)z^2(a,-\infty) - \frac{1}{2}\int_{-\infty}^{a}\alpha^{'}(t)z^2(t,-\infty)dt + \frac{1}{2}\int_{-\infty}^{a}\beta(s)z^2(a,s)ds - \qquad (9)$$

$$-\frac{1}{2}\int_{-\infty}^{a}\int_{s}^{a} H_{ts}^{''}(t,s)z^2(t,s)dtds.$$

Taking into account (6) and applying Dirichlet's formula from (9), we have

$$\frac{1}{2}\alpha(a)z^2(a,-\infty) - \frac{1}{2}\int_{-\infty}^{a}\alpha^{'}(t)z^2(t,-\infty)dt + \frac{1}{2}\int_{-\infty}^{a}\beta(s)z^2(a,s)ds -$$

$$-\frac{1}{2}\int_{-\infty}^{a}\int_{-\infty}^{t} H_{ts}^{''}(t,s)z^2(t,s)dsdt = \int_{-\infty}^{a} f(t)u(t)dt. \qquad (10)$$

Suppose that $f(t)=0$ for $t\in(-\infty,a]$. Then, taking into account conditions (*i*), (*ii*) and (*iii*), we see that (10) implies

$$\int_{-\infty}^{t} u(\tau)d\tau = 0,\ t\in(-\infty,a]\ or\ \int_{s}^{a} u(\tau)d\tau = 0,\ s\in(-\infty,a]$$

or

$$\int_{s}^{t} u(\tau)d\tau = 0,\ (t,s)\in G.$$

Therefore, $u(t)=0$ for all $t\in(-\infty,a]$.

The theorem is proved.

Remark 1. If $B(s,t)=0$ for all $(t,s)\in G$, then the integral equation (1) is Volterra linear integral equation of the first kind. In this case the assertion of the theorem is true for $H(t,s)=A(t,s), \forall(t,s)\in G.$

Remark 2. If $A(t,s)=0$ for all $(t,s)\in G$, then the integral equation (1) is Volterra linear integral equation of the first kind. In this case the assertion of the theorem is true for $H(t,s)=B(s,t), \forall(t,s)\in G.$

3. Examples

Example 1. Consider the integral equation

$$\int_{-\infty}^{t} A(t,s)u(s)ds + \int_{t}^{0} B(t,s)u(s)ds = f(t), t \in (-\infty, 0], \tag{11}$$

where

$$A(t,s) = -\frac{c}{a(b-a)}\left[e^{bt}e^{-a(t-s)} - \frac{2b}{a+b}e^{as}\right], (t,s) \in G, \tag{12}$$

$$B(s,t) = \frac{cd}{ab}(e^{bt} - 2), (t,s) \in G, \tag{13}$$

$G = \{(t,s); -\infty < s \le t \le 0\}, a,b,c$ and d are real parameters, $a > 0, b > 0, c > 0, d < 0, a \ne b.$

Then taking into account (12) and (13) from (3) we have

$$H(t,s) = -\frac{c}{a(b-a)}\left[e^{bt}e^{-a(t-s)} - \frac{2b}{a+b}e^{as}\right] + \frac{cd}{ab}(e^{bt} - 2), (t,s) \in G, \tag{14}$$

$$H_t^{'}(t,s) = -\frac{c}{a}e^{bt}e^{-a(t-s)} + \frac{cd}{a}e^{bt}, (t,s) \in G, \tag{15}$$

$$H_s^{'}(t,s) = -\frac{c}{b-a}\left[e^{bt}e^{-a(t-s)} - \frac{2b}{a+b}e^{as}\right], (t,s) \in G, \tag{16}$$

$$H_{ts}^{''}(t,s) = -ce^{bt}e^{-a(t-s)}, (t,s) \in G, \tag{17}$$

$$\alpha(t) = \lim_{s \to -\infty} H(t,s) = 0, \alpha'(t) = 0, t \in (-\infty, 0], \tag{18}$$

$$\beta(s) = H_s^{'}(0,s) = \frac{c}{a+b}e^{as}, s \in (-\infty, 0]. \tag{19}$$

From (14) and (16), we obtain

$$l \le \frac{c}{|b-a|a}\left(1 + \frac{2b}{a+b}\right) + \frac{2c|d|}{ab}, \tag{20}$$

$$l_1 \le \frac{c(a+3b)}{|b-a|a(a+b)}, \tag{21}$$

$$\gamma(s) = \frac{c}{|b-a|}\left[e^{bs} + \frac{2b}{a+b}e^{as}\right], s \in (-\infty, 0]. \tag{22}$$

Then taking into account (14)-(22), we can verity that conditions (*i*), (*ii*) and (*iii*) are satisfied for the integral equation (11). Therefore the solution of the integral equation (11) is unique in the pace $L_1(-\infty, 0]$.

Example 2. Consider the integral equation

$$\int_{-\infty}^{t} A(t,s)u(s)ds = f(t), \quad t \in (-\infty, 0], \tag{23}$$

where

$$A(t,s) = -\frac{c}{a(b-a)}\left[e^{bt}e^{a(t-s)} - \frac{2b}{a+b}e^{as}\right] + \frac{cd}{a+b}(e^{bt}-2), \tag{24}$$
$$(t,s) \in G = \{(t,s); -\infty < s \le t \le 0\},$$

$a,b,$ cand d real parameters, $a > 0, b > 0, c > 0, d > 0, a \neq b.$ Then taking into account (23), from (3) we have

$$H(t,s) = A(t,s), \quad (t,s) \in G. \tag{25}$$

Then taking into account (24), (25) and (14)-(22), we can verity that conditions (i), (ii) and (iii) are satisfied for the integral equation (23). Therefore the solution of the integral equation (23) is unique in the space $L_1(-\infty, 0]$.

REFERENCES

1. A.S. Apartsyn, Nonclassical linear Volterra equations of the first kind, VSP, Utrecht, The Netherlands, 2003.
2. A. Asanov, Regularization, uniqueness and existence of solutions of Volterra equations of the first kind, VSP, Utrecht, The Netherlands, 1998.
3. A. Asanov, K.B. Matanova, R.A. Asanov. A class of linear and nonlinear Fredholm integral equations of the third kind, Kuwait J. Sci. 44(1) (2017), 17-28.
4. A.L. Bukhgeim. Volterra equations and inverse problems, VSP, Utrecht, The Netherlands, 1999.
5. A.M. Denisov. Elements of the theory of inverse problems, VSP, Utrecht, The Netherlands, 1999.
6. M.I. Imanaliev, A. Asanov. Solutions of system of nonlinear Volterra integral equations of the first kind, Soviet. Math. Dokl. 309:5 (1989), 1052-1055.
7. M.I. Imanaliev, A. Asanov. Solutions of systems of Volterra nonlinear integral equations of the third kind, Doklady Mathematics,76(2007),490-493.
8. Imanaliev M.I., Asanov A. Solutions of system of Fredholm linear integral equations of the third kind, Doklady Matematics, 81(2010), 115-118.

9. M.I. Imanaliev, A. Asanov, R.A. Asanov. Solutions to systems of Linear Fredholm Integral Equations of the Third kind with Multipoint Singularities, Doklady Mathematics,95(3) (2017), 235-239.
10. M.I. Imanaliev, A. Asanov, Z.A. Kadenova. A Class of Linear Integral Equations of the First Kind with Two Independent Variables, Doklady Mathematics, 89(1) (2014), 98-102.
11. M.M. Lavrent'ev. The integral equations of the first kind, Doklady Akademii Nayk, 127(1959), 31-33.
12. M.M. Lavrent'ev, V.G. Romanov,S.P. Shishatskii. Ill-Posed problems of mathematical physics and analysis, Providence R.I., American Mathematical Society, 1986.
13. N.A. Magnitskii. Volterra linear integral equations of the first and third kinds, Zhurnal vychislitel'noi matematiki i matematicheckoi fiziki, 19(1979), 970-989.
14. S.P. Shishatskii, A. Asanov, E.R. Atamanov. Uniqueness Problems for degenerating equations and nonclassical problems, VSP, Utrecht, The Netherlands, 2001.
15. A. Asanov, J. Orozmamatova. About Uniqueness of Solutions of Fredholm Linear Integral Equations of the First Kind in the Axis, Filoma, 33(5) (2019), 1329-1333.

MSC 34K20

BOUNDEDNESS OF SOLUTIONS OF A CLASS OF LINEAR DIFFERENTIAL EQUATIONS OF THE SECOND ORDER ON THE SEMIAXIS

Asanova K. A
Institute of Mathematics of NAS of the Kyrgyz Republic

In this paper based on a formula for one class of second order linear differential equations with variable coefficients were established sufficient conditions of exponential estimate of solutions for one class of second order linear differential equations on the semi axis.
Keywords: Formula, linear differential equations, second order, limitations, semi axis.

Бул макалада коэффициенттери өзгөрмөлүү болгон экинчи тартиптеги сызыктуу дифференциалдык теңдемелердин бир классынын чыгарылыштарынын формуласынын

негизинде жарым октогу экинчи тартиптеги сызыктуу дифференциалдык теңдемелердин бир классынын чыгарылыштарынын чектелгендигинин жетишерлик шарты көрсөтүлдү.
Урунттуу сөздөр: Формула, сызыктуу дифференциалдык теңдемелер, экинчи тартиптеги, чектелген, жарым ок.

В данной работе на основе формулы для одного класса линейных дифференциальных уравнений второго порядка с переменными коэффициентами установлены достаточные условия ограниченности решений одного класса линейных дифференциальных уравнений второго порядка на полуоси.
Ключевые слова: Формула, линейные дифференциальные уравнения, второго порядка, ограниченность, полуось.

Consider the linear differential equation

$$y'' + p(t)y' + q(t)y = f(t), t \geq t_0 \tag{1}$$

with initial conditions

$$y(t_0) = m, y'(t_0) = n, m,\ n \in R \tag{2}$$

where $p(t), q(t), f(t)$ – known features.

The questions of boundedness and asymptotic stability of solutions on the semiaxis for differential and integro-differential equations were studied in [1-7]. In this paper, on the basis of the results of [8] and the method of transformations, we establish sufficient conditions for the boundedness of solutions of the differential equation (1) on the semiaxis.

Assume the following conditions are met: *a)*

$$q(t) = q_0(t) + q_1(t),\ \ t \geq t_0, \tag{3}$$

$$q_0(t) = a(t)[p(t) - a(t)] - l^2(t) + a'(t), t \geq t_0, \tag{4}$$

$$l(t) = r exp\left\{\int_{t_0}^{t}[2a(s) - p(s)]ds\right\}, \tag{5}$$

where $a(t), a'(t), q_1(t), p(t), f(t)$ – known continuous functions on $[t_0, \infty)$,
$a'(t)$ – function derivative $a(t), r \in R, r \neq 0$;

b) $a(t) + l(t) \geq 0$ and $a(t) - l(t) \geq 0$ for all $t \in [t_0, \infty)$,

$|f(t)|\left\{exp\left[-\int_{t_0}^{t}(a(\tau) + l(\tau) - p(\tau))d\tau\right] + exp\left[-\int_{t_0}^{t}(a(\tau) - l(\tau) - p(\tau))d\tau\right]\right\} \leq$
$\leq l_1(t)$ and $|q_1(t)|\left\{exp\left[-\int_{t_0}^{t}(a(\tau) + l(\tau) - p(\tau))d\tau\right] + exp\left[-\int_{t_0}^{t}(a(\tau) - l(\tau) -\right.\right.$
$\left.\left.-p(\tau))d\tau\right]\right\} \leq l_2(t)$ for all $t \in [t_0, \infty)$, где $l_1(t), l_2(t) \in L_1[t_0, \infty), l_1(t) \geq 0$ and

$l_2(t) \geq 0$ for all $t \in [t_0, \infty)$.

Substituting (3) into (1) we have

$$y'' + p(t)y' + q_0(t)y = f(t) - q_1(t)y, t \geq t_0. \quad (6)$$

Taking into account the corollary of Theorem 1 from [7], from (6) we obtain

$$y(t) = \frac{y_1(t)}{2l(t_0)}\left[m\left(l(t_0) - a(t_0)\right) - n\right] + \frac{y_2(t)}{2l(t_0)}[n + m(l(t_0) + a(t_0))] + y_3(t), \quad (7)$$

where $[y_1(s)y_2(t) - y_1(t)y_2(s)]ds, t \geq t_0$

$$y_1(t) = exp\left[-\int_{t_0}^{t}(a(s) + l(s))ds\right], \quad (8)$$

$$y_2(t) = exp\left[-\int_{t_0}^{t}(a(s) - l(s))ds\right], \quad (9)$$

$$y_3(t) = \int_{t_0}^{t}[f(s) - q_1(s)y(s)][2l(s)y_1(s)y_2(s)]^{-1}\ [y_1(s)y_2(t) -$$

$$-y_1(t)y_2(s)]ds, t \geq t_0. \quad (10)$$

Substituting (8), (9), (10) by (7) and taking into account (5) we get

$$y(t) = \frac{1}{2l(t_0)}\left[m\left(l(t_0) - a(t_0)\right) - n\right]exp\left\{-\int_{t_0}^{t}[a(s) + l(s)]ds\right\} +$$

$$+\frac{1}{2l(t_0)}\left[n + m\left(l(t_0) + a(t_0)\right)\right]exp\left\{-\int_{t_0}^{t}[a(s) - l(s)]ds\right\} +$$

$$+\int_{t_0}^{t}\frac{1}{2r}exp\left[\int_{t_0}^{s}p(\tau)d\tau\right][f(s) - q_1(s)y(s)]\{exp\left[-\int_{t_0}^{s}\left(a(\tau) + l(\tau)\right)d\tau\right]$$

$$exp\left[-\int_{t_0}^{t}\left(a(\tau) - l(\tau)\right)d\tau\right] - exp\left[-\int_{t_0}^{t}\left(a(\tau) + l(\tau)\right)d\tau\right]$$

$$exp\left[-\int_{t_0}^{s}\left(a(\tau) - l(\tau)\right)d\tau\right]\}ds,\ t \geq t_0. \quad (11)$$

We introduce the notation:

$$F(t) = \frac{1}{2r}\left[m\left(r - a(t_0)\right) - n\right]exp\left\{-\int_{t_0}^{t}[a(s) + l(s)]ds\right\} +$$

$$+\frac{1}{2r}\left[n + m\left(r + a(t_0)\right)\right]exp\left\{-\int_{t_0}^{t}[a(s) - l(s)]ds\right\} +$$

$$+\frac{1}{2r}\int_{t_0}^{t}f(s)\,\{exp\left[-\int_{t_0}^{s}\left(a(\tau) + l(\tau) - p(\tau)\right)d\tau\right]exp\left[-\int_{t_0}^{t}\left(a(\tau) - l(\tau)\right)d\tau\right] -$$

$$-exp\left[-\int_{t_0}^{s}\big(a(\tau)-l(\tau)-p(\tau)\big)d\tau\right]exp\left[-\int_{t_0}^{t}\big(a(\tau)+l(\tau)\big)d\tau\right]\}ds, \qquad (12)$$

$$K(t,s)=-\frac{q_1(s)}{2r}\{exp\left[-\int_{t_0}^{s}\big(a(\tau)+l(\tau)-p(\tau)\big)d\tau\right]exp\left[-\int_{t_0}^{t}\big(a(\tau)-l(\tau)\big)d\tau\right]-$$

$$-exp\left[-\int_{t_0}^{s}\big(a(\tau)-l(\tau)-p(\tau)\big)d\tau\right]exp\left[-\int_{t_0}^{t}\big(a(\tau)+l(\tau)\big)d\tau\right]\}, \qquad (13)$$

$(t,s)\in G=\{(t,s): t_0\le s\le t<\infty\}$.

Taking into account notations (12) and (13), we write equation (11) in the form

$$y(t)=F(t)+\int_{t_0}^{t}K(t,s)y(s)ds, t\ge t_0, \qquad (14)$$

Theorem. Let conditions a) and b) be satisfied. Then the solution of the Cauchy problem (1)-(2) satisfies the following estimate:

$$|y(t)|\le c_1,\ t\ge t_0, \qquad (15)$$

where

$$c_1=c_2exp\left[\frac{1}{2|r|}\int_{t_0}^{\infty}l_2(s)ds\right],$$

$$c_2=\frac{1}{2|r|}\left[\big|m\big(r-a(t_0)\big)-n\big|+|n+m(r+a(t_0))|+\int_{t_0}^{\infty}l_1(s)ds\right].$$

Proof. Considering condition b), from (12) and (13) we have:

$$|F(t)|\le c_2, \quad t\ge t_0, \qquad (16)$$

$$|K(t,s)|\le\frac{l_2(s)}{2|r|}, \quad (t,s)\in G. \qquad (17)$$

Then, by virtue of (16) and (17) from (14) we obtain:

$$|y(t)|\le c_2+\frac{1}{2|r|}\int_{t_0}^{t}l_2(s)|y(s)|ds,\ t\ge t_0. \qquad (18)$$

Further, due to the Granwall-Bellman inequality, from (18) we have estimate (15). Theorem proven.

Example. Consider problems (1) - (2) for $t_0=0, p(t)=2(1+t)$,

$$q(t)=(1+t)^2+2exp\left[-\frac{1}{2}t^2-4t\right], f(t)=3exp\left[-\frac{1}{2}t^2-3t\right], \quad t\in[0,\infty).$$

In this case, conditions *a)* are satisfied for

$$a(t)=3+t, r=1, q_1(t)=2exp\left[-\frac{1}{2}t^2-4t\right],\ l(t)=1, t\in[0,\infty).$$

In addition, conditions *b)* are satisfied for

$$l_1(t)=6e^{-t},\ l_2(t)=4e^{-2t},\ t\in[0,\infty).$$

Indeed

$$a(t)+\ l(t)=2+t\geq 0, a(t)-\ l(t)=t\geq 0, t\in[0,\infty),$$

$$|f(t)|\{\exp\left[-\int_0^t\left(a(\tau)+l(\tau)-p(\tau)\right)d\tau\right]+\exp[-\int_0^t(a(\tau)-l(\tau)-p(\tau))d\tau]\}=$$

$$=3e^{-(t^2+3t)}\left(e^{\frac{t^2}{2}}+e^{\frac{t^2}{2}+2t}\right)\leq 6e^{-t}=l_1(t),\ t\in[0,\infty),$$

$$|q_1(t)|\left\{exp\left[-\int_0^t(a(\tau)+l(\tau)-p(\tau))d\tau\right]+exp\left[-\int_0^t(a(\tau)-l(\tau)-p(\tau))d\tau\right]\right\}=$$

$$=2e^{-(\frac{1}{2}t^2+4t)}\left[e^{\frac{t^2}{2}}+e^{\frac{t^2}{2}+2t}\right]\leq 4e^{-2t}=l_2(t)\in[0,\infty).$$

Thus, all conditions are met. In this case

$$c_2=\frac{1}{2}\left[|n|+|n+2m|+\int_0^\infty l_1(s)ds\right]=\frac{1}{2}[|n|+|n+2m|+6], \qquad (19)$$

$$c_1=c_2exp\left[\frac{1}{2|r|}\int_0^\infty l_2(s)ds\right]=c_2.$$

Therefore, to solve this Cauchy problem (1) - (2), estimate (15) is valid for , where the number is determined by formula (19).

REFERENCES

1. S. Iskandarov, “On the method of reduction to a system for a linear second-order Volterra integro-differential equation,” Issled. by integro-diffr. equations. – Bishkek: Ilim, 2006. – Issue 35. – pp. 31-35.
2. Iskandarov S. Method of weight and cutoff functions and asymptotic properties of solutions of integro-differential and integral equations of Volterra type – Bishkek: Ilim – 2002. –- 216 p.
3. Iskandarov S., Shabdanov D.N. The method of partial cutting and boundedness of solutions of the implicit Volterra integro-differential equation of the first order // Issled. by integro-differential. equations. – Bishkek: Ilim – 2004. – Issue 33. – pp. 67-71.
4. S. Iskandarov, The Method of Weight and Cutoff Functions and Asymptotic Properties of Solutions to Volterra-Type Equations, Extended Abstract of Cand. dis...doc. Phys.-Math. Sciences: 01.01.02. – Bishkek – 2003. – 34 p.

5. S. Iskandarov, On boundedness and quadratic integrability on the half-line of solutions and their first derivatives of a class of second-order linear integro-differential equations of the Volterra type. //Izv. Academy of Sciences of the Kyrgyz.SSR. – 1983. – No. 1. – pp. 19-23.
6. After all, Yu.A., Pakhyrov 3. Sufficient criteria for the boundedness of solutions of linear integro-differential equations. // Research. by integro-diffr. equations price Kyrgyzstan. – Frunze: Ilim, 1973. – Issue 9. – WITH. 68-103.
7. Imanaliev M.I., Iskandarov S., Asanova K.A. Lemma of Lyusternik-Sobolev and specific asymptotic stability of solutions of a linear homogeneous integro-differential equation of the third order of the Volterra type. // Dokl. Russian. Acad.Sci. – 2016. – T -469, No. 4 – pp. 397-401.
8. Avyt Asanov, Kanykei Asanova. For formulas for solution of One class of linear Differential Equations of the Second Order with the Variable Coefficients // Theory and Practice of Mathematics and Computer Science – 2021 – Vol.10 – Chapter 2 – pp. 17-26.

MSC 45A05, 45B05

ONE CLASS OF FREDHOLM LINEAR INTEGRAL EQUATIONS OF THE THIRD KIND WITH DEGENERATE KERNELS ON THE SEMIAXIS

Asanov R. A.
The International University of Central Asia (*IUCA)*

Based on a new approach, it is shown that solutions for one class of Fredholm linear integral equations of the third kind with degenerate kernels on the semiaxis are equivalent to solving systems of linear algebraic equations. The questions of existence and uniqueness of the solution for this integral equation are studied.

Keywords: Solutions, linear, equations, semi-axially integral, algebraic, Fredholm, third kind, equivalent.

Жаңы ыкманын негизинде жарым октогу Фредгольмдун үчүнчү түрдөгү кубулган сызыктуу интегралдык теңдемелердин бир классын чыгаруу сызыктуу алгебралык теңдемелердин системасынын чыгарууга эквиваленттүү экендиги көрсөтүлдү. Бул интегралдык теңдеменин чыгарылышынын жашашы жана жалгыздыгы далилденди.

Урунттуу сөздөр: Чыгарылыш, сызыктуу, теңдемелер, жарым октогу интегралдар, алгебралык, Фредгольм, үчүнчү түр, эквиваленттүү.

На основе нового подхода показано, что решения для одного класса линейных интегральных уравнений Фредгольма третьего рода с вырожденными ядрами на полуоси эквивалентны решению систем линейных алгебраических уравнений. Исследуются вопросы существования и единственности решения этого интегрального уравнения.
Ключевые слова: Решения, линейные, уравнения, полуосевые интегралы, алгебраические, Фредгольма, третьего рода, эквивалентные.

Consider the following integral equations

$$p(x)u(x) = \lambda \sum_{j=1}^{m} a_j(x) \int_a^{\infty} b_j(y)u(y)dy + f(x),\ x\epsilon[a,\infty), \quad (1)$$

where $p(x)$ a known continuous function on $[a,\infty)$, $a_j(x)$ and $b_j(x)$ known continuous functions on $[a,\infty)$ $(j = 1,\dots,m)$, $f(x)$ a known continuous function on $[a,\infty)$, $u(x)-$ unknown continuous function on $[a,\infty)$, $\lambda-$ a real parameter, $p(x_l) = 0, x_l\, \epsilon[a,\infty),\ l = 1,2,\dots,k.$

Many questions for integral equations were investigated in $[1-12]$. In particular, regularizing operators according to M.M. Lavrentiev were constructed in [3] to solve linear integral Fredholm equations of the first kind. In [5-6], uniqueness theorems were proved for systems of nonlinear Volterra integral equations of the third kind and for systems of linear integral Fredholm equations of the third kind and regularizing operators according to M.M. Lavrentiev were constructed. In this paper, the uniqueness and existence theorems of the solution for linear integral equations (1) are proved.

Denote by $C[a,\infty)$ is the space of all continuous functions on $[a,\infty)$. By $L_p[a,\infty)$ we denote the space of all functions with integrable p-th degree on $[a,\infty)$, p>1.

Everywhere we will assume that

$$p(x) = \prod_{e=1}^{k} p_l(x),\ p_l(x_l) = 0,\ p_l(x)\,\epsilon\, C[a,\infty), \quad (2)$$

$p_l(x) \neq 0$ for $x\,\epsilon\,[a,\infty)$ and $x \neq x_l,\ l = 1,\dots,k.$

Assume the following conditions are met:

a) For everyone $l = 1,\dots,k,$ and $j = 1,\dots,m$ $a_{l,j}(x)-$ functions are continuous functions on $[a,\infty), a_{k,j}(x) \in L_p[a,\infty), p > 1, b_j(x) \in L_q[a,\infty),\ \frac{1}{p}+\frac{1}{q} = 1,$

where $a_{0,j}(x) = a_j(x), a_{l,j}(x) = \frac{1}{p_l(x)}\left[a_{l-1,j}(x) - a_{l-1,j}(x_l)\right], x\epsilon R;$

b) For everyone $l = 1, \dots, k$ $f_l(x) -$ functions are continuous functions on $[a, \infty)$, $f_k(x) \in L_p[a, \infty)$, where $f(x) = f(x)$, $f_l(x) = \frac{1}{p(x)}[f_{l-1}(x) - f_{l-1}(x_l)]$, $x \in R$.

Theorem. Let the conditions (2), a) and b) be satisfied. Then

1) if a system of linear algebraic equations

$$\begin{cases} \lambda \sum_{j=1}^{m} a_j(x_1)c_j + f(x_1) = 0, \\ \lambda \sum_{j=1}^{m} a_{1,j}(x_2)c_j + f_1(x_2) = 0, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \lambda \sum_{j=1}^{m} a_{k-1,j}(x_k)c_j + f_{k-1}(x_k) = 0, \\ c_i = \int_a^{\infty} b_i(y)[\lambda \sum_{j=1}^{m} a_{k,j}(y)c_j + f_k(y)]dy, i = 1, \dots, m, \end{cases} \tag{3}$$

with respect to the unknowns $c_1, c_2 \dots, c_m$ has a unique solution, then the integral equation (1) in the space $C[a, \infty) \cap L_p[a, \infty), p > 1$ has a unique solution that can be represented as

$$u(x) = \lambda \sum_{j=1}^{m} a_{k,j}(x)c_j + f_k(x), \ x \in R; \tag{4}$$

2) if the system of linear algebraic equations (3) are incompatible, then the integral equation (1) in the space $L_p[a, \infty), p > 1$ has no solution;

3) if the system of linear algebraic equations (3) has an infinite number of solutions depending on q parameters, then the integral equation (1) in space $C[a, \infty) \cap L_p[a, \infty), p > 1$ has an infinite number of solutions depending on q parameters. In this case, the general solution of the system (1) is determined by the formula (4).

Proof. First, let $u(t) \in C[a, \infty) \cap L_p[a, \infty)$ is the solutions of the integral equation (1). Then, assuming $x = x_1$ from (1) we have

$$\lambda \sum_{j=1}^{m} a_j(x_1) \int_a^{\infty} b_j(y)\, u(y)dy + f(x_1) = 0, \tag{5}$$

Subtracting (5) from (1) we get

$$\prod_{l=1}^{k} p_l(x)\, u(x) \ = \lambda \sum_{j=1}^{m} [a_j(x) - a_j(x_1)] \int_a^\infty b_j(y)\, u(y) dy + f(x) - f(x_1).$$

Hence, given the conditions a) and b) we have

$$\prod_{l=2}^{k} p_l(x)\, u(x) = \lambda \sum_{j=1}^{m} a_{1,j}(x) \int_a^\infty b_j(y)\, u(y) dy + f_1(x), \quad x \,\epsilon\, [a, \infty). \qquad (6)$$

If k=1, then

$$\prod_{l=2}^{k} p_l(x) = 1,\ x\, \epsilon [a, \infty)\,.$$

In the case when $k \geq 2$ assuming $x = x_2$ from (6) we obtain

$$\lambda \sum_{j=1}^{m} a_{1,j}(x_2) \int_a^\infty b_j(y)\, u(y) dy + f_1(x_2) = 0. \qquad (7)$$

Subtracting (7) from (6) and taking into account the conditions a) and b) we get

$$\prod_{l=3}^{k} p_l(x)\, u(x) = \ \lambda \sum_{j=1}^{m} a_{2,j}(x) \int_a^\infty b_j(y)\, u(y) dy + +f_2(x),\ x \,\epsilon\, [a, \infty). \qquad (8)$$

If $k = 2$, then

$$\prod_{l=3}^{k} p_l(x) = 1,\ x\, \epsilon [a, \infty)\,.$$

In the case when $k \geq 3,$ continuing this process, we will make sure that the solution of equation (1) $u(x)$ satisfies condition (3) and is determined by formula (4).

Conversely, let $u(t)\, \epsilon\, C[a, \infty) \cap L_p[a, \infty)$ is determined by the formula (4) and satisfy the condition (3). Multiplying (4) by $P_k(x)$ and by virtue of (3) we get

$$p_k(x) u(x) = \lambda \textstyle\sum_{j=1}^{m} a_{k-1,j}(x) c_j + f_{k-1}(x),\ x\, \epsilon [a, \infty). \qquad (9)$$

Further multiplying (9) by $p_{k-1}(x)$ and given the conditions (3) we have

$$p_{k-1}(x)\, p_k(x) u(x) = \lambda \textstyle\sum_{j=1}^{m} a_{k-2,j}(x) c_j + f_{k-2}(x),\ x\, \epsilon [a, \infty). \qquad (10)$$

Continuing this process with respect to system (10) and taking into account condition (3), we make sure that $u(t)$ is the solution of the integral equation (1). The theorem is proved.

Example. Consider the integral equation

$$x(x\text{-}3)u(x) = \lambda\int_0^\infty[\tfrac{x}{x^2+6}e^{-y} + (x+2)\,e^{-2y}](y^2+6)u(y)dy + \tfrac{\alpha x}{x^2+6}+\beta x + \mu, \quad (11)$$

$x \in [a,\infty)$.

Where $\lambda,\ \alpha,\beta,\mu -$ real parameters, $\lambda \neq 0$. It is not difficult to verify that for the integral equation (11), the conditions (2), a) and b) are satisfied when

$$p \geq 2,\ \ a = 0,\qquad m = 2, k = 2, x_1 = 0, x_2 = 3,\qquad p_1(x) = x,\ p_2(x) = x - 3,$$

$$a_1(x) = \frac{x}{x^2+6},\qquad a_2(x) = x + 2,\qquad b_1(y) = e^{-y}(y^2+6),$$

$$b_2(y) = e^{-2y}(y^2+6), \mathrm{f}(x) = \tfrac{\alpha x}{x^2+6} + \beta x + \mu,\ a_{1,1}(x) = \tfrac{1}{x^2+6},\ a_{1,2}(x) = 1, a_{2,1}(x) = -\tfrac{x+3}{15(x^2+6)},\ a_{2,2}(x) = 0,\ f_1(x) = \tfrac{\alpha}{x^2+6} + \beta,\ f_2(x) = -\tfrac{\alpha(x+3)}{15(x^2+6)}.$$

Then for the integral equation (11), the conditions (3) are written as follows:

$$\begin{cases} 2\lambda c_2 + \mu = 0, \\ \lambda\left(\frac{1}{15}c_1 + c_2\right) + \frac{\alpha}{15} + \beta = 0, \\ c_1 = -\frac{4}{15}(\lambda c_1 + \alpha), \\ c_2 = -\frac{7}{60}(\lambda c_1 + \alpha). \end{cases} \quad (12)$$

From (4) we have

$$u(x) = -\frac{(\lambda c_1 + \alpha)(x+3)}{15(x^2+6)},\ x \epsilon [a,\infty). \quad (13)$$

Further, from (12) we get

$$\begin{cases} c_2 = -\frac{\mu}{2\lambda},\ c_1 = -\frac{8\mu}{7\lambda},\qquad \lambda c_1 + \alpha = \frac{30\mu}{7\lambda}, \\ \alpha = \frac{2(4\lambda+15)\mu}{7\lambda},\ \beta = \frac{\mu(7\lambda-4)}{14\lambda}. \end{cases} \quad (14)$$

1) Let $\lambda \neq 0,\ \alpha = \frac{2(4\lambda+15)\mu}{7\lambda},\ \beta = \frac{\mu(7\lambda-4)}{14\lambda}$. Then from (14) we have that the integral equation (11) has a unique solution in space $C[a,\infty) \cap L_p[a,\infty)$, defined as at least or

$$u(x) = -\frac{2\mu(x+3)}{7\lambda(x^2+6)}, x \epsilon [a,\infty). \quad (15)$$

2) Let $\lambda \neq 0$, at least $\alpha \neq \frac{2(4\lambda+15)\mu}{7\lambda}$ or $\beta \neq \frac{\mu(7\lambda-4)}{14\lambda}$.

3)

Then the integral equation (11)has no solution in space

$$C[a,\infty) \cap L_p[a,\infty), p > 1.$$

REFERENCES

1. Tsalyuk, Z.B., Volterra integral equations // J. Sov. Math. – 1977 – vol. 12 – p.715.
2. Lavrent'ev, M.M., On integral equations of the first kind // Dokl. Akad. Nauk SSSR – 1959 – vol. 127, no. 1 – pp. 31–33.
3. Magnitskii, N.A., Linear integral equations of the first and third kind // Zh. Vychisl. Mat. Mat. Fiz. – 1979 – vol. 19, no. 4 – pp. 970-989.
4. Imanaliev, M.I. and Asanov, A., On solutions to systems of nonlinear Volterra integral equations of the first kind // Dokl. Akad. Nauk SSSR – 1989 – vol. 309, no. 5 – pp. 1052-1055.
5. Imanaliev, M.I. and Asanov, A., Regularization and uniqueness of solutions of systems of nonlinear Volterra integral equations of the third kind // Dokl. Ross. Akad. Nauk – 2007 – vol. 415, no. 1 – pp. 14-17.
6. Imanaliev, M.I. and Asanov, A., On solutions of systems of linear Fredholm integral equations of the third kind // Dokl. Ross. Akad. Nauk – 2010 – vol. 430, no. 6 – pp. 1-4.
7. Imanaliev, M.I., Asanov, A., and Asanov, R.A., On one class of systems of linear Fredholm integral equations of the third kind // Dokl. Ross. Akad. Nauk – 2011 – vol. 437, no. 5 – pp. 592-596.
8. Apartsyn A.S. Nonclassical linear Volterra Equations of the First Kind. VSP, Utrecht – The Netherlands – 2003 – 168 pages.
9. Asanov A. Regularization, Uniqueness and Existence of Solutions of Volterra Equations of the First Kind. VSP, Utrecht – The Netherlands – 1998 – 276 p.
10. Avyt Asanov, Kalyskan Matanova, Ruhidin Asanov. A class of linear and nonlinear Fredholm integral equations of the third kind //Kuwait Journal of Science – 2017 – Vol.44, No 1 – pp.17-28.

11. Bukhgeim A.L. Volterra Equations and Inverse Problems. VSP, Utrecht – The Netherlands – 1999 – 204 p.
12. Denisov A.M. Elements of the Theory of Inverse Problems. VSP, Utrecht – The Netherlands – 1999 – 272 p.

MSC 45A05, 45B05

THE GALERKIN METHOD FOR CONSTRUCTING SOLUTIONS TO A QUASILINEAR DIFFERENTIAL EQUATION OF THE SECOND ORDER

Bapa kyzy A.
Issyk-Kul State University named after K. Tynystanov

The article considers the problem of constructing a periodic solution to a second-order quasilinear differential equation by the Galerkin method. An algebraic equation is constructed with respect to the coefficients of the Fourier series and its solvability is proved. An estimate of the accuracy between the approximate and exact solution of the differential equation is obtained.
Keywords: Second order quasilinear differential equation, periodic solution, Galerkin method, algebraic equation, approximate and exact solutions.

Макалада экинчи тартиптеги квазисызыктуу дифференциалдык теңдеменин мезгилдик чыгарылышын Галеркиндин методу менен табуу масселеси каралат. Дифференциалдык теңдеме Фурьенин коэффициенттерине карата алгебралык теңдемеге келтирилип, анын чыгарылышынын чечилиши далилденет. Так жана жакындаштырылган чыгарылыштардын ортосундагы айырманын чени аныкталат.
Урунттуу сөздөр: Квазисызыктуу экинчи тартиптеги дифференциалдык теңдеме, мезгилдик чыгарылыш, Галеркиндин методу, алгебралык теңдеме, так жана жакындаштырылган чыгарылыштар.

В статье рассматривается задача постороения периодического решения квазилинейной дифференциальной уравнении второго порядка методом Галеркина. Посторoена алгебраическое уравнение относительно коэффициентов ряда Фурье доказано его разрешимость. Получена оценка точности между приближенным и точным решением дифференциального уравнения.
Ключевые слова: Квазилинейное дифференциальное уравнение второго порядка, периодическое решение, метод Галеркина, алгебраическое уравнение, приближенные и точные решения.

Consider for continuously differentiable with respect to t, 2π-periodic functions $f(t,x)$ the norms

$$|f|_r = \max_{\mathcal{T}\times D}\left\|f^{(r)}(t,x)\right\| \quad x \in D \subset R, \qquad \|f\|_0 = \left[\frac{1}{2}\int_0^{2\pi} \|f\|^2\,dt\right]^{\frac{1}{2}}.$$

Given a second-order differential equation

$$\frac{d^2x}{dt^2} = f(t), \quad (1)$$

where $f(t)$ is a periodic function with a period of 2π, continuous, expanding in a Fourier series of the form

$$f(t) = C_0 + \sqrt{2}\sum_{k=1}^{\infty}(c_k coskt + d_k sinkt). \quad (2)$$

On the set of periodic functions, an operator S_m is defined such that

$$S_m f(t) = C_0 + \sqrt{2}\sum_{k=1}^{m}(c_k coskt + d_k sinkt).$$

aking into account (2), from (1) we have

$$\frac{d^2x}{dt^2} = C_0 + \sqrt{2}\sum_{k=1}^{m}(c_k coskt + d_k sinkt). \quad (3)$$

Theorem 1. Let x=x(t) be the solution of equation (1). If a

$$C_0 = 0, \qquad x(0) = \sqrt{2}\sum_{k=1}^{m}\frac{c_k}{k^2}, \qquad \frac{dx(0)}{dt} = \sqrt{2}\sum_{k=1}^{m}\frac{d_k}{k^2},$$

then the 2π -periodic solution $x = x(t)$ will be represented as the formula

$$x(t) = \sqrt{2}\sum_{k=1}^{\infty}\frac{1}{k^2}(-c_k coskt - d_k sinkt). \quad (4)$$

Proof. Integrating both parts of the equality of equation (3)

$$\int_0^t \frac{d^2x(t)}{dt^2}dt = \int_0^t d\left(\frac{dx}{dt}\right) = \frac{dx(t)}{dt} - \frac{dx(0)}{dt} =$$

$$= C_0 t + \sqrt{2}\sum_{k=1}^{\infty}\left(c_k\int_0^t coskt\,dt + d_k\int_0^t sinktdt\right) =$$

$$= C_0 t + \sqrt{2}\sum_{k=1}^{\infty}\left(\frac{c_k}{k}sinkt - \frac{d_k}{k}(coskt - 1)\right).$$

$$\frac{dx(t)}{dt} = \frac{dx(0)}{dt} + C_0 t + \sqrt{2}\sum_{k=1}^{\infty}\frac{1}{k}(c_k sinkt - d_k(coskt - 1)),$$

$$\int_0^t \frac{dx(t)}{dt}dt = \int_0^t dx(t) = x(t) - x(0) = \frac{dx(0)}{dt}\cdot t + C_0\frac{t^2}{2} +$$

$$+\sqrt{2}\sum_{k=1}^{\infty}\frac{1}{k}\left(c_k\int_0^t sinktdt - d_k\int_0^t (coskt-1)dt\right) =$$

$$= \frac{dx(0)}{dt}\cdot t + \frac{C_0}{2}t^2 + \sqrt{2}\sum_{k=1}^{\infty}\frac{1}{k^2}(c_k(1-coskt) - d_k(sinkt - t)),$$

$$x(t) = x(0) + \frac{dx(0)}{dt}\cdot t + \frac{C_0}{2}t^2 + \sqrt{2}\sum_{k=1}^{m}\frac{c_k}{k^2} + \sqrt{2}\sum_{k=1}^{m}\frac{d_k}{k^2}t$$

$$+ +\sqrt{2}\sum_{k=1}^{\infty}\frac{1}{k^2}(-c_k coskt - d_k sinkt),$$

or

$$x(t) = x(0) + \sqrt{2}\sum_{k=1}^{\infty}\frac{c_k}{k^2} + \left(\frac{dx(0)}{dt} + \sqrt{2}\sum_{k=1}^{m}\frac{d_k}{k^2}\right)t + \frac{C_0}{2}t^2 +$$

$$+\sqrt{2}\sum_{k=1}^{\infty}\frac{1}{k^2}(-c_k coskt - d_k sinkt).$$

Hence, taking into account the condition of the theorem, we obtain

$$x(t) = \sqrt{2}\sum_{k=1}^{\infty}\frac{1}{k^2}(-c_k coskt - d_k sinkt).$$

The theorem has been proven.

Let us estimate the difference between the exact and approximate solutions of equation (1).

Theorem 2. The difference $\bar{x}(t) - x_m(t)$ satisfies the estimate

$$|\bar{x}(t) - \bar{x}_m(t)|_0 \le \sigma(m)|f)|_0,$$

$$\|\bar{x}(t) - \bar{x}_m(t)\| \le \sigma_1(m)\|f\|_0,$$

where

$$\sigma(m) = \left[\frac{2}{(m+1)^4} + \frac{2}{(m+2)^4} + \cdots \ldots\right] \quad ,\sigma_1(m) = \frac{\sqrt{2}}{(m+1)^2}.$$

Proof. From decomposition (4) we obtain

$$\bar{x}(t) - S_m\bar{x}(t) = \bar{x}(t) - \bar{x}_m(t) = \sum_{k=1}^{\infty} \frac{\sqrt{2}}{k^2}(-c_k coskt - d_k sinkt), \tag{5}$$

from which, applying the Bunyakovskii-Schwartz inequality, we have

$$|\bar{x}(t) - \bar{x}_m(t)|_{\mathbf{0}}^{\mathbf{2}} \le \left\| \sum_{k=m+1}^{\infty} \frac{\sqrt{2}}{k^2}(-c_k coskt - d_k sinkt), \right\|^{\mathbf{2}} \le$$

$$\le \sum_{k=m+1}^{\infty} \frac{2}{k^4} \sum_{k=m+1}^{\infty} \frac{\sqrt{2}}{k^2}[\|-c_k coskt - d_k sinkt\|^2] \le$$

$$\le \sum_{k=m+1}^{\infty} \frac{2}{k^4} \sum_{k=m+1}^{\infty} \frac{\sqrt{2}}{k^2}[\|c_k\|^2 + \|d_k\|^2]. \tag{6}$$

Taking into account the Parseval equality from (6)

$$\sum_{k=m+1}^{\infty} [\|c_k\|^2 + \|d_k\|^2] = |f|_0^2. \tag{7}$$

As

$$\sigma^2(m) = \sum_{k=m+1}^{\infty} \frac{2}{k^4} = \frac{2}{(m+1)^4} + \frac{2}{(m+2)^4} + \cdots \ldots.$$

hen, taking into account (7), from (6) follows the estimate

$$|\bar{x}(t) - \bar{x}_m(t)|_{\mathbf{0}}^{\mathbf{2}} \le \sigma^2(m)|f|_0^2.$$

From here we find

$$|\bar{x}(t) - \bar{x}_m(t)|_{\mathbf{0}} \le \sigma(m)|f|_0.$$

Applying the Parseval formula to relation (5), we obtain

$$|\bar{x}(t) - \bar{x}_m(t)|_{\mathbf{0}}^{\mathbf{2}} = \sum_{k=m+1}^{\infty} \frac{\sqrt{2}}{k^2}[\|c_k\|^2 + \|d_k\|^2] \le$$

$$\le \frac{2}{(m+1)^2} \sum_{k=m+1}^{\infty} \frac{\sqrt{2}}{k^2}[\|c_k\|^2 + \|d_k\|^2] \le \sigma_1^2(m)|f|_0^2$$

From here we find

$$\|\bar{x}(t) - \bar{x}_m(t)\|_0 \le \sigma_1(m)\|f\|_0.$$

The theorem has been proven.

Consider a second-order quasilinear differential equation of the form

$$\frac{d^2x}{dt^2} = Ax + f(t,x), \tag{8}$$

where, A is a real number, $f(t,x)$ is a 2π-periodic function in t.

The periodic solution of Eq. (8) is sought in the form

$$x_m(t) = a_0 + \sqrt{2}\sum_{k=1}^{m}(a_k coskt + b_k sinkt), \tag{9}$$

The coefficients of which are found from the system of algebraic equations

$$\frac{d^2x_m(t)}{dt^2} = Ax_m(t) + S_m f\big(t, x_m(t)\big). \tag{10}$$

Hence we have

$$\sqrt{2}\sum_{k=1}^{m}(-k^2 sinkt - k^2 b_k sinkt) = Aa_0 + \sqrt{2}\sum_{k=1}^{m}(Aa_k coskt + Ab_k sinkt) +$$

$$+A_0^{(m)}\sqrt{2}\sum_{k=1}^{m}\left(A_k^{(m)} coskt + B_k^{(m)} sinkt\right), \tag{11}$$

where

$$A_0^{(m)} = \frac{1}{2\sqrt{2}\pi}\int_0^{2\pi} f\big(t, x_m(t)\big)dt,$$

$$A_k = \frac{1}{2\sqrt{2}\pi}\int_0^{2\pi} f\big(t, x_m(t)\big)cosktdt, \quad B_k = \frac{1}{2\sqrt{2}\pi}\int_0^{2\pi} f\big(t, x_m(t)\big)sinktdt,$$

For expansion coefficients (9), we obtain the system of equations

$$\begin{cases} Aa_0 + \quad A_0 = 0, \\ Aa_k + k^2 a_k + A_k = 0, \\ Ab_k + k^2 b_k + B_k = 0, \qquad\qquad k = \overline{1,m}. \end{cases}$$

Representing the system of equations in the form

$$\begin{cases} Aa_0 + 0\cdot a_k + 0\cdot b_k + A_0^{(m)} = 0, \\ 0\cdot a_0 + (A+k^2)a_0 + 0\cdot b_k + A_k^{(m)} = 0, \\ 0\cdot a_0 + 0\cdot a_k + (A+k^2)b_k + B_k^{(m)} = 0. \end{cases}$$

and write it in matrix form

$$\begin{pmatrix} A & 0 & 0 \\ 0 & A+k^2 & 0 \\ 0 & 0 & A+k^2 \end{pmatrix}\begin{pmatrix} a_0 \\ a_k \\ b_k \end{pmatrix} + \begin{pmatrix} A_0^{(m)} \\ A_k^{(m)} \\ B_k^{(m)} \end{pmatrix} = 0\,, \tag{12}$$

or

$$D^{(m)}\alpha + F^{(m)}(\alpha) = 0\,. \tag{13}$$

where

$$D^m = \begin{pmatrix} A & 0 & 0 \\ 0 & A+k^2 & 0 \\ 0 & 0 & A+k^2 \end{pmatrix}, \quad F^{(m)}(\alpha) = \begin{pmatrix} A_0^{(m)} \\ A_k^{(m)} \\ B_k^{(m)} \end{pmatrix}, \ \alpha = \begin{pmatrix} a_0 \\ a_k \\ b_k \end{pmatrix}. \quad k = \overline{1,m}.$$

Let us assume that the function $x = \bar{x}(t),\ 2\pi$ is a periodic solution of equation (3), then

$$\bar{x}(t) = \bar{a}_0 + \sqrt{2}\sum_{k=1}^{m}\left(\bar{a}_k coskt + \bar{b}_k sinkt\right).$$

Putting this function into equation (8) and taking into account the operator S_m, we obtain the following equality

$$S_m\ \frac{d^2\bar{x}(t)}{dt^2} = \frac{d^2\bar{x}_m(t)}{dt^2} =$$

$$= A\bar{x}_m(t) + S_m f\left(t, \bar{x}_m(t)\right) + S_m\left(f(t,\bar{x}(t)) - f\left(t, \bar{x}_m(t)\right)\right)$$

This equality is equivalent to an algebraic equation of the form

$$D^{(m)}\bar{\alpha} + F^{(m)}(\bar{\alpha}) = -S_m\left(f(t,\bar{x}(t)) - f\left(t,\bar{x}_m(t)\right)\right) = -\rho(m). \tag{14}$$

Imagine the difference $f(t,\bar{x}(t)) - f\left(t,\bar{x}_m(t)\right)$ as

$$f\left(t,\bar{x}(t)\right) - f\left(t,\bar{x}_m(t)\right) = \frac{\partial f\left(t, \bar{x}_m + \theta(\bar{x} - \bar{x}_m)\right)}{\partial x}\left(\bar{x}(t) - \bar{x}_m(t)\right), \quad 0 < \theta < 1.$$

Hence we have

$$\left|S_m\left(f\left(t,\bar{x}(t)\right) - f\left(t,\bar{x}_m(t)\right)\right)\right|_0 \le \left|f\left(t,\bar{x}(t)\right) - f\left(t,\bar{x}_m(t)\right)\right|_0 \le$$

$$\le \left|\frac{\partial f\left(t, \bar{x}_m + \theta(\bar{x} - \bar{x}_m)\right)}{\partial x}\right|_0 |\bar{x}(t) - \bar{x}_m(t)|_0.$$

Further, taking into account the results of theorem 2, from (14) we obtain

$$\|\rho(m)\| \leq |f|_1 |\bar{x}(t) - \bar{x}_m(t)|_0 \leq \sigma(m)|f|_1|f|_0, \tag{15}$$

or

$$\|\rho(m)\| \leq |f|_1 |\bar{x}(t) - \bar{x}_m(t)|_0 \leq \sigma_1(m)|f|_1|f|_0. \tag{16}$$

As $detD^{(m)} = A(A+1)^2(A+2)^2 \cdot \cdots \cdot (A+m)^2 \neq 0$, (12) should

$$\alpha + \left(D^{(m)}\right)^{-1} F^{(m)}(\alpha) = 0, \tag{17}$$

and if $\alpha = \bar{\alpha} = \left(\bar{a}_0, \bar{a}_1, \bar{b}_1, \cdots, \bar{a}_m, \bar{b}_m\right)$, then from (17) we get

$$\bar{\alpha} + \left(D^{(m)}\right)^{-1} F^{(m)}(\bar{\alpha}) = -\rho(m). \tag{18}$$

We solve equations (17) by the method of successive approximations

$$\alpha_{k+1} = -\left(D^{(m)}\right)^{-1} F^{(m)}(\alpha_k), \quad k = 0,1,2,\ldots \quad , \tag{19}$$

take, for the initial approximation, the number $\alpha_0 = \bar{\alpha}$.

Let us show the convergence of the sequence (17).

Let us estimate the difference $\alpha_1 - \alpha_0 = \alpha_1 - \bar{\alpha}$:

$$\alpha_1 - \alpha_0 = \alpha_1 - \bar{\alpha} = -\left(D^{(m)}\right)^{-1} F^{(m)}(\bar{\alpha}) - \bar{\alpha} = \left(D^{(m)}\right)^{-1} \rho(m). \tag{20}$$

Taking into account estimates (15) and (16), from (20) we obtain

$$\|\alpha_1 - \alpha_0\| = \|\alpha_1 - \bar{\alpha}\| \leq \left\|\left(D^{(m)}\right)^{-1} F^{(m)}(\bar{\alpha}) + \bar{\alpha}\right\| = \left\|\left(D^{(m)}\right)^{-1} \rho(m)\right\| \leq$$

$$\leq \left|\left(D^{(m)}\right)^{-1}\right| \|\rho(m)\| \leq \sigma(m) K |f|_1 |f|_0,$$

or

$$\|\alpha_1 - \alpha_0\| \leq \sigma_1(m) K |f|_1 |f|_0, \quad \text{где } K = \left\|\left(D^{(m)}\right)^{-1}\right\|.$$

Imagine the difference $\alpha_{k+1} - \alpha_k$ as

$$\alpha_{k+1} - \alpha_k = -\left(D^{(m)}\right)^{-1} \left(F^{(m)}(\alpha_k) - F^{(m)}(\alpha_{k-1})\right) =$$

$$= -\left(D^{(m)}\right)^{-1} \frac{\partial F^{(m)}\left(\alpha_{k-1} + \theta(\alpha_k - \alpha_{k-1})\right)}{\partial x} (\alpha_k - \alpha_{k-1}).$$

Suppose that for a larger m_0, when $m \geq m_0$, the condition

$$\left\|\frac{\partial F^{(m)}(\alpha)}{\partial x}\right\| \leq \frac{\chi}{K}, \quad \text{при } 0 \leq \chi \leq 1.$$

Then from (21) we obtain the estimate

$$\|\alpha_{k+1}-\alpha_k\| \le \left\|\left(D^{(m)}\right)^{-1}\right\| \left\|\frac{\partial F^{(m)}\left(\alpha_{k-1}+\theta(\alpha_k-\alpha_{k-1})\right)}{\partial x}\right\| \|\alpha_k-\alpha_{k-1}\| \le$$

$\le K\frac{\chi}{K_1}|\alpha_k-\alpha_{k-1}| \le \chi|\alpha_k-\alpha_{k-1}|,$ at $k=0,1,2,\ldots$

Hence, by induction, we obtain

$\|\alpha_{k+1}-\alpha_k\| \le \chi\|\alpha_k-\alpha_{k-1}\| \le \chi^2\|\alpha_{k-1}-\alpha_{k-2}\| \le \cdots \le \chi^k\|\alpha_1-\alpha_0\| \le$

$\le \chi^k\sigma(m)K|f|_1|f|_0 \le \chi^k\sigma(m_1)K|f|_1|f|_0,$ at $k=0,1,2,\ldots$ (21)

Further, taking into account (21), we have

$$\left\|\alpha_{k+p}-\alpha_k\right\| = \left\|\alpha_{k+p}-\alpha_{k+p-1}+\alpha_{k+p-1}-\alpha_{k+p-2}+\cdots+\alpha_{k+1}+\alpha_k\right\| \le$$

$$\le \left\|\alpha_{k+p}-\alpha_{k+p-1}\right\| + \left\|\alpha_{k+p-1}-\alpha_{k+p-2}\right\| + \cdots + \|\alpha_{k+1}-\alpha_k\| \le$$

$$\le \chi^k(\chi^{p-1}+\chi^{p-2}+\cdots+\chi+1 \le \sigma(m_0)K|f|_1|f|_0 \le$$

$$\le \chi^k(1+\chi+\cdots+\chi^p+\cdots)\sigma(m_0)K|f|_1|f|_0 \le \frac{\chi^k}{1-\chi}\sigma(m_0)|f|_1|f|_0.$$

Hence, as $p\to\infty$, we obtain the estimate

$$\|\alpha-\alpha_k\| \le \frac{\chi^k}{1-\chi}\sigma(m_0)|f|_1|\|f\|_0|. \tag{22}$$

For k=0, from (22) we obtain

$$\|\alpha-\alpha_0\| = \|\alpha-\bar{\alpha}\| \le \frac{\sigma(m_0)|f|_1|f|_0}{1-\chi} \le \frac{\sigma_1(m)|f|_1|f|_0}{1-\chi}. \tag{23}$$

Inequality (22) implies uniform convergence of sequence (19) to the solution of equation (17) as $k\to\infty$.

Let us estimate the difference $\|x_m(t)-\bar{x}_m(t)\|_0$:

$$\|x_m(t)-\bar{x}_m(t)\|_0^2 = \frac{1}{2\pi}\int_0^{2\pi}\|x_m(t)-\bar{x}_m(t)\|^2dt =$$

$$\frac{1}{2\pi}\int_0^{2\pi}\left[a_0+\sqrt{2}\sum_{k=1}^{m}\left((a_k-\bar{a}_k)coskt+(b_k-\bar{b}_k)sinkt\right)\right]^2dt = \frac{(a_0-\bar{a}_0)^2}{2}+$$

$$+\frac{1}{2}\sum_{k=1}^{m}\left(\sqrt{2}(a_k-\bar{a}_k)^2+\sqrt{2}(b_k-\bar{b}_k)^2\right) = \frac{\|(a_0-\bar{a}_0)\|^2}{2}+$$

$$+\frac{1}{2}\sum_{k=1}^{m}\left(\left\|(\sqrt{2}a_k-\sqrt{2}\bar{a}_k)\right\|^2+\left\|(\sqrt{2}b_k-\sqrt{2}\bar{b}_k)\right\|^2\right) = \frac{1}{2}\|\alpha-\bar{\alpha}\|^2.$$

Hence, taking into account (21), we obtain

$$\|x_m(t) - \bar{x}_m(t)\|_0 = \frac{1}{2}\|\alpha - \bar{\alpha}\| \le \frac{1}{\sqrt{2}}\sigma_1(m)\frac{|f|_1|f|_0}{1-\chi}. \qquad (24)$$

Let us estimate the difference $|x(t) - \bar{x}_m(t)|$:

$$|\bar{x}(t) - x_m(t)|_0 = |\bar{x}(t) - \bar{x}_m(t) + \bar{x}_m(t) - x_m(t)|_0 \le |\bar{x}(t) - \bar{x}_m(t)|_0 + |x_m(t) - \bar{x}_m(t)|_0 \qquad (25)$$

Since, according to theorem 2

$$|\bar{x}(t) - x_m(t)|_0 \le \sigma(m)|f|_0,$$

then from (25), taking into account inequality (24), we obtain the estimate

$$|\bar{x}(t) - x_m(t)|_0 \le \sigma(m)|f|_0 + \frac{1}{\sqrt{2}}\sigma_1(m)\frac{|f|_1|f|_0}{1-\chi}. \qquad (26)$$

Further, since $\frac{\sqrt{2}}{(m+1)^2} < \sigma(m) < \frac{\sqrt{2}}{m^2}$ and $\sigma_1(m) = \frac{\sqrt{2}}{(m+1)^2} < \frac{\sqrt{2}}{m^2}$,

then from (26) follows the estimate

$$|\bar{x}(t) - x_m(t)|_0 \le \frac{\sqrt{2}}{m^2}|f|_0 + \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{m^2}\frac{|f|_1|f|_0}{1-\chi} = \frac{\sqrt{2}}{m^2}|f|_0\left(1 + \frac{|f|_1|f|_0}{\sqrt{2}(1-\chi)}\right) \le$$

$$\le \frac{1}{m^2}|f|_0\frac{\sqrt{2} + |f|_1}{1-\chi} = \frac{|f|_0(\sqrt{2} + |f|_1)}{m^2(1-\chi)}, \quad as\, m \to \infty$$

Thus, the main assertion is proved.

Theorem 3. Let the second-order differential equation (8) have a 2π-periodic solution and satisfy the following requirements:

a) the requirement of theorem 2 is satisfied;

б) $\left\|\bar{\alpha} + \left(D^{(m)}\right)^{-1}F^{(m)}(\bar{\alpha})\right\| \le \sigma(m)K|f|_1|f|_0$, $K = \left\|\left(D^{(m)}\right)^{-1}\right\|$;

в) $\left\|\frac{\partial F^{(m)}(\alpha)}{\partial x}\right\| \le \frac{\chi}{K}$, $\quad 0 < \chi < 1$, $\sigma(m) < \frac{\sqrt{2}}{m^2}$, $\sigma_1(m) < \frac{\sqrt{2}}{m^2}$.

Then, algebraic equation (17) has a unique solution $\alpha = \left(a_0, \sqrt{2}a_1, \sqrt{2}b_1, \cdots, \sqrt{2}a_m, \sqrt{2}b_m\right)$ such that between the exact $\bar{x}(t)$ and the approximate solution $x_m(t)$ found by the Galerkin method, the estimate

$$|\bar{x}(t) - x_m(t)|_0 \le \frac{|f|_0(\sqrt{2} + |f|_1)}{m^2(1-\chi)}.$$

REFERENCES

1. Urabe M. Galerkin's method for nonlinear periodic systems. - Mechanics, 1966, 97, No. 3, p. 3-34.
2. Kondratieva A.A. Numerical-analytical methods of localization of limit cycles in mathematical models of nonlinear dynamics. Tutorial. -M.: publishing house "Kind word", 2019, 48s.
3. Alymbaev A.T., Nurzhanov O.A. Numerical-analytical method for studying autonomous systems of integro-differential equations // Ukr.mat.zhurn., 1979, v.31, No. 5, -p.540-547.
4. Alymbaev A.T. Numerical, numerical-analytical and asymptotic methods for studying boundary value problems. - Bishkek: KNU Publishing House, 2015, - 205 p.

MSC 35C20, 35K05

SOME A PRIORI ESTIMATES FOR A SYSTEM OF QUASI-LINEAR PARABOLIC EQUATIONS

Turkmanov J.K. [1], **Agybaev A.S.** [2], **Karynbaeva M.M.** [3]

[1,3] *Bishkek state university named after K. Karasaev*

[2] *Kyrgyz State Technical University named after I. Razzakov*

In this article we deduce some integral and uniform estimates for Sdutions of the Cauchy problem associated with a quasi-linear system of singularly perturbed equations parabolic type. In particular, these estimates characterize the behavior of a solution and it's derivatives as a small parameter tends to zero and the time derivative increases infinitely. They are proved to be useful in investigating the properties of solutions of model problems of gas dynamics.

Key words: Quasi-Linear, parabolic equation, degenerate problem, solution, several lines, asymptotic expansion, function, continuos derivatives, point generally speaking, standard algorithms, hogt's, estimetes, constant.

Бул макалада параболалык типтеги теңдемелердин квази сызыктуу тутумуна байланыштуу кенейтилген Коши маселесинин интегралдык жана бир чендеги баалоолорун карайбыз. Тактап айтканда, бул баалоолор чечимдин абалын жана анын туундуларын мүнөздөйт, анткени кичинекей параметр нөлгө умтулат жана убакыт боюнча туундусу чексиз көбөйөт. Газ динамикасынын моделдик маселелеринин чыгарылыштарынын касиеттерин изилдөөдө пайдалуу экени далилденет.

Урунттуу сөздөр: квази сызыктуу, параболалык теңдеме, козголгон маселе, чечим, бир нече сызык, асимптотикалык ажыроо, функция, үзгүлтүксүз туундулар, чекит, жалпысынан айтканда, стандарттуу алгоритмдер, баалоолор, турактуулар.

В этой статье выводим некоторые интегральные и равномерные оценки расширенной задачи Коши, связанной с квазилинейной системой сингулярно возмущенных уравнений параболического типа. В частности, эти оценки характеризуют поведение решения и его производных, поскольку малый параметр стремится к нулю, а производная по времени увеличивается бесконечно. Доказано, что они полезны при исследовании свойств решений модельных задач газовой динамики.
Ключевые слова: квазилинейное, параболическое уравнение, вырожденная задача, решение, несколько линий, асимптотическое разложение, функция, непрерывные производные, точка, вообще говоря, стандартные алгоритмы, оценки хогта, константа.

In the strip $\Pi_T = \{(t,x)|0<t<T, -\infty<x<\infty\}$, let us consider the Cauchy problem:

$$\varepsilon\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial t}=\frac{d}{dx}\varphi(t,x,v), \tag{1}$$

$$\varepsilon\frac{\partial^2 v}{\partial x^2}-\frac{\partial v}{\partial t}=-\frac{\partial u}{\partial x}, \tag{2}$$

$$u|_{t=0}=u_0(x),\ v|_{t=0}=v_0(x), \tag{3}$$

where ε is a non – negative constant, and $u_0(x)$ and $v_0(x)$ are continuous bounded functions possessing bounded derivatives of the first and second order. We will assume that $0<m_0\le v_0(x)$, where m_0 is a constant not depending on the parameter ε. Suppose that the function $\varphi(t,x,v)$ is continuos and bounded in each of the domains $D_a=\Pi_\infty\times\{a\le v\le\infty\}$, where a is an arbitrary positive constant, and possesses in D_a the continuous uniformly bounded derivatives up to the fourth order, inclusive, with respect to either variable, $\varphi_v^{'}(t,x,v)<0$ and $\varphi_{vv}^{''}(t,x,v)\ge 0$, and the function $F(t,x,v)=-\int_{m_0}^{v}\sqrt{-\varphi_s^{'}(t,x,s)}ds$ increases infinitely as $v\to+0$.

For $\varphi(t,x,v)\equiv(2v)^{-2}$, $\varepsilon=0$ the problem (1)-(3) describes motion of shallow water and isentropic gas motion in terms of the Lagrange coordinates in the case $\frac{C_p}{C_v}=2$.

In [1], for the case $\varphi(t,x,v)\equiv\varphi(v)$, T.D. Ventzel by means of the change of variables $f^{\pm}=-F(t,x,v)\pm u(t,x)$ has shown that in the strip Π_T the inequalities $|u(t,x)|\le M, v(t,x)\ge m>0$ $|u(t,x)|\le M, v(t,x)\ge m>0$ hold, where the constants M, m do not depend on ε. In the same work in has been proved that under the above formulated conditions for the function $u_0(x), v_0(x), \varphi(v)$ the solution of the problem exists everywhere in Π_T.

Making the change of varioles $\tau = \frac{t}{\varepsilon}$, $\xi = \frac{x}{\varepsilon}$ and denoting again the independent variobles by *t*, *x*, we obtain the problem

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u_1}{\partial t} = \frac{d\varphi(t,x,v_1)}{dx} \tag{4}$$

$$\frac{\partial^2 v_1}{\partial x^2} - \frac{\partial v_1}{\partial t} = -\frac{\partial u_1}{\partial x} \tag{5}$$

$$u_1\big|_{t=0} = \tilde{u}_1(x), \; v_1\big|_{t=0} = \tilde{v}_1(x), \tag{6}$$

where $\tilde{u}_1(x)$, $\tilde{v}_1(x)$ are given functions. In what follows, the index in notation of the solution of the problem (4)-(6) will be omuted and for the solution of the equations (4), (5) the use will be made of the conventional notation i.e., $u(t,x)$ and $v(t,x)$. The solution of the problem (4)-(6) will be assumed to exit everywhere in Π_∞ and the inequalities $v(t,x) \geq m \succ 0$, $|u(t,x)| \leq M$ for that solution to be fulfilled everywhere in Π_∞.

If $G(t,x,\xi,\tau) = [4\pi(t-\tau)]^{-\frac{1}{2}} \times \exp\{-(x-\xi)^2 / [4(t-\tau)]\}$, then the problem (4)-(6) can be written in the form

$$u(t,x) = \int_{-\infty}^{\infty} G(t,x,\xi,o) U_0(\xi) d\xi + \frac{1}{2}\int_0^1 dt \int_{-\infty}^{\infty} \varphi(\tau,\xi,v)(t-\tau)^{-1}(x-\xi) G(t,x,\xi,\tau) d\xi, \tag{7}$$

$$v(t,x) = \int_{-\infty}^{\infty} G(t,x,\xi,o) v_0(\xi) d\xi - \frac{1}{2}\int_0^1 d\tau \int_{-\infty}^{\infty} (t-\tau)^{-1}(x-\xi) G(t,x,\xi,\tau) u(\tau,\xi) d\xi. \tag{8}$$

The estimate

$$v(t,x) \leq M\sqrt{t+1} \tag{9}$$

can be immediately obtained from (8). Substituling (7), (8) and changing the order of integration, after elementary transformations we obtain

$$v(t,x)=\int_{-\infty}^{\infty}G(t,x,\xi,0)v_0 d\xi=\frac{1}{2}\int_{-\infty}^{\infty}(x-\xi)G(t,x,\xi,o)u_0(\xi)d\xi+$$
$$+\frac{1}{2}\int_0^1 d\tau\cdot\int_{-\infty}^{\infty}\varphi(\tau,\xi,v)\left[1-\frac{(x-\xi)^2}{2(t-\tau)}\right]G(t,x,\xi,\tau)d\xi=\mathrm{I}_1-\frac{1}{2}\mathrm{I}_2+\frac{1}{2}\mathrm{I}_3. \tag{10}$$

From (10) we readily obtain the estimate for the derivative of the function $v(t,x)$ with respect to the variable x:

$$\left|v_x^{'}(t,x)\right|\le M\sqrt{t+1} \tag{11}$$

Theorem 1. Let the functions $U_0(x), V_0(x)$ have the limits u^-, v^- respectively, as $x\to-\infty$ and the limits u^+, v^+ as $x\to+\infty$, and the functions $\varphi(t,-x,u^-)$, $\varphi(t,x,u^+)$ have as $x\to\pm\infty$ the limits φ^-, φ^+. Then for every fixed value $t=t_0$ the functions $u(t,x)$, $v(t,x)$ have the same limits at infinity as they have for $t=0$.

Proof. Consider first function $v(t,x)$ defined by equation (10). For the definiteness let $x\to+\infty$. Obviously,

$$\mathrm{I}_1=\int_{-\infty}^{x/2}v_0(\xi)G(t,x,\xi,o)d\xi+\int_{x/2}^{\infty}\left[v_0(\xi)-v^+\right]\cdot G(t,x,\xi,o)d\xi+v^+\int_{x/2}^{\infty}G(t,x,\xi,o)d\xi=\mathrm{I}_{1,1}+\mathrm{I}_{1,2}+\mathrm{I}_{1,3}.$$

For $x\to+\infty$, we shall estimate each summand on the right-hand side of the last equation. For values x such that $\frac{x}{2t}\gg 1$, we have,

$$\left|\mathrm{I}_{1,1}\right|\le M\cdot\int_{-\infty}^{x/2}G(t,x,\xi,o)d\xi\le M\cdot\int_{x/(4\sqrt{t})}^{\infty}\exp(-z^2)dz.$$

Using Millse's relation [2], we obtain the inequality

$$\left|\mathrm{I}_{1,1}\right|\le M\exp\left[-x^2/(4t^2)\right]\left[x/(4\sqrt{t})+\sqrt{\omega+x^2(16-t)}\right]^{-1},$$

where $4/\pi\le\omega\le 2$. It is not difficult to see that $\mathrm{I}_{1,3}\to v^+$ as $x\to\infty$, where

$$\left|\mathrm{I}_{1,3}-v^+\right|=\left|v^+\right|\exp\left[-x^2/(4t^2)\right]\left\{\sqrt{\pi}\left[x/(4\sqrt{t})+\sqrt{\omega+x^2/(16t)}\right]\right\}^{-1}.$$

For the integral $\mathrm{I}_{1,2}$ we can easily obtain the inequality $\left|\mathrm{I}_{1,2}\right|\le\sup_{\xi\ge x/2}\left|v_0(\xi)-v^+\right|$.

Further,

$$\mathrm{I}_2=\int_{-\infty}^{x/2}G(t,x,\xi,o)(x-\xi)\left[u_0(\xi)-u^+\right]d\xi+\int_{x/2}^{\infty}G(t,x,\xi,o)(x-\xi)\left[u_0(\xi)-u^+\right]d\xi+$$

$$+\sqrt{t\pi}(u^- - u^+)\exp\left[-x^2/(16t)\right] = \mathrm{I}_{2,1} + \mathrm{I}_{2,2} + \mathrm{I}_{2,3}.$$

For the integral $\mathrm{I}_{2,1}$ the estimate $\left|\mathrm{I}_{2,1}\right| \le M\sqrt{t}\exp\left[-x^2/(16t)\right]$ is valid. Passing to the estimation of the summand $\mathrm{I}_{2,2}$ get

$$\mathrm{I}_{2,2} = 2\sqrt{t/\pi}\int_{-x/(4\sqrt{t})}^{-x/(8\sqrt{t})}\left[u_0(x+2z\sqrt{t}) - u^+\right]z\cdot\exp(-z^2)dz -$$

$$-2\sqrt{t/\pi}\int_{-x/(8\sqrt{t})}^{\infty}\left[u_0(x+2z\sqrt{t}) - u^+\right]z\cdot\exp(-z^2)dz - x/(8\sqrt{t}) -$$

$$-\int_{-x/(8\sqrt{t})}^{\infty}\left[u_0(x+2z\sqrt{t}) - u^+\right]z\cdot\exp(-z^2)dz .$$

Obviously, for $x/(4\sqrt{t}) \ge 1$ the fallowing inequalities hold:

$$\left|\int_{-x/(4\sqrt{t})}^{-x/(8\sqrt{t})}\left[u_0(x+2z\sqrt{t}) - u^+\right]z\cdot\exp(-z^2)dz \le M\exp\left(-\frac{x^2}{64t}\right)\right| ,$$

$$\left|\int_{-x/(8\sqrt{t})}^{\infty}\left[u_0(x+2z\sqrt{t}) - u^+\right]z\cdot\exp(-z^2)dz \le M\sup_{\xi\ge\frac{3x}{4}}|u_0(\xi) - u^+|\right|.$$

Finally we pass to the limit I_3. Partition it in three summands

$$\mathrm{I}_3 = \int_0^t d\tau\int_{-x}^{x-\alpha(x)}\varphi(\tau,\xi,v)\left[1 - \frac{(x-\xi)^2}{2(t-\tau)}\right]G(t,x,\xi,\tau)d\xi + \int_0^t d\tau\int_{x+\alpha(x)}^{\infty}\varphi(\tau,\xi,v)\left[1 - \frac{(x-\xi)^2}{2(t-\tau)}\right]G(t,x,\xi,\tau)d\xi +$$

$$+\int_0^t d\tau\int_{x-\alpha(x)}^{x+\alpha(x)}\varphi(\tau,\xi,v)\left[1 - \frac{(x-\xi)^2}{2(t-\tau)}\right]G(t,x,\xi,\tau)d\xi = \mathrm{I}_{3,1} + \mathrm{I}_{3,2} + \mathrm{I}_{3,3},$$

where $\alpha(x)$ a monotonically increasing function, $\alpha(x) \prec \frac{x}{2}$ for $x \succ 0$ and $\lim_{x\to\infty}\alpha(x) = \infty$.

By siple calculations, for $\frac{x}{\sqrt{t}} \gg 1$ we obtain the estimate

$$\left|\mathrm{I}_{3,1}\right| \le 4\cdot M\alpha(x)\sqrt{t}\exp\left\{-\frac{\left[\alpha(x)\right]^2}{16t}\right\}\frac{\sqrt{16t\omega + \left[\alpha(x)\right]^2} - \alpha(x)}{\sqrt{16t\omega + \left[\alpha(x)\right]^2} + \alpha(x)},\ \frac{4}{\pi} \le \omega \le 2.$$

Hence $\mathrm{I}_{3,1} \le M\cdot t^{\frac{3}{2}}\left[\alpha(x)\right]^{-1}\exp\left\{-\left[\alpha(x)\right]^2/(16t)\right\}$.

An analogous estimate can be obtained for the integral $\mathrm{I}_{3,2}$. Thus, using the above-obtained estimates, we can write equality (10) as follws:

$$z(t,x)=v(t,x)-v^{+}=g(t,x)+2^{-1}\int_0^t d\tau\int_{x-\alpha(x)}^{x+\alpha(x)}\left[\varphi(\tau,\xi,v)-\varphi(t,\xi,v^{+})\right]\left[1-\frac{(x-\xi)^2}{2(t-\tau)}\right]G(t,x,\xi,\tau)d\xi \quad (12)$$

here $g(t,x)$ is the function, tending to zero as $x\to+\infty$. on considering equality (12) as the integral equation with regard to the function $z(t,x)$ we solveit by the method of successive approximations; hot that as the initial approximation $z_0(t,x)$ we shall take function $g(t,x)$.

Let inequality $x\succ 2N$ hold, where the number N is chosen in such a way that for $x\geq N$ the relation $|g(t,x)|\leq\mu$ is fulfilled. Supposing $x-\alpha(x)\geq N$, we shall have

$$|z_1(t,x)-z_0(t,x)|\leq 2^{-1}\int_0^1 d\tau\int_{x-\alpha(x)}^{x+\alpha(x)}\int_0^1\left|\frac{\partial\varphi(\tau,\xi,v^{+}+\Theta z_0(t,\xi)}{\partial v}\right|dx\times$$

$$\times\left|v_0(\tau,\xi)-v^{+}\Big/1-\frac{(x-\xi)^2}{2(t-\tau)}\right|G(t,x,\xi,\tau)d\xi e M_v t\frac{\mu}{2},\ M_v=\sup_{(t,x)\in\Pi_t,m\leq v\leq M\sqrt{t+1}}\left|\varphi_v^{'}(t,x,v)\right|,$$

and this inequality is valid for all $x\geq N$. The inequality $|v_2(t,x)-v_1(t,x)|\leq\left(M_v\frac{t}{2}\right)^2\cdot\mu$ can be obtained analogously. If $\left(M_v\frac{t}{2}\right)\prec 1$, then the sequence $\{z_n(t,x)\}$ converges, and the terms of that sequence are uniformly bounded by some constant like $M\cdot\mu$. Hence, for the function $v(t,x)$ for $t\prec M_v^{-1}$ and $x\geq 2N$ the relation $\left|v(t,x)-v^{+1}\right|\leq M\cdot\mu$ is fulfilled. Repeating our reasoning successively for $k\cdot M_v^{-1}\leq t\leq(k+1)\cdot M_v^{-1}$, we shall get the validity of the assertion of the theorem with respect to the function $v(t,x)$ for all $t\succ 0$.

Estimate now the difference $u(t,x)-u^{+}$. It can be easily seen that the first summand in the expression (7) is investigated exactly in the same manner as the first one in the expression (10). Let us consider the second summand for $x\geq 2N$:

$$2^{-1}\int_0^1 d\tau\int_{-\infty}^{\infty}\varphi(\tau,\xi,v)(t-\tau)^{-1}(x-\xi)G(t,x,\xi,\tau)d\xi=-\text{I}_1-\tilde{\text{I}}_2=-\int_0^t d\tau\int_{-\infty}^{-\alpha(x)/2\sqrt{t-\tau}}\varphi(\tau,x+2z\sqrt{t-\tau},v)\times$$

$$\times(t-\tau)^{1/2}zG(z,4^{-1},0,0)dz-\int_0^t d\tau\int_{-\alpha(x)/2\sqrt{t-\tau}}^{\infty}\varphi(\tau,x,+2z\sqrt{t-\tau},v)(t-\tau)^{-1/2}zG(z,4^{-1},0,0)dz.$$

It is not difficult to see that for sufficiently large values of N the inequality

$$\left|\tilde{I}_1\right| \le \int_0^t (t-\tau)^{-1/2} \exp\left[-\frac{x^2}{16(t-\tau)}\right] d\tau \le M\sqrt{t}\exp\left[-\frac{x^2}{16t}\right]$$

is valid. The integral $\tilde{I}_2$ in the following way:

$$\int_0^1 (t-\tau)^{-1/2} d\tau \int_{-\alpha(x)/(2\sqrt{t})}^{\infty} \left[\varphi(\tau, x+2z\sqrt{t-\tau}, v^+) - \varphi^+\right] \cdot zG(z, 4^{-1}, 0, 0)dz + \varphi^+ \int_0^t G(x, t, x-\alpha(x), \tau)d\tau +$$

$$+\int_0^t (t-\tau)^{-1/2} d\tau \int_{-\alpha(x)/(2\sqrt{t})}^{\infty} \left[\varphi(\tau, x+2z\sqrt{t-\tau}, v) - \varphi(\tau, x+2z\sqrt{t-\tau}, v^+)\right] \cdot zG(z, 4^{-1}, 0, 0)dz =$$

$$= \tilde{I}_{2,1} + \tilde{I}_{2,2} + \tilde{I}_{2,3}.$$

Using the estimates for the function $v(t,x) - v^+$, we obtain for $x \ge 2N$:

$$\left|\tilde{I}_{2,1}\right| \le \sup_{t \ge N, 0 \le \tau \le t} \left|\varphi(\tau, \xi, v^+) - \varphi^+\right|, \ \left|\tilde{I}_{2,2}\right| \le M\sqrt{t}\exp\left\{\frac{-[\alpha(x)]^2}{4t}\right\}, \ \left|\tilde{I}_{2,3}\right| \le \sup_{\xi \ge N, 0 \le \tau \le t} \left|\varphi(t,\xi) - v^+\right|.$$

From these inequalities follows the relation $\lim_{x\to\infty}\left|u(t,x) - u^+\right| = 0$. Obviously, the case $x \to -\infty$ is considered analogously.

Remark 1. From the above estimates follow the estimates for the rate of convergence of the functions $u(t,x)$, $v(t,x)$ to the corresponding limiting values as $|x| \to \infty$. This rate depends on t and on the rate of convergence of the functions $u_0(t,x)$, $v_0(t,x)$, $\varphi(t,x,v^-)$, $\varphi(t,x,v^+)$ to their limiting values.

Remark 2. It follows from the above reasoning that the behavior of the function $v(t,x)$ as $x \to \infty$ does not depend on the character of variation of the function $u_0(x)$ as $|x| \to \infty$ and of the function $v_0(x)$ as $x \to -\infty$.

The proof of the theorem below is the same as that of theorem 1.

Theorem 2. If $\lim_{|x|\to\infty}\left(\left|u_0'(x)\right| + \left|v_0'(x)\right|\right) = 0$, $\lim_{|x|\to\infty}\left(\left|\varphi'(t,-x,v^-\right| + \left|\varphi'(t,x,v^+)\right|\right) = 0$, then $\lim_{|x|\to\infty}\left(\left|u_x'(t,x)\right| + \left|v_x'(t,x)\right|\right) = 0$.

Theorem 3. If the conditions of Theorem 1 and 2 are fulfilled and, moreover, if the integrals:

$$I_1(t,x) = \int_{-\infty}^{x} \left[v(t,\xi) - v^-\right] d\xi, \ I_2(t,x) = \int_x^{\infty} \left[v(t,\xi) - v^+\right] d\xi, \ I_3(t,x) = \int_{-\infty}^{x} \left[u(t,\xi) - u^-\right] d\xi,$$

$I_4(t,x)=\int_x^{\infty}\left[u(t,\xi)-u^{+}\right]d\xi$ exist for $t=0$, then they do exist for any $t\succ 0$ and the following equalities hold:

$$\begin{cases} I_1(t,0)+I_2(t,0)=I_1(0,0)+I_2(0,0)+(u^{+}-u^{-})t, \\ I_3(t,0)+I_4(t,0)=I_3(0,0)+I_4(0,0)+(\varphi^{+}-\varphi^{-})t \end{cases}. \quad (13)$$

Theorem 4 is proved in the same way as Lemma 4 in [3].

Corollary 1. If $v^{-}\neq v^{+}$, then

$$I_{1,a}(t,0)+I_{2,a}(t,0)=\int_{-\infty}^{0}\left[v(t,x-at)-v^{-}\right]dx+\int_{0}^{\infty}\left[v(t,x-at)-v^{+}\right]dx=I_{1,a}(0,0)+I_{2,a}(0,0),$$

where $a=(u^{+}-u^{-})(v^{+}-v^{-})^{-1}$; if $u^{-}\neq u^{+}$, then

$$I_{3,b}(t,0)+I_{4,b}(t,0)=\int_{-\infty}^{0}\left[u(t,x-bt)-u^{-}\right]dx+\int_{0}^{\infty}\left[u(t,x-bt)-u^{+}\right]dx=I_{3,b}(0,0)+I_{4,b}(0,0),$$

where $b=(\varphi^{-}-\varphi^{+})(u^{+}-u^{-})^{-1}$.

Theorem 4. Let the function $\varphi(t,x,v^{-})-\varphi^{-}$ be absolutely integrable with respect to the variable x for $x\in(-\infty;0)$, and the function $\varphi(t,x,v^{+})-\varphi^{+}$ be absolutely integrable for $x\in[0,\infty)$. Let, moreover, the integrals

$$I_1(t)=\int_{-\infty}^{0}\left(v(t,x)-v^{-}\right)dx,\ I_2(t)=\int_{-\infty}^{0}\left(v(t,x)-v^{+}\right)dx,\ I_3(t)=\int_{-\infty}^{0}\left(u(t,x)-u^{-}\right)dx,$$

$$I_4(t)=\int_{-\infty}^{0}\left(u(t,x)-u^{+}\right)dx.$$

Converge for $t=0$. Then these integrals converge for any $t\succ 0$ and the inequalities hold

$$I_1(t)+I_2(t)\leq Me^{t}(1+t^{3/2}),\ I_3(t)+I_4(t)\leq Me^{t}(1+t^{3/2}) \quad (14)$$

Proof. Consider first the integral $I_1(t)$. Denote the function $\left[1-2^{-1}(x-\xi)^2/(t-\tau)\right]G(t,x,\xi,\tau)$ by $Q(t,x,\xi,\tau)$. We have

$$v(t,x)-v^{-}=\int_{-\infty}^{\infty}\left[v_0(\xi)-v^{-}\right]G(t,x,\xi,o)d\xi-2^{-1}\int_{-\infty}^{\infty}u_0(\xi)(x-\xi)G(t,x,\xi,0)d\xi+2^{-1}\int_0^t d\tau\int_{-\infty}^{\infty}\varphi(\tau,x,\xi,v)\times$$
$$\times Q(t,x,\xi,\tau)d\xi=P_1-2^{-1}\cdot P_2+2^{-1}\cdot P_3.$$

Obviosly,

$$\mathrm{P}_1 = \int_{-\infty}^{0} \left[v_0(\xi) - v^- \right] G(t,x,\xi,o) d\xi + \int_{0}^{\infty} \left[v_0(\xi) - v^- \right] G(t,x,\xi,o) d\xi = \mathrm{P}_{1,1}(t,x) + \mathrm{P}_{1,2}(t,x),$$

whence

$$\int_{-\infty}^{0} \left| \mathrm{P}_{1,1}(t,x) \right| dx \le \int_{-\infty}^{0} \left| v_0(\xi) - v^- \right| \left\{ \int_{-\infty}^{0} G(t,x,\xi,0) dx \right\} d\xi \le \mathrm{I}_1(0),$$

$$\int_{-\infty}^{0} \left| \mathrm{P}_{1,2}(t,x) \right| dx \le \int_{0}^{\infty} \left\{ \int_{-\infty}^{0} G(t,x,\xi,0) dx \right\} d\xi \le M \cdot \sqrt{t}.$$

Pass now to the estimate of the integral P_2. Let us write it in the form

$$\int_{-\infty}^{0} \left[u_0(\xi) - u^- \right] (x-\xi) G(t,x,\xi,o) d\xi + \int_{0}^{\infty} \left[u_0(\xi) - u^+ \right] (x-\xi) G(t,x,\xi,o) d\xi + (u^+ - u^-) \sqrt{\frac{t}{\pi}} \exp\left[-\frac{x^2}{4t} \right] =$$

$= \mathrm{P}_{2,1}(t,x) + \mathrm{P}_{2,2}(t,x) + \mathrm{P}_{2,3}(t,x)$ and hence

$$\int_{-\infty}^{0} \left| \mathrm{P}_{2,1}(t,x) \right| dx \le M \sqrt{t} \mathrm{I}_3(0), \quad \int_{-\infty}^{0} \left| \mathrm{P}_{2,2}(t,x) \right| dx \le M \sqrt{t} \mathrm{I}_4(0), \quad \int_{-\infty}^{0} \left| \mathrm{P}_{2,3}(t,x) \right| dx = \left| u^+ - u^- \right| \cdot t.$$

Finally, let us estimate the integral $\int_{-\infty}^{0} |\mathrm{P}_3(t,x)| dx$. We represent the function $\mathrm{P}_3(t,x)$ as

$$\mathrm{P}_3(t,x) = \int_0^t d\tau \int_{-\infty}^{0} \left[\varphi(\tau,\xi,v^-) \right] Q(t,x,\xi,\tau) d\xi + \int_0^t d\tau \int_0^{\infty} \left[\varphi(\tau,\xi,v) - \varphi(t,\xi,v^+) \right] Q(t,x,\xi,\tau) d\xi +$$

$$+ \left\{ \int_0^t d\tau \int_{-\infty}^{0} \left[\varphi(\tau,\xi,v^-) \right] Q(t,x,\xi,\tau) d\xi + \int_0^t d\tau \int_0^{\infty} \left[\varphi(\tau,\xi,v^+) \right] Q(t,x,\xi,\tau) d\xi \right\} =$$

$$= \mathrm{P}_{3,1}(t,x) + \mathrm{P}_{3,2}(t,x) + \mathrm{P}_{3,3}(t,x) + \mathrm{P}_{3,4}(t,x).$$

For the function $\mathrm{P}_{3,1}(t,x)$ we shall have

$$\int_{-\infty}^{0} \left| \mathrm{P}_{3,1} \right| dx \le \int_0^t d\tau \int_{-\infty}^{0} \left| \varphi(\tau,\xi,v) - \varphi(\tau,\xi,v^-) \right| \int_{\xi-\sqrt{2(t-\tau)}}^{-\infty} Q(t,x,\xi,\tau) dx d\xi + \int_0^t d\tau \int_{-\infty}^{0} \left| \varphi(\tau,\xi,v) - \varphi(\tau,\xi,v^-) \right| \times$$

$$\times \int_0^{\xi+\sqrt{2(t-\tau)}} Q(t,x,\xi,\tau) dx d\xi + \int_0^t d\tau \int_{-\infty}^{0} \left| \varphi(t,\xi,v) - \varphi(t,\xi,v^-) \right| \int_{\xi-\sqrt{2(t-\tau)}}^{\xi+\sqrt{2(t-\tau)}} Q(t,x,\xi,\tau) dx d\xi = \mathrm{P}_{3,1,1} + \mathrm{P}_{3,1,2} + \mathrm{P}_{3,1,3}$$

Calculating the integral of the function $Q(t,x,\xi,\tau)$, we obtain

$$\mathrm{P}_{3,1,1} \le M \int_0^t d\tau \int_{-\infty}^{0} \left| v(r,x) - v^- \right| dx, \quad \mathrm{P}_{3,1,2} \le M \int_0^t d\tau \int_{-\infty}^{0} \left| v(t,\xi) - v^- \right| d\xi.$$

For the integral $\mathrm{P}_{3,1,3}$ we can easily get the inequality

$$\mathrm{P}_{3,1,3} = \left\{ \frac{2}{2\sqrt{\pi}} \int_{-1/2}^{1/2} e^{-s^2} ds + \frac{1}{\sqrt{2\pi}} e^{-1/2} \right\} \int_0^t d\tau \int_{-\infty}^{0} \left| \varphi(\tau,\xi,v) - \varphi(t,\xi,v^-) \right| d\xi \le M \int_0^t d\tau \int_{-\infty}^{0} \left| v(\tau,x) - v^- \right| d\xi.$$

The integral of the function $\mathrm{P}_{3,2}(t,x)$ is estimated analogously:

$$\int_{-\infty}^{0}\left|\mathrm{P}_{3,2}(t,x)\right|dx \le M\int_0^t d\tau\int_0^\infty \left|v(t,x)-v^{+}\right|dx .$$

Let us now pass to the integral $\mathrm{P}_{3,3}$:

$$\mathrm{P}_{3,3}=\int_0^t d\tau\int_{-\infty}^{0}\left[\varphi(t,\xi,v^{-})-\varphi^{-}\right]Q(t,x,\xi,\tau)d\xi+\varphi^{-}\int_0^t d\tau\int_0^\infty Q(t,x,\xi,\tau)d\xi=\mathrm{P}_{3,3,1}(t,x)+\mathrm{P}_{3,3,2}(t,x) .$$

Obviously,

$$\int_{-\infty}^{0}\left|\mathrm{P}_{3,3,1}(t,x)\right|dx \le M\int_0^t d\tau\int_0^\infty\left|\varphi(t,x,v^{-})-\varphi^{-}\right|dx , \qquad \int_{-\infty}^{0}\left|\mathrm{P}_{3,3,2}(t,x)\right|dx=\frac{2}{3\sqrt{\pi}}t^{3/2}\varphi^{-} .$$

We can easily see that the integral of the function $\mathrm{P}_{3,4}$ is calculated exactly in the same manner.

Thus

$$\int_{-\infty}^{0}\left|v(t,x)-v^{-}\right|dx \le M(1+\sqrt{t})+M\left\{\int_0^t d\tau\int_{-\infty}^{0}\left|\varphi(\tau,\xi,v^{-})-\varphi^{-}\right|d\xi+\int_0^t d\tau\int_0^\infty\left|\varphi(t,\xi,v^{+})-\varphi^{+}\right|d\xi\right\}+$$

$$+M\int_0^t d\tau\int_{-\infty}^{0}\left|v(t,x)-v^{-}\right|dx+\left|u^{+}-u^{-}\right|\cdot t+(9\pi)^{-\frac{1}{2}}\left|\varphi^{+}-\varphi^{-1}\right|\cdot t^{3/2} .$$

Similarly,

$$\int_{-\infty}^{0}\left|v(t,x)-v^{+}\right|dx \le M(1+\sqrt{t})+M\left\{\int_0^t d\tau\int_{-\infty}^{0}\left|\varphi(\tau,\xi,v^{-})-\varphi^{-}\right|d\xi+\int_0^t d\tau\int_0^\infty\left|\varphi(t,\xi,v^{+})-\varphi^{+}\right|d\xi\right\}+$$

$$+M\int_0^t d\tau\int_0^\infty\left|v(t,x)-v^{+}\right|dx+\left|u^{+}-u^{-}\right|\cdot t+(9\pi)^{-\frac{1}{2}}\left|\varphi^{+}-\varphi^{-}\right|\cdot t^{3/2} .$$

Adding term by term the last two inequalities and using the Gronwall Bellmans lemma, we obtain the first inequality (14). The second one is proved in a similar manner.

The following assertion is proved without any changes.

Theorem 5. If $v_0(x)-v^{-}$, $u_0(x)-u^{-}$, $\varphi(t,x,v)-\varphi^{-}\in L_{p,x}(-\infty;0)$, $v_0(x)-v^{+}$, $u_0(x)-u^{+}$, $\varphi(t,x,v^{+})-\varphi^{+}\in L_{p,x}(0;+\infty)$, $p\ge 1$, then $v(t,x)-v^{-}$, $u(t,x)-u^{-}\in L_{p,x}(-\infty;0)$, $v(t,x)-v^{+}$, $u(t,x)-u^{+}\in L_{p,x}(0;\infty;)$

Under appropriate assumptions on the initial data of the problem we can formulate analogous statements for the derivatives of the functions $u(t,x)$, $v(t,x)$.

Let us pass now to the estimates of the functions under consideration in the uniform norm.

Theorem 6. Everywhere in the half-plane $t \succ 0$ the estimates

$$\left|u'_x(t,x)\right| + \left|u'_t(t,x)\right| + \left|v'_x(t,x)\right| + \left|v'_t(t,x)\right| + \left|u''_{xx}(t,x)\right| \le M \ln(e+t),$$

$$\left|v''_{xx}(t,x)\right| \le M\sqrt{\ln(e+t)}$$

are valid.

Proof. As follows from equality (8),

$$\frac{\partial^2 v}{\partial x^2} = -(2t)^{-1}\int_{-\infty}^{\infty} v_0(\xi)Q(t,x,\xi,o)d\xi + (4t)^{-1}\int_{-\infty}^{\infty} u_0(\xi)\left[3(x-\xi) - t^{-1}(x-\xi)^3\right]G(t,x,\xi,o)d\xi -$$

$$-4^{-1}\int_0^t d\tau \int_{-\infty}^{\infty} (t-\tau)^{-1}\varphi(t,\xi,v)\left[3 - 3(t-\tau)^{-1}(x-\xi)^2 + +4^{-1}(t-\tau)^{-2}(x-\xi)^4\right]G(t,x,\xi,\tau)d\xi =$$

$$= -2^{-1}(K_1 - K_2 + K_3).$$

In proving the theorem we may assume $t \gg 1$. Evidently, $|K_1| \le M\cdot t^{-1}$, $|K_2| \le M\cdot t^{-\frac{1}{2}}$, and hence to estimate the second derivative of the function $v(t,x)$ with respect to the variable x it sufficiently to estimate the integral K_3:

$$\frac{1}{2}\int_0^t d\tau \int_{-\infty}^{\infty} \tau^{-1}\varphi(t-\tau, x+\xi), v(t-\tau, x+\xi)\left[3 - 3\tau^{-1}\xi^2 + 4^{-1}\tau^{-2}\xi^4\right]G(\tau,\xi,0,0)d\xi =$$

$$= 2^{-1}\int_0^{\delta} d\tau \int_{-\infty}^{\infty} \{...\}d\xi + 2^{-1}\int_{\delta}^{t} d\tau \int_{-\infty}^{\infty} \{...\}d\xi = K_{3,1} + K_{3,2}$$

where δ is some positive value will be defined below. Since $\left|v'_x(t,x)\right| \le M\sqrt{t+1}$, we can easily obtain the estimate $\left|K_{3,1}\right| \le M\sqrt{\delta(t+1)}$. In the integral $K_{3,2}$, maxing the change of the variable

$\xi = 2z\sqrt{\tau}$, we get $\left|K_{3,2}\right| \le M\left|\ln t - \ln\delta\right|$.

Choosing $\delta = t^{-1}$, we obtain the intermediate estimate

$$\left|v''_{xx}(t,x)\right| \le M \ln(e+t).$$

The estimates

$$\left|u'_x(t,x)\right| \le M\ln(e+t), \quad \left|v'_x(t,x)\right| \le M\ln(e+t)$$

can be found analogously formulas (7) and (8).

Estimate now the function $u''_{xx}(t,x)$. From (7) we have

$$u''_{xx}(t,x) = L_1(t,x) + L_2(t,x) = -(2t)^{-1}\int_{-\infty}^{\infty} u_0(\xi)Q(t,x,\xi)d\xi + 4^{-1}\int_0^t d\tau \int_{-\infty}^{\infty} \varphi(\tau,\xi,v)(t-\tau)^{-2} \times$$

$$\times\left[2^{-1}(t-\tau)^{-1}(x-\xi)^3 - 3(x-\xi)\right]G(t,x,\xi,\tau)d\xi\ .$$

Obviously, $|L_1(t,x)| \le M \cdot t^{-1}$. The integral $L_2(t,x)$ can be represented in terms of

$$L_2(t,x) = \int_0^{t-\delta} d\tau \int_{-\infty}^{\infty} \{...\} d\xi + \int_{t-\delta}^{t} d\tau \int_{-\infty}^{\infty} \{...\} d\xi = L_{2,1}(t,x) + L_{2,2}(t,x)\,.$$

Integrating once by parts and using the obtained estimates for the derivatives of the function $v(t,x)$, we readily obtain the inequality $|L_{2,2}(t,x)| \le M\sqrt{\delta}\left\{1 + [\ln(t+1)]^2\right\}$. It is easily seen that the inequality $|L_{2,1}(t,x)| \le M\delta^{-\frac{1}{2}}$ holds. Choosing δ from the equality $\delta = [\ln(t+e)]^2$, we find the estimate $|u''_{xx}(t,x)| \le M\ln(t+e)$. Estimates for the derivatives of the functions under consideration with respect to the variable t follows from equations (4) and (5). Getting back to the estimate of the function $v''_{xx}(t,x)$, we are able, with regard for inequality (14), to find from (10) the required estimate for that derivative.

In conclusion, we can formulate an analogue of theorem 6 in the form applicable to the problem (1)-(3).

Theorem 7. Everywhere in the half-plane $t \succ 0$, for the solution of the problem (1)-(3) the estimates

$$|\varepsilon u'_x(t,x)| + |\varepsilon u'_t(t,x)| + |\varepsilon v'_x(t,x)| + |\varepsilon v'_t(t,x)| + |\varepsilon^2 u''_{xx}(t,x)| \le M\ln(e + \frac{t}{\varepsilon}),$$

$$|\varepsilon^2 u''_{xx}(t,x)| \le M\sqrt{\ln(e + \frac{t}{\varepsilon})}$$

are valid.

It should (1) be noted that the results of the above theorem improve the results obtained in [1] for the cases $\varepsilon \to 0$ and $t \to 0$.

REFERENCES

1. Amman H., Exitence and multiplicity theorems for semi-linear boundary value problems. Mat. Zametki (1976) No.150.281-295.(in Russian)
2. Dzhuraev T.D., Boundary value problems for equations of mixed type. Fan, Tashkent, 1973 (in Russian)
3. Moiseev N.N., Asymtotic methods in nonlinear mechanics. Nauka, Moskow,1969.(in Russian)
4. Sushko V.G., Asymptotic form with respect to a small parameter for the solutions of a differential equation with discontinuous coefficients.Vestn. Mosk. Nauk 38(1983),No.5,166 (in Russian)
5. Oleinik O.A. and Kruzhkov S.N., Quasilinear parabolic equations of second order with several independent variables. Uspekin Mat/Nauk 16(1961), №5, 115-155.(in Russian)
6. Mitrinovic D.S. and Vasic P.V., Analiticke nejednakosti. Gradevinska knjiga, Belgrad, 1970.
7. Turkmanov J.K. A priori estimates of solutions of the Cauchy problem for a quasi-linear parabolic eqution // Herald of Institute of Mathematics of NAS KR, Bishkek, 2021, №1. –Pp. 105-114.

MSC 45D05

ALGORITHM TO INVESTIGATE LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS WITH PROPORTIONAL RETARDING OF ARGUMENT

V.T. Muratalieva
Institute of Mathematics of NAS of KR

Supra the author constructed and implemented the following algorithms on a computer. Given a Volterra integro-differential equation with power coefficients by integral summands; Volterra integral equations with proportional retarding of argument, the algorithm presents data to detect existence of solutions and occurrence of arbitrary constants in it. In this paper such items are considered for a Volterra integro-differential equation integral equation with proportional retarding of argument.

Keywords: integro-differential equation, unbounded domain, Volterra equation, algorithm, analytical function

Мурда, автор төмөнкү алгоритмдерди түзүп жана компьютерде жүзөгө ашырган. Даражалуу көбөйтүндүлүү интегралдык кошулуучулары бар вольтерралык интегро-дифференциалдык теңдеме; аргументинин кечигүүсү пропорциялуу болгон интегралдык теңдеме берилген. Алгоритм чыгарылышынын жашоосун аныктоо жолуна мүмкүндүгүн жана анда каалагандай турактуу сан бар экендигин аныктоо үчүн малыматты берет. Бул макалада бул сыяктуу маселелер аргументинин кечигүүсү пропорциялуу болгон интегро-дифференциалдык теңдеме үчүн каралат.
Урунттуу сөздөр: интегро-дифференциалдык теңдеме, чектелбеген аймак, Вольтерра тибиндеги теңдеме, алгоритм, аналитикалык функция

Ранее автор построила и реализовала на компьютере следующие алгоритмы. Даны вольтерровское интегро-дифференциальное уравнение со степенными сомножителями при интегральных слагаемых; интегральное уравнение с пропорционально запаздывающим аргументом. Алгоритм представляет данные для определения существования решения и наличия в нем произвольных постоянных. В данной статье такие вопросы рассматриваются для интегро-дифференциального уравнения с пропорционально запаздывающим аргументом.
Ключевые слова: интегро-дифференциальное уравнение, неограниченная область, уравнение типа Вольтерра, алгоритм, аналитическая функция.

Introduction

Algorithmization of some problems in various branches of mathematics is one of main directions of modern investigations.

Before our publications [1-5], we did not find algorithms on conditions of existence of solutions of Volterra equations with analytical functions. Supra we constructed and implemented the following algorithms on a computer. Given a linear equation with power coefficients by integral summands, the algorithm presents data to detect existence of solutions and occurrence of arbitrary constants in it. In [6] we consider integral equations with proportional retarding of argument.

In this paper such items are considered for an integro-differential equation with proportional retarding of argument. We will use denotations $\boldsymbol{R} := (-\infty;\infty)$; $\boldsymbol{R}_+ := [0;\infty)$; $\boldsymbol{R}_{++} := (0;\infty)$; $\boldsymbol{N}_0 := \{0, 1, 2, 3, ...\}$; $\boldsymbol{N} := \{1, 2, 3, ...\}$.

Remark. We use the term ”Algorithm” as it is usually understood in Analysis: arithmetical operations and comparison over numbers in $\boldsymbol{R}$ (for rational numbers this definition coincides with the strict one).

We write discrete arguments in brackets to bring denotations nearer to algorithmic ones and to bypass the common ambiguity of expressions such as b_{2j}.

1. Equations considered

We will consider equations of type

$$tu'(t) + \Sigma_{a,p} bt^p \int_0^{at} u(s)ds = f(t), 0 < a \leq 1, \qquad (1)$$

$p \geq 0$, $u(t), f(t)$ are analytic functions, $f(0) = 0$.

Suppose that there are not two terms with equal p and a in (1).

We will consider real-valued analytical functions in the form

$$f(t) = f[1]t + f[2]t^2 + \ldots, \qquad (2)$$

$$u(t) = u[0] + u[1]t + u[2]t^2 + \cdots.$$

Substituting (2) in (1) we obtain

$$t(u[1] + 2u[1]t + 3[2]t^2 + \cdots) + \Sigma_{a,p} bt^p \int_0^{at} (u[0] + u[1]s + \cdots)ds =$$

$$= f[1]t + f[2]t^2 + \cdots \;;$$

$$t(u[1] + 2u[1]t + 3[2]t^2 + \cdots) +$$

$$+\Sigma_{a,p} bt^p (atu[0] + a^2t^2u[1]/2 + a^3t^3u[2]/3 + \cdots) = f[1]t + f[2]t^2 + \cdots;$$

$$u[1]t + 2u[1]t^2 + 3u[2]t^3 + \cdots +$$

$$+\Sigma_{a,p} (abu[0]t^{p+1} + a^2bu[1]/2 \cdot t^{p+2} + a^3bu[2]/3 \cdot t^{p+3} + \cdots) =$$

$$= f[1]t + f[2]t^2 + \cdots . \qquad (3)$$

Equating coefficients by $t, t^2, t^3, \ldots$ terms we obtain an infinite system (denote it as (4)) of linear equations for unknown $u[0], u[1], \ldots$.

2. Description of algorithm

Input:

2.1) The number (K) of integral operators in (1).

2.2) non-negative integer numbers p_k (in increasing order), $k=1..K$;

2.3) rational numbers $a_k \in (0,1]$ (in increasing order for equal values of p_k), $k=1..K$;

2.4) non-zero rational numbers b_k $k=1..K$;

Output:

2.5) display of the integral equation for the function $u(t) = u[0] + u[1]t + \ldots$ (by custom, the cases $b_k=-1$ and $b_k=1$ are demonstrated individually; the cases $p_k=0$ and $p_k=1$ are demonstrated individually).

Calculation:

2.6) Estimation of the maximal value (m_0) of exponents after which the total coefficients by elder unknowns will be non-zero (see Theorem 1 below);

Output:

2.7) The system (4) for exponents $1..\, m_0+1$ of *t*. (There may be "0" or "0*u[k]" in the left-hand sides of some equations).

For a human:

2.8) Investigate the system of linear algebraic equations (4). What of $u[0], u[1], \dots$ can be found under conditions on *f[0], f[1], …?*

3. Estimation

Theorem 1. There exists such value (m_0) of exponents that the total coefficients by elder unknowns are non-zero after it.

Proof. First case. *p[K]=0* (all *p[k]=0*). The relation (2) is following:

$$u[1]t + 2u[2]t^2 + 3u[3]t^3 + \cdots + \Sigma_{k=1}^{K}\left(a[k]b[k]u[0]t + \frac{a[k]^2 b[k]u[1]}{2\cdot t^2} + \cdots\right) = = f[1]t + f[2]t^2 + \cdots . \quad (5)$$

u[0] is arbitrary; t^m ($m\geq 1$): $mu[m] + \Sigma_{k=1}^{K} a[k]^m b[k]u[m-1]/m = f[m]$.

All *u[1], u[2],…* can be found.

II case. *p[K]=1* (*p[0]=…=p[L]=0*; *p[L+1]=…=p[K]=1)*.

The relation (3) is following:

$$u[1]t + 2u[2]t^2 + 3u[3]t^3 + \cdots +$$
$$+\Sigma_{k=1}^{L}\left(a[k]b[k]u[0]t + a[k]^2 b[k]u[1]/2\cdot t^2 + \cdots\right) +$$
$$+\Sigma_{k=L+1}^{K}\left(a[k]b[k]u[0]t^2 + a[k]^2 b[k]u[1]/2\cdot t^3 + \cdots\right) =$$
$$= f[1]t + f[2]t^2 + \cdots . \quad (6)$$

u[0] is arbitrary; t^m($m\geq 2$):

$$mu[m] + \Sigma_{k=1}^{L} a[k]^m b[k]u[m-1]/m +$$
$$+\Sigma_{k=L+1}^{K} a[k]^{m-1} b[k]u[m-2]/(m-1) = f[m].$$

The general case is similar to this case.

4. An example of integro-differential equation

We consider the equation

$$tu'(t) - 5\int_0^{t/2} u(s)ds = f(t). \qquad (7)$$

Here

K=1; p[1]=0; a[1]=1/2; b[1]=–12.

Display of the equation by 2.5):

t u(t) – 5 int_0^(1/2) t u(s)ds = f(t).*

Substituting:

$$u[1]t + 2u[2]t^2 + \ldots - 5\int_0^{t/2} (u[0] + u[1]s + ..)ds = f[1]t + f[2]t^2 + \ldots$$

$$u[1]t + 2u[2]t^2 + \ldots - 5(u[0]t/2 + u[1]t^2/3/4 + \ldots) = f[1]t + f[2]t^2 + ..$$

Hence, a solution exists.

5. Conclusion

We hope to construct such algorithms for various classes of integro-differential equations with proportional retardation of argument.

REFERENCES

1. Мураталиева В.Т. Спектральные свойства линейных вольтерровских интегро-дифференциальных уравнений третьего рода второго порядка // Наука вчера, сегодня, завтра: сборник статей по материалам XXXIV междунар. научно-практ. конф. № 5(27). Часть I. – Новосибирск: СибАК, 2016. – С. 57-61.
2. Панков П.С., Мураталиева В.Т. Спектральные свойства линейных задач с аналитическими функциями // Доклады Национальной академии наук Кыргызской Республики, 2016, № 1. – С.11-14.
3. Мураталиева В.Т. Алгоритм для исследования спектральных свойств линейных задач с аналитическими функциями // Вестник Жалал-Абадского государственного университета, 2016, № 1 (32). – С. 55-59.
4. Muratalieva V.T. Method and algorithm to investigate integro-differential

equations with analytical functions // Тезисы докладов Международной научной конференции «III Борубаевские чтения», посвященной 35-летию со дня образования Института математики НАН КР. - Бишкек: Институт математики, 2019. - С. 48.
5. Muratalieva V.T. Spectral properties of non-linear Volterra integral equations with analytic functions // Herald of Institute of Mathematics of NAS of KR, 2020, No. 1. - Pp. 76-82.
6. Muratalieva V.T. Algorithm to investigate linear Volterra integral equations with proportional retarding of argument // Herald of Institute of Mathematics of NAS of KR, 2021, No. 2. - Pp. 26-31.

MSC 18C05

CATEGORY OF CONTROLLED PROCESSES IN COMPUTATIONAL MATHEMATICS AND ALMOST-ISOLATED SYSTEMS

Pankov P.S.[1], Kenenbaeva G.M.[2]
[1]*Institute of Mathematics of NAS of KR*
[2]*Kyrgyz State University named after J. Balasagyn*

Supra, the notion of category of equations was introduced by the second author with assistance of the notion "predicate" on the base of the principle of preservation of solution while transformations; elements of the category of equations and its subcategories were constructed on the base of well-known categories. As a specification of the second law of thermodynamic for isolated systems, supra the first author introduced new definitions, proposed general hypotheses and derived estimations from below on increasing of entropy while motion of a material point both without friction and with friction over definite distance on depending on time in permanently unstable (affectable) systems. Such definitions are united in this paper.
Keywords: category, entropy, control, differential equation, affectable system, friction, motion.

Мурда экинчи автор, өзгөртүүлөрдө чыгарылышты сактоо принцибинин негизинде "предикат" түшүнүгүнүн жардамы менен теңдемелердин категориясынын түшүнүгүн киргизген; теңдемелердин категориясынын жана анын категориячаларынын элементтери белгилүү категориялардын негизинде курулган. Четтетилген системада термодинамиканын экинчи законун тактоо катары, биринчи автор жаңы аныктамаларды киргизген, жалпы гипотезаларды сунуштап, таасир этилүүчү системада материалдык чекитти сүрүлүүсүз жана сүрүлүүнүн негизинде кандайдыр бир аралыкка жылдырууда убакыттан көз каранды болгон энтропиянын өсүүсүнүн төмөнкү баасын алган. Бул макалада бул аныктамалар айкалыштырылган.
Урунттуу сөздөр: категория, энтропия, башкаруу, дифференциалдык теңдеме, таасир этилүүчү система, сүрүлүү, кыймылдоо.

Ранее вторым автором было введено понятие категории уравнений с помощью понятия "предикат" на основе принципа сохранения решения при преобразованиях; построены элементы категории уравнений и ее подкатегорий на основе известных категорий. Как уточнение второго закона термодинамики для изолированных систем, первый автор ввел новые определения, предложил общие гипотезы и получил оценки снизу для возрастания энтропии при передвижении материальной точки без трения и с трением на определенное расстояние в зависимости от времени в перманентно неустойчивых системах. В данной статье эти определения объединены.

Ключевые слова: категория, энтропия, управление, дифференциальное уравнение, перманентно неустойчивая система, трение, движение.

1. Introduction

Investigations in the category of topological spaces in Kyrgyzstan were initiated [1].

The notion of the category of equations was introduced [2] with assistance of the notion “predicate” on the base of the principle of preservation of solution while transformations; elements of the category of equations and its subcategories were constructed on the base of well-known categories [3], [4].

As a specification of the second law of thermodynamic for isolated systems, on the base of [5], [6], [7], in [8] a general hypothesis on estimations from below on increasing of entropy during motion (transformation) was put.

By using new definitions of almost closed systems and permanently unstable (affectable) systems in [9] some estimations from below on increasing of entropy while motion of a material point both without friction and with friction over definite distance on depending on time in such systems were obtained [10]-[12].

Such definitions and results are united in this paper.

Remark. Our as well as other authors’ “Definitions” below are not strongly mathematical because they mean real objects and processes too.

Section 2 contains necessary definitions.

Section 3 presents definition of the category of energy-entropy-optimization processes.

2. Definitions

2.1. The basic category is the category of sets *Set*. *Ob(Set)* are sets; *Mor(Set)* are functions.

2.2. Category of functions *Func* (synonyms in various branches of mathematics: maps, operators, transformations). It is mentioned in publications but we did not find its formal description. *Ob(Func) = Mor(Set), Mor(Func)* are transformations of functions.

2.3. Category of topological spaces *Top*. *Ob(Top)* are topological spaces, *Mor(Top)* are continuous functions.

This category is used in building of the category *Equa-Par-Top*.

We proposed

2.4. Category of equations *Equa*. *Ob(Equa)* contains tuples

{Non-empty sets *X, Y,* predicate $P(x)$ on X, transformation $B: X \to Y$}.

If $(\exists\, x \in X)(P(x) \wedge (y = B(x)))$ then $y \in Y$ is said to be a solution of the equation $\{X, Y, P, B\}$. Particularly, if B is the identity operator I, then we obtain the equation *"P(x)"*only. *Mor(Equa)* are such transformations of tuples $\{X, Y, P, B\}$ that solutions (or their absence) preserve.

Some subcategories for the category *Equa*.

2.5. Category of equations for functions *Equa-Func*. *Ob(Equa-Func)* contains tuples $\{X \in Ob(Func), Y \in Ob(Func),$ predicate $P(x)$ on X, transformation $B: X \to Y\}$.

Mor(Equa-Func) contains invertible transformations of functions inherited from *Mor(Equa)* and specific transformations.

2.6. Category of equations with parameters *Equa-Par*. *Ob(Equa-Par)* are tuples {non-empty sets *X, F, Y,* predicate $P(x,f)$ on $X \times F$, transformation $B: X \to Y$}.

If $(\exists x \in X)(P(x,f) \wedge (y = B(x)))$ then then $y \in Y$ is said to be a solution of the equation $\{X, F, Y, P, B\}$.

Mor(Equa-Par) are such transformations of tuples $\{X, Y, P, B\}$ (except F) that solutions (or their absence) preserve.

2.7. Category of correct equations *Equa-Par-Top*

By our approach «correctness» can by a parameter only, hence the category of correct equations *Equa-Par-Top* is a subcategory of the category *Equa-Par*.

Ob(Equa-Par-Top) are tuples {topological spaces *X, F, Y,* predicate $P(x,f)$ on $X \times F$, continuous transformation $B: X \to Y$} such that 1) $(\forall f \in F)(\exists! y \in Y)(\exists x \in X)(P(x,f) \wedge (y = B(x)))$; 2) the element y depends on the element f continuously.

Mor(Equa-Par-Top) are transformations preserving properties 1) and 2).

2.8. Category of equations for functions with parameters *Equa-Func-Par*.

3. Category of energy-entropy-optimization processes

Denote the physical dimension as *dim; dim(time)=* τ; *dim(length)*=λ; *dim(mass)*= μ; *dim* (*absolute temperature* Θ) = ϑ. Then *dim(energy)*= $\mu\lambda^2/\tau^2$.

By one of definitions, entropy is the measure of a system's thermal energy per unit temperature that is unavailable for doing useful work. In processes considered below increment of entropy is defined by the formula $\Delta H = \sum \Delta Q/\Theta$ where ΔQ is a quantity of transferred heat energy or energy that was converted to heat irreversibly [5]. $dim(\Delta H)= \mu\lambda^2/\tau^2/\vartheta$.

Definition 1. If low energetic outer influences can cause sufficiently various reactions and changings of the inner state of the system then it is said to be an "almost isolated", "permanently unstable", "affectable" system (A-system).

Such outer influences are said to be commands (these reactions and changings are implemented by means of inner energy of the object or of outer energy entering into object besides of commands).

The second law of thermodynamic for isolated systems states increment of entropy $\Delta H > 0$ but does not give quantitative estimations.

One estimation was proven in [6]: the minimal energy (increment of entropy) to treat one bit of information is the Shannon-von Neumann-Landauer boundary: $\Delta H_{bit} = k_B \ln 2$ where k_B is the Boltzmann constant.

A hint to such estimation in general case was in [7]: "In any mechanical system the energy that must be expended to work against friction is equal to the product of the frictional force and the distance through which the system travels. Hence the faster a swimmer travels between two points, the more energy he or she will expend, although the distance traveled is the same whether the swimmer is fast or slow."

Basing on the notion of economical (cruising) speed, taking into account ideas of Ecological Rallies for cars we specified the second law of thermodynamic for A-systems. We proved some estimations in mathematical models describing such concrete systems.

Let there is an A-system. Let it is in any stationary state *A* now and there it can pass to any other stationary state *B*.

Hypothesis 1. There exists such time T_0 (the adiabatic time of the system), depending only on the initial state of the system, that ΔH is not less than any positive value for any transition from the state *A* to the state *B* during $T<T_0$. Moreover, there also exists such positive constant C_0 that $\Delta H \geq C_0 / T^2; dim(C_0)= \mu\lambda^2/\vartheta$.

By the principle of determinism, there is only scenario of the future for any isolated system, that is, there cannot be different possibilities of transitions. Hence, the system is to be A-system: different possible actions, transforming it from the state *A* to the state *B* are controlled by any outer impetus of sufficiently small energy.

Give a more concrete hypothesis. Let any point of mass *m* does not move in any inertial coordinate system at the moment t_1 and it is at the distance *d* from its initial state and does not move at the moment $t_2 = t_1 + T$.

Hypothesis 2. There exists such time T_0 (the adiabatic time of the system), depending only on the initial state of the system, that $\Delta H \geq G_0\ m\ d^{\ 2}/T^{\ 2}$ for any transition from the state *A* to the state *B* during $T<T_0$; $dim(C_0)= 1/\vartheta$.

Denote ΔH for adiabatic time as ΔH_0.

Substantiation. 1) To move the point has to acquire velocity ~ d/T, i.e. kinematical energy $E_{kin} \sim md^2/T^2$. As the point is motionless in the final state, this energy has to pass into other kinds of energy. But possibilities of kinematical energy to pass into potential, chemical etc. ones during bounded time are bounded. Hence a greater part of kinematical energy has to pass into heat, i.e. the increment of entropy must be $\Delta H \sim md^2/T^2/\Theta$. 2) If any estimation on ΔH exists then it must be of dimension of entropy.

Consider the category *Energy-Entropy-Opt-Pro* of processes.

There is an energy reserve in an A-system. A body has a position, a velocity and an acceleration at each moment. Energy is used for increasing of velocity.

Braking is being made at the constant temperature Θ.

REFERENCES

1. Борубаев А.А. О категорных характеристиках компактных, полных равномерных пространств и полных по Райкову топологических групп // Известия Академии наук, вып. 4, 2007. - С. 1-6.
2. Кененбаева Г.М., Аскар кызы Л., Бейшебаева Ж.К., Маматжан уулу Э. Элементы категории уравнений // Вестник Института математики НАН КР, 2018, № 1. - С. 88-95.
3. Кененбаева Г.М., Аскар кызы Л. Элементы категории корректных уравнений // Вестник Института математики НАН КР, 2019, № 1. - С. 69-74.
4. Kenenebaeva G.M., Askar kyzy L. Connections of category of correct equations with other categories // Herald of Institute of Mathematics of NAS of KR, 2020, No. 1.
5. Мартин Н. Ингленд Дж. Математическая теория энтропии. – Москва: Мир, 1988. – 350 с.
6. Landauer R., Bennett C. H. The Fundamental Physical Limits of Computation // Scientific American, July, 1985, pp. 48-56.
7. Landauer R. Irreversibility and heat generation in the computing process // IBM Journal of Research and Development, vol. 5, 1961, pp. 183-191.
8. Панков П.С. Адиабатические показатели замкнутых систем // Вестник КНУ им. Ж.Баласагына. Серия 3. Естественно-технические науки. Физика и физическое образование, 2003. – С. 146-147.
9. Панков П., Баячорова Б., Жураев М. Кыргыз тилин компьютерде чагылдыруу. – Бишкек: Турар, 2010. – 172 б.
10. Pankov P., Akerova Dzh. Minimization in mathematical model of increment entropy in almost closed systems // Abstracts of V Congress of the Turkic World Mathematicians (Kyrgyzstan, Bulan-Sogottu, 5-7 June, 2014) / Ed. Acad. A.Borubaev. – Bishkek: Kyrgyz Mathematical Society, 2014. – P. 252.
11. Кененбаева Г. М., Акерова Дж.А., Кененбаев Э. Математические модели для оценки приращения энтропии // Тезисы докладов II Борубаевских чтений. - Бишкек: Кыргызское математическое общество, 2018.

12. Панков П.С., Акерова Дж.А. Дифференциальные уравнения с управлением в модели возрастания энтропии в почти замкнутых системах с трением // Вестник Института математики НАН КР, 2018, № 1. - С. 23-30.

MSC 34A26, 91F20

FUNCTIONAL RELATIONS AND MATHEMATICAL MODELS OF TRANSFORMING VERBS

Pankov P.[1], Kenenbaev E.[2], Chodobaev S.[3]
[1,2]Institute of Mathematics of NAS of KR,
[3]KNU named after J.Balasagyn

In frames of developing general methods for independent interactive computer presentation of natural languages, B. Bayachorova and S. Karabaeva proposed to present transforming verbs. Such verbs are more complex than “sequences of shifts”. In the paper mathematical and computer models of some verbs are constructed by functional relations between points of virtual objects. Examples in Kyrgyz and English languages are given.

Keywords: language, transforming verb, computer presentation, mathematical model, independent presentation

Табигый тилдерди компьютерде көз карандысыз интерактивдик чагылдыруу үчүн жалпы усулдугун өнүктүрүүнүн ичинде Б.Баячорова жана С.Карабаева өзгөртүүчү этиштерди чагылдырууну сунуш кылды. Мындай этиштер “жылдыруулардын удаалыштыгынан” татаалыраак. Бул макалада кээ бир этиштердин математикалык жана компьютердик моделдери элестетилген объекттердин чекиттеринин арасында функционалдык өз ара байланыштар аркылуу курулду. Кыргызча жана англисче мисалдар берилген.

Урунттуу сөздөр: тил, өзгөртүүчү этиш, компьютердик чагылдыруу, математикалык модель, көз карандысыз чагылдыруу

В рамках разработки общей методики независимого интерактивного компьютерного представления естественных языков Б. Баячорова и С. Карабаева предложили представить преобразующие глаголы. Такие глаголы - более сложные, чем «последовательность сдвигов». В данной статье построены математические и компьютерные модели при помощи функциональных соотношений между точками виртуальных объектов. Даны примеры на кыргызском и английском языках.

Ключевые слова: язык, преобразующий глагол, компьютерное представление, математическая модель, независимое представление.

1. Introduction

At all times, travellers learned languages from inhabitants which did not know the traveller’s native language. Such method (demonstration of objects and actions with comments) is described in details in “Gulliver's travels” by J. Swift, 1726. In 1878 M. Berlitz proposed a method with is considered as “the first-known immersive teaching method”. Such method was improved as “Total physical response” [1].

Winograd T. [2] proposed giving commands to a robot with such words as "table", "box", "block", "pyramid", "ball", "grasp", "moveto", "ungrasp".

Using these ideas, since [3] general methods for interactive computer presentations of natural languages are being developed [4-12]. If a computer presentation does not depend on the user's knowledge and skills on similar objects then it is called independent.

Such presentations are more effective because the user can learn a language inductively, and they begin thinking in it, without translation in mind.

For further developing of such presentations a corresponding classification of notions (nouns and verbs) of languages was proposed [13], with a proposal to develop presentations of transforming verbs.

2. Definitions for independent presentation of notions

Definition 1. If low energetic outer influences can cause sufficiently various reactions and changing of the inner state of the object (by means of inner energy of the object or of outer energy entering into object besides of such influences) at any time then such (permanently unstable) object is an *affectable object*, or a *subject*, and such outer influences are *commands*.

Definition 2. A system of commands such that any subject can achieve desired efficiently various consequences from other one is a *language*.

Hypothesis 1. A human's genuine understanding of a text in a natural language can be clarified by means of observing the human's actions in real life situations corresponding to the text.

Definition 3. *Simple* mathematical models consist of fixed (F_i) and movable (M_j) sets and temporal sequence of conditions of types $(M_j \subset F_i)$, $(M_j \cap F_i = \varnothing)$, $(M_j \cap F_i \neq \varnothing)$.

They present verbs consisting of a "series of shift".

Transformation mathematical models also include *transforming* objects (tools) and *transformable* objects [13]. We propose, as extending of definitions [14]:

Definition 4. A transformable object is a (varying) set or union of sets, some points of it are connected with *functional relations*.

Transformation of objects can be presented by a controlled differential equation. Firstly, we consider motion of a flat object without inertia and not-self-moving objects. Let $S \in R^2$ be initial position of the object; *S(t)* be the position and shape of the object at time *t*.

$$y'(t,z)=F(t,\ S(t),u(t)),\ 0 \le t < \infty \qquad (1)$$

with initial condition $y(0)= z \in S$,

where *u(t)* is a control function (given by the user), the function *F(s,u)* (to be implemented by the programmer) is bounded, $y(t,z)\colon [0,\ \infty) \times S \to R^2$ is the trajectory of the point $z \in S$.

Let *S* and *S(t)* be defined by points $z_1, z_2, \ldots, z_k \in S$ and their images correspondingly. Then $F=F(t, y_1, y_2, \ldots, y_k, u(t))$. Values $y_1, y_2, \ldots, y_k$ are to be connected with functional relations.

Computer interactive presentations are built on the base of mathematical models.

Definition 5. Let any *notion* (word of a language) be given. If an algorithm acting at a computer: generates (randomly) a sufficiently large amount of instances covering all essential aspects of the *notion* to the user, gives a command involving this notion in each situation, perceives the user's actions and performs their results clearly on a display, detects whether a result fits the command, then such algorithm is said to be a *computer interactive presentation* of the *notion.*

Certainly, commands are to contain other words too. But these words must not give any definitions or explanations of the notion.

Definition 6. If all words being used in Definition 5 are unknown to the user nevertheless s/he is be able to fulfill the meant action (because it is the only natural one in this situation) then the notion (word of a language) is said to be *primary*. If the user has to know supplementary words to complete the action then the notion is said to be *secondary*. Thus, there arises a natural hierarchy of notions.

Hypothesis 3. Any notion has a minimalistic mathematical model (involving minimal number of *entities* in Occam's sense).

3. Mathematical models for transformable and transforming objects and verbs

3.1. $k=1$. The object is not transformable. It can be used for (non-transforming) verbs ЖЫЛДЫРУУ-MOVE,SHIFT; КОЙ-PUT; АЛ-TAKE.

3.2. $k=2$, the functional relation is $dist(y_1,y_2)=const$. It can be used for verbs БУР-ROTATE,TURN; ТАРТ-PULL.

3.3. $k=3$, the functional relations are $y_1=const$, $y_2=const$, $dist(y_2,y_3)=const$. It can be used for the verb ИЙ-BEND.

3.4. $k>3$, the functional relations are

$dist(y_1,y_2)= dist(y_2,y_3)=\ldots= dist(y_{k-1},y_k)= const$ (ЧЫНЖЫР-CHAIN)

The verb ТАРТ-PULL. It was implemented in [15].

3.5. $k\geq 2$. Firstly, two S_1 and S_2. $y_1 \in S_1$ and $y_2 \in S_2$. The relation is $y_1=y_2$ for any $t>t_1$. (Or more than two distinct objects).

■■■ ■■ ■ ■■■	■■■ ■■■ ■■■■■■■■ ■ ■ ■■	■■■ ■■■ ■■■■■■ ■■■■ ■
Чарчыны ТҮЗ! MAKE a square!	СИММЕТРИЯЛА! SYMMETRIZE!	БАЙЛА! CONNECT!

3.6. Transforming object (tool).

◊ ■■■ ■■■ ■■■--------------■■■ ■■■ ■■■
Жипти бычак МЕНЕН КЕС! CUT the needle WITH the knife!

3.7. $k=2$. The functional relations are $y_1=const$, $dist(y_1,y_2)$ *increases*. The verb КЕР-STRETCH.

4. Conclusion

This paper enlarges the scope of verbs and nouns of objects which can be presented interactively in the frames of general project of developing mathematical models of various notions for independent presentation of natural languages. We hope that such software would be interesting and useful for people to learn languages.

REFERENCES

1. Asher J. The strategy of total physical response: An application to learning Russian // International Review of Applied Linguistics. 1965, no. 3.
2. Winograd T. Understanding Natural Language. Massachusetts Institute of Technology, New York, 1972.
3. Pankov P.S., Aidaraliyeva J.Sh., Lopatkin V.S. Active English on computer. Conference "Improving Content and Approach in the Teaching of English Language in the Context of Educational Reform", Kyrgyz State Pedagogical University, Bishkek, 1996. - pp. 25-27.
4. Выготский Л.С., Сахаров Л.С. Исследование формирования понятий: методология двойного стимулирования // Хрестоматия по общей психологии. Психология мышления. – Москва: изд. МГУ, 1981.
5. Pankov P.S., Alimbay E. Virtual Environment for Interactive Learning Languages. Human Language Technologies as a Challenge for Computer Science and Linguistics: Proceedings of 2nd Language and Technology Conference, Poznan, Poland, 2005. – Pp. 357-360.
6. Bayachorova B.J., Pankov P. Independent Computer Presentation of a Natural Language // Varia Informatica. – Lublin: Polish Information Processing Society, 2009. – Pp. 73-84.
7. Панков П., Баячорова Б., Жураев М. Кыргыз тилин компьютерде чагылдыруу. – Бишкек: Турар, 2010.
8. Pankov P., Karabaeva S. Proposal to Develop Interactive Computer Presentation of Natural Languages in Various Media // Proceedings of the 7th International

Conference on Control and Optimization with Industrial Applications, 2020. Baku, Azerbaijan. - Volume I, pp. 320-322.

9. Pankov P., Dolmatova P. Software for Complex Examination on Natural Languages // Human Language Technologies as a Challenge for Computer Science and Linguistics: Proceedings of 4th Language and Technology Conference, 2009, Poznan, Poland. – P. 502-506.
10. Pankov P.S., Bayachorova B.J., Juraev M. Mathematical Models for Independent Computer Presentation of Turkic Languages // TWMS Journal of Pure and Applied Mathematics, Volume 3, No.1, 2012. – Pp. 92-102.
11. Pankov P., Karabaeva S. Proposal to Develop Interactive Computer Presentation of Natural Languages in Various Media // Proceedings of the 7th International Conference on Control and Optimization with Industrial Applications, 2020. Baku, Azerbaijan. - Volume I, pp. 320-322.
12. Pankov P.S., Bayachorova B.J., Karabaeva S.Zh. Mathematical models of human control, classification and application // Herald of Institute of Mathematics of NAS of KR, 2020, No. 1. - Pp. 88-95.
13. Bayachorova B.J., Karabaeva S.Zh. Classification of verbs by their mathematical models // Herald of Institute of Mathematics of NAS of KR, 2021, No. 2. - Pp. 7-13.
14. Kenenbaeva G.M., Kenenbaev E. Category of objects with functional relations for differential equations // Herald of Institute of Mathematics of NAS of KR, 2020, No. 2. - Pp. 80-86.
15. Панков П.С., Барыктабасов К.К. Инварианты при бесконечномерных геометрических преобразованиях и их компьютерная реализация // Вестник КГПУ им. И.Арабаева, 2004, серия 3, выпуск 2. – С. 56-60.

MSC 68T30

COMPUTER PRESENTATION OF MATHEMATICAL KNOWLEDGE AS TASKS

Pankov P.S.[1], Burova E.S.[2]
[1,2]*Institute of Mathematics of NAS of KR*

General problem is considered: what mathematical knowledge can be presented as tasks? Proposed kind of presentations has the following features: no preliminary knowledge on the object is necessary; the user masters the object while treating with a computer mouse; mathematical objects are treated as real ones with peculiarities; each presentation is also a task; after successful solution of the task the soft announces: "Congratulations! You have mastered the notion ...".

Keywords: mathematics, task, computer presentation, independent presentation, interactive presentation.

Төмөндөгү көйгөй каралат: кайсы математикалык билим маселелер катары көрсөтүлө алат? Сунуш кылынган чагылдыруулардын түрлөрүнүн касиеттери төмөндөгү: объект боюнча алдын ала билим кереги жок; колдонуучу объектти компьютердик маус менен өздөштүрөт; математикалык объекттер өзгөчөлүктөрү болгон чыныгы объекттер катары бар; ар бир чагылдыруу ошондой эле маселе бар; маселени ийгиликтүү чыгаруудан кийин программалык жабдуу жарыялайт: «Куттуктоо! Сиз … түшүнүгүнө ээ болдуңуз!» деп.

Урунттуу сөздөр: математика, маселе, компьютердик чагылдыруу, көз карандысыз чагылдыруу, интерактивдүү чагылдыруу.

Рассматривается следующая общая проблема: какие математические знания могут быть представлены в виде задач? Особенности предложенных представлений следующие: не требуются предварительные знания об объекте; пользователь осваивает объект с помощью компьютерной мыши; математические объекты представляются, как реальные объекты с особенностями; каждое представление является также задачей; после успешного решения задачи софт объявляет: «Поздравляю! Вы освоили понятие …»

Ключевые слова: математика, задача, компьютерное представление, независимое представление, интерактивное представление.

1. Introduction

We consider the following general problem: what mathematical knowledge can be presented as tasks?

We proposed a definition of independent computer presentation of an object [1]. Particularly, it means that the user is able to master foundations of the subject by using corresponding software (with interactive actions with feedback) without any preliminary knowledge, regardless or with minimal use of their native language.

Also, we introduced definitions of "almost-closed or affectable objects" (including both humans and computers), of "commands" (low-energetic outer influences on affectable objects causing sufficiently various high-energetic reactions and consequences).

We propose a kind of presentations with the following features: no preliminary knowledge on the object is necessary; the user masters the object while treating with a computer mouse; mathematical objects are treated as real ones with peculiarities; each presentation is also a task; after successful solving of the task the soft announces: “Congratulations! You have mastered the notion ...”.

Probably, the first known publication on the problem of evident presentations of mathematical objects and processes with feedback was [2]. For instance, “pull the four-dimensional solid through the two-dimensional surface”.

Remark. Because of lot of publications, of softs etc. it is difficult to distinguish and insist on novelty in such publications.

2. Classification of presentations

Presentations can be classified as avatar (A-presentation) and non-avatar (N-presentation).

Remark. There is duality in perception of N-presentations. If the user watches some changes on the screen then they can imagine either motion of themselves in the space or motion of space (of objects in space). The first is used in computer games and the second one is in mathematical software “Mathematica”, “Matlab”, “MathCad”.

Both for examination and for interest each run of the software generates slightly different environment within the same mathematical object.

The unified denotations for the user in proposed software.

Background is in the spectrum from white till black; sometimes chess color (light grey and dark grey) for 2D-spaces is used. Drag-and-Drop object, or Avatar object is green and is denoted as A-object below. Function, or result of A-object is red and is denoted as F-object below. Target for F-object is yellow and is denoted as T-object below. Approaching T-object is accompanied by music of “hot-cold” type too. Tracks of A-object (light green) and F-object (light red) can also stay while 2D-motion.

4. Examples of presentations

1) Solving of the equation *F(x)=0.* A-point can move along the abscissa axis only. T-object is the abscissa axis.

2) Searching for *min F(x)*. A-point can move along the abscissa axis only. T-object is gradient of yellow color down.

3) Solving of the system of equations *F(x,y)=u*, *G(x,y)=v* (firstly, linear ones; the user discovers linearity). A-point is *(x,y)*, F-point is *(F,G)*, T-point is *(u,v)*.

3a) Solving of the equation $\sqrt{z}=w\neq 0$ for complex numbers. The origin $z=0$ repels A-point. The user discovers the following: to reach T-point going around the origin is necessary [3].

3b) The item 3) is interpreted as transformations of the plane. For example, F-point is the mirror reflection of A-point.

4) Searching for *min F(x,y)*. The value of *F* and T-object as gradient of yellow color down are on a separate part of a display.

Measures as invariants.

5) Length of a curve. A-object with F-object is a red curve with the leading green endpoint. While pulling its length preserves. T-objects are several curves of various lengths. The user is to detect the T-object with corresponding length and pull A-object on this T-object [5].

6) Area of a figure. A-object with F-object is a red rounded figure with the green boundary. While pulling its area preserves. T-objects are several figures of various areas. The user is to detect the T-object with corresponding area and pull A-object on this T-object.

Remark. Programming of preserving area while continuous transformations of a figure is an interesting task itself.

7) N-presentations of non-Euclidean spaces filled with T-objects and brown Obstacles [3]. The user drives a green car, with additional possibilities to put marks etc. The screen is the windshield of the car. The task is to find and gather T-objects without breaking Obstacles.

7a) Moebuis band. The user can verify that a right boot left on a street will be met as a left boot after passing half of the street.

7b) Topological torus (a square with opposite sides glued). This space used to be discovered by many programmers independently. Motion in arbitrary direction will led to the initial position someday.

7c) Riemann surface of the function $\sqrt{z}$, with the third coordinate up. (3a) above).

7d) Riemann surface of the function $\sqrt{(z^2 - a^2)}$, with the third coordinate up. Passing between two unbreakable pillars only leads to another part of the space.

7e) Motion with creating the Riemann surface of the function $H(z,w)=0$, H is a given polynomial. This is the only way to investigate its branching points and general structure.

Remark. Collective investigations of mathematical objects can be involved too. These spaces allow multiple users who can see and meet others naturally.

7f) Projective plane with the third coordinate up. While motion along the street trees on this side move to us as usually but trees on the opposite side move from us.

Remark. This space does not permit multiple users.

8) N-presentations for 4D-space filled with 4D-solids [4].

8a) The 3D-coordinates are presented as usually, the fourth coordinate (call it "deep") is denoted with continuous darkening of the environment. We look at the space through 3D-slit and can "deep" and "undeep". The task is to detect 4D-solids. For instance, the 4D- "deepical" cone is seen as little ball … enlarging ball … none while motion "deep".

8b) [6] Denote coordinates as *X*, *Y*, *Z,W*. The user can choose each of 2D-subspaces *XY, XZ, XW, YZ, YW, ZW,* see projections of 4D-solids and rotate them around this plane. The task: to extract a right boot from 3D-space and return it to same 3D-space as a left boot.

5. Conclusion

We hope that successful implementation of proposed and other such presentations of mathematical objects would distinguish new essential features of various mathematical objects and be interesting both for programmers and for users regardless their relation to mathematics.

REFERENCES

1. Pankov P.S. Independent learning for Open society. Collection of papers as results of seminars conducted within the frames of the program «High Education Support». Foundation «Soros-Kyrgyzstan», Bishkek, 1996.
2. Ulam S.M. A Collection of Mathematical Problems. Interscience, New York, 1960, Chapter VIII. Computing machines.
3. Борубаев А.А., Панков П.С. Компьютерное представление кинематических топологических пространств. - Бишкек: КГНУ, 1999. – 131 с.
4. Pankov P. S., Bayachorova B. J., Terehin A. V. Computer Presentation of Four-Dimensional Spaces. The 8-th International Conference on Computer Graphics and Visualization "GraphiCon-98", Moscow, 1998.
5. Панков П.С., Барыктабасов К.К. Инварианты при бесконечномерных геометрических преобразованиях и их компьютерная реализация // Вестник КГПУ им. И.Арабаева, 2004, серия 3, выпуск 2. – С. 56-60.
6. Панков П.С., Асанов Д.Т. Интерактивное компьютерное представление математических объектов и кинематические преобразования // Природа университетского образования и исследования: Сборник материалов межвузовской конференции. - Бишкек: АУЦА, 2004. – С. 55-61.

MSC 18C05

IMPLEMENTATION OF ALGORITHM TO DETECT PATTERNS IN IRGÖÖ-TYPE PROCESSES

Tagaeva S.B.
Institute of Mathematics of NAS of KR

Algorithm to detect simple patterns on a plane is implemented. Processes of generation of order from chaos (irgöö-type processes by the name of the first process of local diminishing of entropy due to dissolving of energy mentioned in literature) are considered. Main features of such processes: they are real (including running computers) and random, are defined by some components (states of computer) at each moment. Supra, by the authors' definition, appearance of phenomena in systems only with large number of components is said to be the effect of numerosity. The least number of components preserving such phenomenon was said to be the constant related to it.
Keywords: algorithm, order, chaos, irgöö, numerosity, effect, phenomenon, differential equation, difference equation.

Тегиздикте жөнөкөй саймаларды аныктап таануучу алгоритм ишке ашырылган. Башаламандыктан тартип пайда болгону процесстери (адабиятта биринчи белгилүү болгон энергия тарткатылганынан энтропиянын жергилик азаюу процессинин аталышы боюнча алганда, «иргөө» тибиндеги процесстер) каралат. Мындай процесстердин негизги өзгөчөлүктөрүн төмөнкүчө белгилешет: алар чыныгы (анын ичинде компьютерлердин аракеттери) жана кокустан, ар бир учурда бир нече компоненттер (компьютердин абалдары) менен аныкталат. Мурда, авторлор сунуштаган аныктама боюнча, көп элементтерден турган системалар гана үчүн пайда болуучу кубулуштар «көпчө» эффектиси деп аталган. Мындай кубулушка алып келүүчү, ошол кубулуш менен байланышкан, эң кичине сан турактуу болуп саналат.
Урунттуу сөздөр: алгоритм, ирет, башаламандык, иргөө, «көпчө», эффект, кубулуш, дифференциалдык теңдеме, айырмалуу теңдеме

Реализован алгоритм для выявления простых узоров на плоскости. Рассматриваются процессы возникновения порядка из хаоса (процессы типа «иргөө» по названию первого такого процесса локального уменьшения энтропии вследствие диссипации энергии, известного в литературе). Основные признаки таких процессов: они - реальные (включая действия компьютеров) и случайны, определяются несколькими компонентами (состояниями компьютера) в каждый момент. Ранее, по определению авторов, возникновение явлений только для систем с большим количеством компонент названо эффектом «множественности». Самое малое число, вызывающее такое явление, названо постоянной, связанной с этим явлением.
Ключевые слова: алгоритм, порядок, хаос, иргөө, множественность, эффект, явление, дифференциальное уравнение, разностное уравнение.

1. Introduction

Algorithm to detect simple patterns on a plane is implemented in the paper.

Processes of generation of order from chaos (irgöö-type processes by the name of the first such process of local diminishing of entropy due to dissolving of energy mentioned in literature [1]) are considered in the paper. Main features of such processes: they are real (including running computers), random, are defined by some components (states of computer) at each moment.

Considering a computer as a real object and computer presentations as real processes was noted in [2]. Our as well as other authors' "Definitions" below are not strongly mathematical because they mean real objects and processes.

Discoveries of new "phenomena" and "effects" used to be sufficient steps in developing science but there were not definitions of these notions before [4] with corresponding definitions and examples, methodic to search new "phenomena" as consequences of "effects".

Differential equations (excluding some simplest ones) are not "processes" in our sense because differential equations in general cannot be "solved" to detect any "phenomena". Difference equations (excluding some simplest ones) are not "processes" in our sense because they in general cannot be "solved" to detect any "phenomena". Because of computational errors and instability numerical experiments with them can give other results. Hence, a program run on a concrete computer only is a "process".

Section 2 presents definition of effect of numerosity.

There is the example of some known process in Section 3.

In Section 4 we propose an algorithm to detect a pattern.

2. Effect and constants of numerosity

The law of large numbers can be considered as some phenomena in statistic.

Supra, by our definition, appearance of phenomena in systems only with large number of components was said to be the effect of numerosity.

We found some phenomena due to this effect not related to statistic.

Definition 1. Appearance of phenomena in systems only with large number of components is said to be the effect of *numerosity*.

Definition 2. If a phenomenon occurs less often for number of components less than N and does more often for number of components greater than N then the number N is said to be the *constant of numerosity* for this phenomenon.

3. Example of irgöö-type process

We searched self-ordering of discrete electrical charges in viscous media [3]. Motion of equal, repelling by the Coulomb law electrical charges from a random initial distribution on a topological torus (bounded surface without an edge) formed a final regular grid was modeled by computer.

Motion of N electrical charges can be described by a system of N two-dimensional differential equations. These differential equations were approximated by a system of difference equations.

4. Algorithm and program

Construction of an algorithm was proposed [5].

For detecting patterns we propose

Algorithm. Let a finite set of K distinct points $\{z[1].. z[K]\}$ in a bounded metrical (locally Euclidean) space be given. Choose a constant $v>1$.

A) Found the minimum $M[i]:=min\{|z[i]-z[j]|: j\neq i \}, 1\leq i\leq K$.

B) Calculate numbers of neighbors and the average

$C[i]:=card\{j: M\leq | z[i]-z[j]|\leq Mv\}, i=1..K; S:=\Sigma\{ C[i]: i=1..k\}/K.$

C) If most of numbers $C[i]$ are equal (let their common value be C^*) then a pattern exists.

For example, in R^2: if $C^*=3$ then a hexagonal grid exists;

if $C^*=4$ then a hexagonal grid exists;

if $C^*=6$ then a triangular grid exists.

The following program with graphical demonstration of the initial distribution and of the final one was written in *pascal* (with $N=256$), $v=1.18$.

```
program sab_alg; uses crt, graph, math;
var hxy,vx,vy,dx,dy,dxy,dxy1,hxy1,z,z2,xj,yj,dxy2,dxyd,
d_xy2, mn, rel_xy, scous: double;
i, j, nxy, it, nt, np, ihand, n_time, ik: longint;
ncount: array[1..500] of longint;
sxy,smn,s_nc,r_xy: string; var drv, mode, f, n: integer;
```

```
x,y:array[1..500] of double; xn,yn:array[1..500] of integer;
function dxy_2(ii,jj:longint): double;
begin xj:=x[jj]; if xj>x[ii]+z2 then xj:=xj-z; if xj<x[ii]-z2 then xj:=xj+z;
yj:=y[jj]; if yj>y[ii]+z2 then yj:=yj-z; if yj<y[ii]-z2 then yj:=yj+z;
dxy_2:=sqr(x[ii]-xj)+sqr(y[ii]-yj); end;
begin {main} drv:=0; mode:=VgaHi;
InitGraph(drv,mode,'c:\tp\bgi'); randomize;
SetTextStyle(0,0,2);
OutTextXY(30,20,'Tagaeva, 2022. Repelling charges on torus and algorithm');
nxy:=256; str(nxy,sxy);
OutTextXY(70,40,'  '+sxy+' (Wait a little)  ');
z:=700.; z2:=z/2.0; np:=10; hxy:=1.0; hxy1:=hxy; nt:=1000;
for ik:=1 to nxy do begin  x[ik]:=z*random; y[ik]:=z*random;
xn[ik]:=round(x[ik]); yn[ik]:=round(y[ik]);
Setcolor(green); circle(xn[ik]+80,yn[ik]+70,2); end;
   for it:=0 to nt do  begin {it} if it>np then hxy:=2.0*hxy1;
  if it>2*np then hxy:=4.0*hxy1;
         for i:=1 to nxy do begin {i=ix} vx:=0.; vy:=0.; for j:=1 to nxy do
begin if j<>i then begin dxy2:=dxy_2(i,j)+1.;
dxy1:=z/(dxy2*sqrt(dxy2)); if dxy1<sqr(z)/nxy{*0.5} then begin
dx:=(x[i]-xj)*dxy1; dy:=(y[i]-yj)*dxy1;
vx:=vx+dx; vy:=vy+dy; end; end; end;
 x[i]:=x[i]+vx*hxy; if x[i]>z then x[i]:=x[i]-z; if x[i]<0. then x[i]:=x[i]+z;
 y[i]:=y[i]+vy*hxy; if y[i]>z then y[i]:=y[i]-z; if y[i]<0. then y[i]:=y[i]+z;
       end {i=ix};
for ik:=1 to nxy do begin xn[ik]:=round(x[ik]);
yn[ik]:=round(y[ik]) end; end {it};
Setcolor(white);
repeat
for ik:=1 to nxy do begin circle(xn[ik]+80,yn[ik]+70,8);
```

```
circle(xn[ik]+80,yn[ik]+70,6); circle(xn[ik]+80,yn[ik]+70,4);
circle(xn[ik]+80,yn[ik]+70,2) end;
delay(100); scous:=0.0;
for i:=1 to nxy do begin mn:=100000.0;
for j:=1 to nxy do
begin if i<>j then mn:=Min(mn, dxy_2(i,j)) end;
ncount[i]:=0; for j:=1 to nxy do
begin if (i<>j) and (dxy_2(i,j)<mn*1.3) then ncount[i]:=ncount[i]+1 end;
scous:=scous+ncount[i]; end;
scous:=scous/nxy; str(scous:8:1,smn);
OutTextXY(480,40,' average neighbors '+smn);
until keypressed; END.
```

5. Results of experiment

This program was run five times to calculate *S* for each *N* being a square of integer.

N=25: 2.9, 2.6, 2.4, 2.8, 2.7   S=2.7

N=36: 3.5, 3.3, 3.4, 2.9, 2.8   S=3.2

N=49: 3.5, 3.0, 3.3, 3.5, 3.7   S=3.4

N=64: 3.7, 3.9, 3.7, 3.8, 3.8 S=3.8

N=81: 3.9, 4.3, 4.6, 5.5, 3.7 S=4.4

N=100: 4.0, 3.9, 3.8, 3.8, 3.7 S=3.8

N=121: 3.8, 4.1, 4.8, 5.0, 4.2 S=4.4

N=144: 4.0, 3.9, 3.8, 4.0, 4.3 S=4.0

N=169: 3.8, 4.1, 4.8, 5.0, 4.2 S=4.4

N=196: 3.9, 4.0, 3.9, 3.9, 4.0 S=3.9

N=225: 5.0, 4.3, 5.0, 5.0, 4.7 S=4.8

N=256: 4.0, 4.0, 4.9, 4.0, 4.0 S=4.2

N=289: 5.3, 5.0, 4.9, 5.4, 5.0 S=5.1

Hence, the constant of numerosity is ~ 64 (beginning of alternation).

Also, when the number of charges is a square of even number then the grid is square in most of experiments; when it is a square of odd number then the grid is triangular in most of experiments.

Conclusion

We hope that proposed definitions would yield new phenomena in reality and in computational experiments and constants of numerosity would be found for other real and virtual processes. The general problem: what kinds of patterns can be detected by any algorithms?

REFERENCES

1. Pankov P.S., Kenenbaeva G.M. The phenomenon of irgöö as the first example of dissipative system and its implementation on computer (in Kyrgyz) // Proceedings of the National Academy of Sciences of Kyrgyz Republic, no. 3, 2012, pp. 105-108.
2. Borubaev A.A., Pankov P.S. Computer presentation of kinematic topological spa-ces. - Bishkek: Kyrgyz State National University, 1999 (in Russian).
3. Pankov P.S., Tagaeva S.B. The phenomenon of self-ordering of many repelling electrical charges on a topological torus // Herald of Institute of Mathematics of NAS of KR, 2018, No. 1. - Pp. 12-17 (in Russian).
4. Kenenbaeva G. Theory and methodic of searching new effects and phenomena in the theory of perturbed differential and difference equations (in Russian). - Bishkek: "Ilim", 2012, 203 p.
5. Kenenbaeva G.M., Tagaeva S.B. Category of irgöö-type processes in computati-onal mathematics and algorithms to detect patterns // Herald of Institute of Mathe-matics of NAS of KR, 2021, No. 2. - Pp. 13-20.

MSC 34A26, 35A16

ALGORITHMS TO ENLARGE DOMAINS OF SOLUTIONS BY MEANS OF FUNCTIONAL RELATIONS

Kenenbaev E.
Institute of Mathematics of NAS of KR

New task for sets is considered: to enlarge domains of solutions by means of functional relations. Solutions of some differential equations have values connected by functional relations. Asgeirsson's identity for partial differential equations of hyperbolic type is used. An algorithm for rectangular domains is implemented.

Keywords: functional relation, ordinary differential equation, partial differential equation, solution, domain, algorithm, pascal.

Көптүктөр үчүн жаңы маселе каралат: функционалдык өз ара байланыштар чыгарылыштарды аныктоо аймактарын кеңейтүү. Ар кандай типтеги айрым дифференциалдык теңдемелердин чыгарылыштарында функционалдык өз ара байланыштарга байланыштуу маанилер бар. Гиперболалык типтеги дифференциалдык теңдемелер үчүн Асгейрссон бирдейлиги колдонулду. Тик бурчтуу аймактар үчүн алгоритм ишке ашылды.

Урунттуу сөздөр: функционалдык өз ара байланыш, чыгарылыш, кадимки дифференциалдык теңдеме, айрым туундулуу дифференциалдык теңдеме, чыгарылыш, аныктоо аймагы, алгоритм, pascal.

Рассматривается новая задача для множеств: расширить область определения решений с использованием функциональных соотношений. Решения некоторых дифференциальных уравнений различных типов имеют значения, связанные функциональными соотношениями. Применено тождество Асгейрссона для дифференциальных уравнений гиперболического типа. Реализован алгоритм для прямоугольных областей.

Ключевые слова: функциональное соотношение, решение, обыкновенное дифференциальное уравнение, дифференциальное уравнение в частных производных, решение, область определения, алгоритм, pascal.

1. Introduction

New task for sets is considered: to enlarge domains of solutions by means of functional relations. It is known that solutions of some types of differential equations have functional relations (in our terminology) connecting their values in different points. By given values of solutions in several points one can find their values in other points. In the paper we use functional relations for enlarging of domains of solutions.

Sometimes known values of function in some points (multi-point value problem) define it within all the domain. Otherwise, we proposed

Definition. If the function $f(x):X\to F$ is known on a set $X_0 \subset X$, there is a functional relation (FR) on values of the function $f(x)$ and the function $f(x)$ can be defined on a set X_1 by means of (*), $X_0 \subset X_1 \subset X$ then X_1 is said to be an (FR)-enlarging of X_0.

The second section contains examples of functional relations with one-step enlargings. All references are given in [2].

The third section contains an algorithm for enlarging of pairs of rectangular domains for Asgeirsson's identity and propositions on using this algorithm.

The fourth section contains a program in pascal for enlarging of rectangular domains implementing this algorithm.

2. Examples of functional relations with enlarging

In this paper we will use functional denotations of type *x[n]* instead of x_n. Denote the functional relation number F for every equation as the minimal number of connected points (if it exists). We will give either mention of k-point value problem (k-PVP) or a formula for (FR)-enlarging.

2.1. The linear differential equation of the k-th order $y^{(k)}(x)=0$, or a polynomial of $(k-1)$-th order: $F=k+1$. Let numbers $x[1], x[2],\ldots,x[k+1], y[1], y[2],\ldots,y[k+1]$ be given. Construct the Lagrange interpolation polynomial of the $(k-1)$-th order by the values $x[1], x[2],\ldots,x[k]$ и $y[1], y[2],\ldots,y[k]$ then (*) $L(x[k+1])-y[k+1]=0$. k-PVP.

2.2. The first result on functional relations (in our terms) for a linear ordinary differential equation was obtained by C. J. de la Vallée Poussin (for instance see [1]): the k-PVP $y^{(k)}(x)+p_1(x)\, y^{(k-1)}(x)+\ldots+ p_k(x)\, y(x) = 0,\ \ a\le x\le b,$

$p_k(x)\in C[a,b],\ \ y(x[i]) = c[i],\ i=1,\ldots, k$ has a unique solution when

$\| p_1\|_{[a,b]}(b-a)+ \| p_2\|_{[a,b]} (b-a)^2/2! +\ldots + \| p_n\|_{[a,b]} (b-a)^n/n! \ < 1.$

2.3. A solution of the hyperbolic equation $\frac{\partial^2}{\partial x_1\, \partial x_2} u(x_1,x_2) = 0$ fulfills the Asgeirsson's identity $(F=4)$: (FR) $u(w_1, v_1)+ u(w_2, v_2)- u(w_1, v_2)- u(w_2, v_1)\equiv 0.$

It is considered in the next section.

2.4. A solution of the wave equation $\frac{\partial^2}{\partial x_1^2}u(x_1,x_2)=\frac{\partial^2}{\partial x_2^2}u(x_1,x_2)$ fulfills the similar Asgeirsson's identity *(F=4)*: for four vertices of a rectangle obtained by means of rotation of the rectangle (6) on 45^0.

3. Algorithm with using Asgeirsson's identity

This algorithm was announced in [2].

Let $X_1 = \{(x,y)\in R^2 \mid (\exists (x_0, y_0)\in R^2)$

$(((x_0, y_0)\in X_0) \wedge ((x_0, y)\in X_0) \wedge ((x, y_0)\in X_0)))\}$.

Let the set X_0 be a union of several boxes B_k.

Consider the domain being the union of two rectangles

$X_0 := (U_1\times V_1)\cup(U_2\times V_2)$.

Algorithm 1. *Given* $B_1 := (U_1\times V_1)$ *and* $B_2 := (U_2\times V_2)$.

A) If $((U_1\cap U_2 = \varnothing)\wedge(V_1\cap V_2 = \varnothing))$ *then no additional points appear*.

B) If $(B_1\cap B_2 \neq \varnothing)$ *i.e.* $((U_1\cap U_2 \neq \varnothing)\wedge(V_1\cap V_2 \neq \varnothing))$ *then*

$\{U_1 := U_1\cup U_2;\ V_1 := V_1\cup V_2;$ *exclude* $B_2\}$.

C) If $((U_1\cap U_2 = \varnothing)\wedge(V_1\cap V_2 \neq \varnothing))$ *then* $V_1 := V_2 := V_1\cup V_2$.

D) If $((U_1\cap U_2 \neq \varnothing)\wedge(V_1\cap V_2 = \varnothing))$ *then* $U_1 := U_2 := U_1\cup U_2$.

Algorithm 2 (briefly). After enlarging all possible pairs of boxes consider triples of boxes for future enlarging.

4. Text of program

```
PROGRAM elaman; uses crt, math;
var b: array[1..4,1..20] of integer; k, mab: integer;
procedure enl(i,j:integer);
begin {no intersection}
if (((b[1,j]>b[2,i]) or (b[2,j]<b[1,i])) and
((b[3,j]>b[4,i]) or (b[4,j]<b[3,i]))) then writeln('  no change');
{intersection}
if ((((b[1,j]>=b[1,i]) and (b[1,j]<=b[2,i]))) or
(((b[2,j]>=b[1,i]) and (b[2,j]<=b[2,i])))) and
((((b[3,j]>=b[3,i]) and (b[3,j]<=b[4,i]))) or
```

```
(((b[4,j]>=b[3,i]) and (b[4,j]<=b[4,i])))) then
begin b[1,i]:=Min(b[1,i],b[1,j]); b[2,i]:=Max(b[2,i],b[2,j]);
b[3,i]:=Min(b[3,i],b[3,j]); b[4,i]:=Max(b[4,i],b[4,j]);
writeln('  exclude B[',j:2,']'); end;
{v-intersection}
if ( ((b[1,j]>b[2,i]) or (b[2,j]<b[1,i])) and
(not ((b[3,j]>b[4,i]) or (b[4,j]<b[3,i])))   )  then
begin b[3,i]:=Min(b[3,i],b[3,j]); b[4,i]:=Max(b[4,i],b[4,j]);
b[3,j]:=b[3,i]; b[4,j]:=b[4,i]; end;
{u-intersection}
if (  ((b[3,j]>b[4,i]) or (b[4,j]<b[3,i])) and
(not ((b[1,j]>b[2,i]) or (b[2,j]<b[1,i])))   ) then
begin b[1,i]:=Min(b[1,i],b[1,j]); b[2,i]:=Max(b[2,i],b[2,j]);
b[1,j]:=b[1,i]; b[2,j]:=b[2,i]; end; end;
{main program}
begin
writeln('  E.Kenenbaev, 2022. Enlarging');
write('  Input ul[1],um[1],vl[1],vm[1]: ');
readln(b[1,1],b[2,1],b[3,1],b[4,1]);
write('  Input ul[2],um[2],vl[2],vm[2]: ');
readln(b[1,2],b[2,2],b[3,2],b[4,2]);
enl(1,2); writeln(' B[1]: ',b[1,1]:3,b[2,1]:3,b[3,1]:5,b[4,1]:3);
writeln(' B[2]: ',b[1,2]:3,b[2,2]:3,b[3,2]:5,b[4,2]:3);
readln end.
```

5. Conclusion

We hope that such methods would yield new properties of solutions of ordinary and partial differential equations, would promote a unified classification of multidimensional partial differential equations and without existing classifications which are based on formal writings of them.

REFERENCES

1. Vallée-Poussin Ch.J. Sur l'equation différentielle linéaire du second ordre. Détermination d'une intégrale par deux valeurs assignées. Extension aux équations d'ordre // Journal Math. Pures Appl., 8, 1929, pp. 125–144.
2. Kenenbaev E. Enlarging of domains of solutions by means of functional relations // Herald of Institute of Mathematics of NAS of KR, 2021, No. 1. - Pp. 32-38.

MSC 49M37

DETERMINATION OF THE OPTIMAL AREA AND PROJECT OF CONSTRUCTION OF A HOUSING BUILDING

Asankulova M.[1], Jolborsova A.J.[2], Iskandarova G.S.[3]

[1,3]*Institute of Mathematics of NAS of KR,*

[2]*KGU named after I.Arabaeva*

In the article [1], a mathematical model of the problem of choosing the optimal area and the project for the construction of a residential building was formulated. This paper shows the performance of the mathematical model on the given numerical example.

Keywords: mathematical model, selection problem, construction project, performance, numerical example.

Буга чейин макалада [1] турак жай имаратын куруунун оптималдуу аянтын жана долбоорун тандоо маселеси үчүн математикалык модель түзүлгөн. Бул макалада келтирилген сандык мисал аркалуу математикалык моделдин аткаруу жөндөмү көрсөтүлгөн.

Урунттуу сөздөр: математикалык модель, тандоо маселеси, куруу долбоору, аткаруу жөндөмү, сандык мисал.

В статье [1] была сформулирована математическая модель задачи выбора оптимального района и проекта строительства жилищного дома. В данной работе показана работоспособность математической модели на приведенном числовом примере.

Ключевые слова: математическая модель, задача выбора, проект строительства, работоспособность, числовой пример.

Recently, more and more attention has been paid to the scientific approach to all issues related to the development of the national economy of our country, including the construction organization. The construction organization is a branch of material production, covering the most important issues and the most complex processes that satisfy material social needs.

Optimization of the development of the production of a construction organization, in particular a construction company engaged in housing construction, is the choice of the most profitable option for locating a construction object from possible sets and a project that will deliver maximum profit to it.

We present the problem statement. Let there be a construction company that is engaged in the construction of multi-apartment residential buildings in 3 possible districts of the city, s = 1, 2, 3. For the construction of a monolithic multi-apartment residential building in these areas, 3 projects are proposed, μ = 1, 2, 3.
In each μ-th project for the district, i-th apartments are provided in the amount p_{is}^{μ}, i = 1, 2, 3, μ = 1, 2, 3, s = 1, 2, 3, т.е.

$$\left|p_{is}^{\mu}\right|_{3,s=1} = \begin{pmatrix} p_{11}^{1} & p_{11}^{2} & p_{11}^{3} \\ p_{21}^{1} & p_{21}^{2} & p_{21}^{3} \\ p_{31}^{1} & p_{31}^{2} & p_{31}^{3} \end{pmatrix} = \begin{pmatrix} 8 & 5 & 5 \\ 5 & 8 & 5 \\ 5 & 5 & 8 \end{pmatrix}, \qquad \left|p_{is}^{\mu}\right|_{3,s=2} = \begin{pmatrix} p_{12}^{1} & p_{12}^{2} & p_{12}^{3} \\ p_{22}^{1} & p_{22}^{2} & p_{2}^{3} \\ p_{32}^{1} & p_{32}^{2} & p_{32}^{3} \end{pmatrix} = \begin{pmatrix} 8 & 10 & 5 \\ 10 & 5 & 5 \\ 8 & 5 & 8 \end{pmatrix},$$

$$\left|p_{is}^{\mu}\right|_{3,s=3} = \begin{pmatrix} p_{13}^{1} & p_{13}^{2} & p_{13}^{3} \\ p_{23}^{1} & p_{23}^{2} & p_{23}^{3} \\ p_{33}^{1} & p_{33}^{2} & p_{33}^{3} \end{pmatrix} = \begin{pmatrix} 5 & 8 & 10 \\ 5 & 8 & 5 \\ 5 & 5 & 5 \end{pmatrix}.$$

The needs of the population of the city in the i-th type of apartments are known and equal to the value $b_i = (b_1, b_2, b_3)$ = (25, 20.25).

The following materials are used in construction: concrete, rebars of various sizes, plastic materials for windows and doors, i.e. j = 1, 2, 3. The total area of the i-th apartment in the project of an apartment building under construction and its cost, depending on the area (in soms) are given in Table 1.

Table 1

Kinds quart	square (sq.m.)	S=1 (district)			S=2 (district)			S=3 (district)		
		μ=1	μ=2	μ=3	μ=1	μ=2	μ=3	μ=1	μ=2	μ=3
i=1	33	29000	25000	26000	27000	28000	29000	27000	28000	29000
i=2	48-50	31000	29000	28000	31000	32000	33000	31000	32000	33000
i=3	91.2	36000	34000	38000	37000	38000	39000	37000	38000	39000

To determine the volume of concrete used for each i-th apartment, the dimensions given in table. 2, tab. 3, tab. 4.

Table 2

№	Name of the apartments	Dimensions apartments (M)	Volume of concrete without openings windows and doors (m^3)	Volume of concrete with openings for windows and doors (m^3)	Rebar volume d=10 (mm) and d=8 (mm) (tons)	Plastic materials (M)
1	three-room	10*10	29,048	26,69≈27	1.5	1150
2	two-room	8*6	19.84≈20	16.0	1.3	950
3	one-room	5,6*6	13.6	12.58	1.1	350

Table 3

№	Name of the apartments	walls	Length (m)	Height (m)	Thickness of the wall (mm)	Volume concrete (m^3)
1	three-room	external	39.04	3	0.15	17.73
		internal	24.88	3	0.12	8.96
2	two-room	external	29.1	3	0.15	13.09
		internal	15	3	0.12	6.75
1	one-room	external	24	3	0.15	10.04
		internal	10	3	0.12	3.6

To calculate the volume of window and door openings for all apartments, standard sizes were used, which are given in Table. 4.

Table 4

№	Name of the apartments		Width (m)	Height (m)	Thickness of the walls (mm)	Quantity (n)	Volume opening (m^3)
1	three-room	Doors	1	2	0.15	6	1.8
		Windows	2	1.70	0.15	4	2.04
2	two-room	Doors	1	2	0.15	5	1.5
		Windows	2	1.70	0.15	3	1.53
3	one-room	Doors	1	2	0.15	3	0.9
		Windows	2	1.70	0.15	2	1.02

Based on the data in Table 2, Table 3, Table 4, the volume of building material for each *i*-th apartment $j = 1, 2, 3$, $s = 1, 2, 3$, i.e.

$$\left|q_{js}^{\mu}\right|_{3,3} = \begin{vmatrix} q_{1s}^{1} & q_{2s}^{2} & q_{3s}^{3} \\ q_{2s}^{1} & q_{2s}^{2} & q_{2s}^{3} \\ q_{3s}^{1} & \mathrm{q}_{3s}^{2} & q_{3s}^{3} \end{vmatrix} = \begin{vmatrix} 12.58 & 16.00 & 26,69 \\ 1.1 & 1.3 & 1.5 \\ 70.00 & 190.00 & 230.00 \end{vmatrix}.$$

The total volume of building material used for each project is determined by the formulas:

$$\bar{q}_{js}^{\mu} = q_{js}^{\mu} \times n, \quad j=1,2,3, \qquad (*)$$

where n is the number of floors in an apartment building.

Then $\bar{q}_{js}^{\mu} = \begin{pmatrix} 62.9 & 80.00 & 133.45 \\ 5.5 & 6.6 & 7.5 \\ 350 & 950 & 1150 \end{pmatrix}$, μ=1,2,3, s=1,2,3.

The cost per unit volume of the j-th type of building material c_2, j = 1, 2, 3, i.e. $c_j = (c_1, c_2, c_3)$, i.e. c_1 = 5000 soms price of one cubic meter of reinforced concrete, c_2 = 45 soms price of one kilogram of rebar d = 10 mm and d = 8 mm, c_3 = 300 soms price of one linear meter of plastic material.

It is assumed that overhead costs and payment for services of a residential building under construction for the i-th type of apartments for all projects μ = 1, 2, 3 and districts s are known and equal to each other (in soms), i.e.

$$\varepsilon_s^{\mu} = \begin{pmatrix} i=1 & i=2 & i=3 \\ 1168960 & 1059600 & 1200600 \\ 1123850 & 1105960 & 1259060 \\ 1099650 & 1159900 & 1269900 \end{pmatrix}, \ \mu=1,2,3, \quad \delta_s^{\mu} = \begin{pmatrix} i=1 & i=2 & i=3 \\ 105000 & 130000 & 135000 \\ 125000 & 135000 & 140000 \\ 130000 & 140000 & 143000 \end{pmatrix}, \ \mu=1,2,3.$$

It is required to choose such a housing construction project and their location among the possible locations of city districts in such a way as to satisfy the needs of the city population for each type of room apartments and at the same time of the construction, company would receive the maximum profit.

Let us calculate the coefficients of the variables according to the formula

$$c_s^{\mu} = \sum_{i=1}^{3} c_{is}^{\mu} p_{is}^{\mu} - \sum_{j=1}^{3} c_j q_{js}^{\mu} - (\varepsilon_s^{\mu} + \delta_s^{\mu}), \quad \mu=1,2,3, \ s=1,2,3.$$

$c_1^1 = 961840$, $c_1^2 = 697906$, $c_1^3 = 785900$, $c_2^1 = 866134$, $c_2^2 = 951274$, $c_2^3 = 956950$,

$c_3^1 = 885334$, $c_3^2 = 970474$, $c_3^3 = 976150$, $c_1^1 = 1189900$, $c_1^2 = 1017900$, $c_1^3 = 931900$,

$c_2^1 = 1138540$, $c_2^2 = 1224540$, $c_2^3 = 1310540$, $c_3^1 = 1079600$, $c_3^2 = 1165600$, $c_3^3 = 1251600$,

$c_1^1 = 1392996$, $c_1^2 = 1220824$, $c_1^3 = 1557342$, $c_2^1 = 1415622$, $c_2^2 = 1493882$, $c_2^3 = 1579968$,

$c_3^1 = 1401782$, $c_3^2 = 1480042$, $c_3^3 = 1566128$.

Then the numerical model of the problem has the form.

Find a maximum

$$L(y) = 961840\, y_1^1 + 697906\, y_1^2 + 785900\, y_1^3 + 866134\, y_2^1 + 951274\, y_2^2 + 956950\, y_2^3 +$$
$$+ 885334\, y_3^1 + 970474\, y_3^2 + 976150\, y_3^3 + 1189900\, y_1^1 + 1017900\, y_1^2 + 931900\, y_1^3 +$$
$$+ 1138540\, y_2^1 + 1224540\, y_2^2 + 1310540\, y_2^3 + 1079600\, y_3^1 + 1165600\, y_3^2 +$$
$$+ 1251600\, y_3^3 + 1392996\, y_1^1 + 1220824\, y_1^2 + 1557342\, y_1^3 + 1415622\, y_2^1 +$$
$$+ 1493882\, y_2^2 + 1579968\, y_2^3 + 1401782\, y_3^1 + 1480042\, y_3^2 + 1566128\, y_3^3 \qquad (1)$$

Under conditions

$$8\, y_1^1 + 5\, y_1^2 + 5\, y_1^3 + 8\, y_2^1 + 10\, y_2^2 + 5\, y_2^3 + 5\, y_3^1 + 8\, y_3^2 + 10\, y_3^3 \geq 25,$$
$$5\, y_1^1 + 8\, y_1^2 + 5\, y_1^3 + 10\, y_2^1 + 5\, y_2^2 + 5\, y_2^3 + 5\, y_3^1 + 8\, y_3^2 + 5\, y_3^3 \geq 15, \qquad (2)$$
$$5\, y_1^1 + 5\, y_1^2 + 8\, y_1^3 + 8\, y_2^1 + 5\, y_2^2 + 8\, y_2^3 + 5\, y_3^1 + 5\, y_3^2 + 5\, y_3^3 \geq 21,$$

$$\sum_{\mu=1}^{3} y_s^{\mu} = 1, \quad s = 1,2,3, \qquad (3)$$

$$y_s^{\mu} = \begin{cases} 0, \\ 1, \ \mu = 1,2,3, \ \ s = 1,2,3. \end{cases} \qquad (4)$$

Let's solve problem (1)-(4) by the method given in [2] and get the optimal plan for choosing a project from among the possible ones:

$$X = \{ y_1^1 = 8,\ y_2^2 = 4,\ y_2^3 = 3,\ y_3^3 = 10,\ y_2^1 = 5,\ \ y_2^3 = 5,\ \ y_2^3 = 5,\ \ y_1^3 = 8,\ \ y_2^3 = 8,\ \ y_3^3 = 5 \}.$$

With such a choice of projects for the construction of apartments, the maximum profit of the construction company is 11348148 soms, i.e.

$$L(y) = 11348148 \text{ soms.}$$

REFERENCES

1. Asankulova M., Zholborsova A., Eshenkulov P. Mathematical model for optimizing the choice of a residential building project. Izvestiya vuzov Kyrgyzstana. Bishkek, 2016, No. 5. - P.100-103.
2. Asankulova M., Zhusupbaev A. Optimization of production and distribution of raw materials between consumers depending on the period // Problems of modern science and education. 2016. N 4 (46), -p.7-12.

ISSN 2304-2338 (printed version), ISSN 2413-4635 (electronic version)

MSC 68D18

APPLICATION OF THE MAXIMA MATHEMATICAL PACKAGE FOR CREATING 2D AND 3D GRAPHICS FOR THE PROBLEM OF HIGHER MATHEMATICS

To make the plots, Maxima

Yunusov Sh.E. [1], **Yunusov I.E.** [2]
[1] *Kyrgyz National University named after J. Balasagyn*
[2] *Kyrgyz State University I. Arabaev*

In the present article touches, upon the issues of using the Maxima package for solving some typical problems of higher mathematics.
Key words: package Maxima, matrix, differentiation, integration.

Бул макалада Maxima пакетин жогорку математиканын айрым негизги маселелеринин 2D жана 3D графиктерин тургузууда колдонуу каралган.
Урунттуу сөздөр: Maxima пакети, 2D жана 3D, график.

В данной работе затрагиваются вопросы использования пакета Maxima для решения некоторых типовых задач высшей математики.
Ключевые слова: пакета Maxima, матрица, дифференцирование, интегрирование.

The Maxima is a descendant of Macsyma, the legendary computer algebra system developed in the late 1960s at the Massachusetts Institute of Technology. It is the only system based on that effort still publicly available and with an active user community, thanks to its open source nature. Macsyma was revolutionary in its day, and many later systems, such as Maple and Mathematica, were inspired by it.

The Maxima branch of Macsyma was maintained by William Schelter from 1982 until he passed away in 2001. In 1998 he obtained permission to release the source code under the GNU General Public License (GPL) [2]. It was his efforts and skill which have made the survival of Maxima possible, and we are very grateful to him for volunteering his time and expert knowledge to keep the original DOE Macsyma code alive and well. Since his death, a group of users and developers has formed to bring Maxima to a wider audience.

Maxima is updated very frequently, to fix bugs and improve the code and the documentation. Maxima is a system for the manipulation of symbolic and numerical expressions, including differentiation, integration, Taylor series, Laplace transforms, ordinary differential equations, systems of linear equations, polynomials, sets, lists, vectors, matrices and tensors. Maxima yields high precision numerical results by using exact fractions, arbitrary-precision integers and variable-precision floating-point numbers. Maxima can plot functions and data in two and three dimensions [1-4].

The Maxima source code can be compiled on many systems, including Windows, Linux, and MacOS X. The source code for all systems and precompiled binaries for Windows and Linux are available at the SourceForge file manager.

Consider the issues of using the Maxima package to solve some typical problems of higher mathematics.

Consider the application of the Maxima mathematical package to create 2d and 3d graphics for the task of higher mathematics

To make the plots, Maxima can use an external plotting package or its own graphical interface Xmaxima (see the section on Plotting Formats). The plotting functions calculate a set of points and pass them to the plotting package together with a set of commands specific to that graphic program. In some cases those commands and data are saved in a file and the graphic program is executed giving it the name of that file to be parsed.

When a file is created, it will begiven the name maxout_xxx.format, where xxx is a number that is unique to every concurrently-running instance of Maxima and format is the name of the plotting format being used (gnuplot, xmaxima, mgnuplot or geomview).

There are commands to save the plot in a graphic format file, rather than showing it in the screen. The default name for that graphic file is maxplot.extension, where extension is the extension normally used for the kind of graphic file selected, but that name can also be specified by the user. The maxout_xxx.format and maxplot.extension files are created in the directory specified by the system variable maxima_tempdir. That location can be changed by assigning to that variable (or to the environment variable MAXIMA_TEMPDIR) a string that represents a valid directory where Maxima can create new files. The output of the Maxima plotting command will be a list with the names of the file(s) created, including their complete path, or empty if no files are created. Those files should be deleted after the maxima session ends.

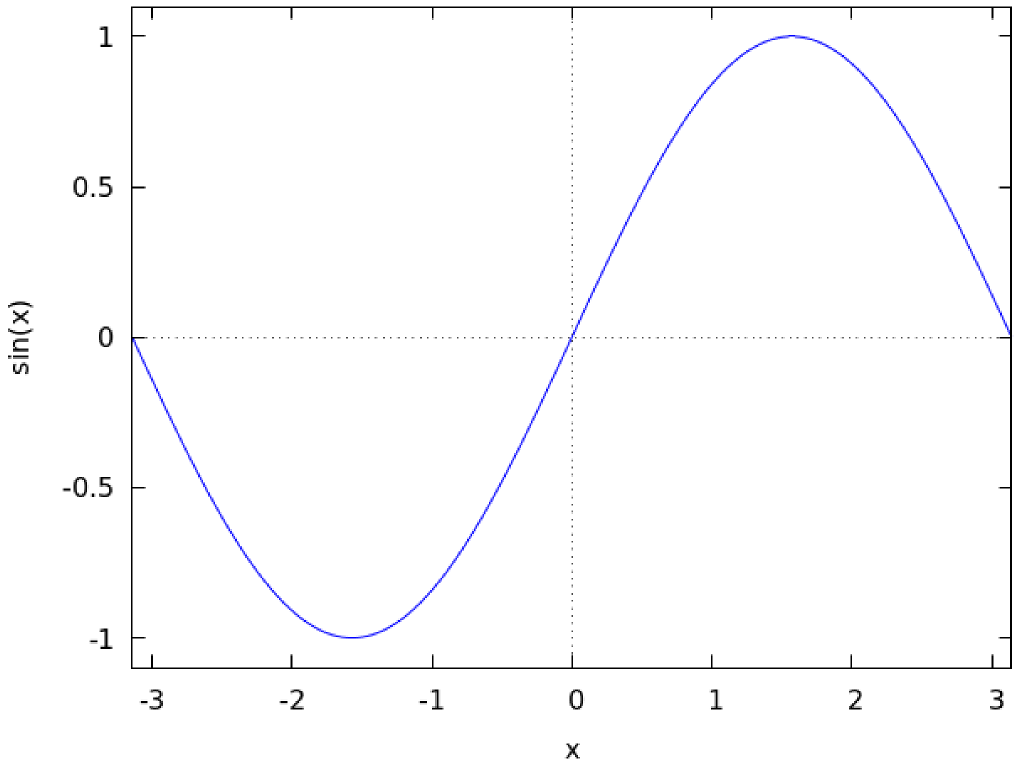
If the format used is either gnuplot or xmaxima, and the maxout_xxx.gnuplot or maxout_xxx.xmaxima was saved, gnuplot or xmaxima can be run, giving it the name of that file as argument, in order to view again a plot previously created in Maxima. Thus, when a Maxima plotting command fails, the format can be set to gnuplot or xmaxima and the plain-text file maxout_xxx.gnuplot (or maxout_xxx.xmaxima) can be inspected to look for the source of the problem.

The additional package draw provides functions similar to the ones described in this section with some extra features, but it only works with gnuplot. Note that some plotting options have the same name in both plotting packages, but their syntax and behavior is different. To view the documentation for a graphic option opt, type ?? opt in order to choose the information for either of those two packages.

Examples:

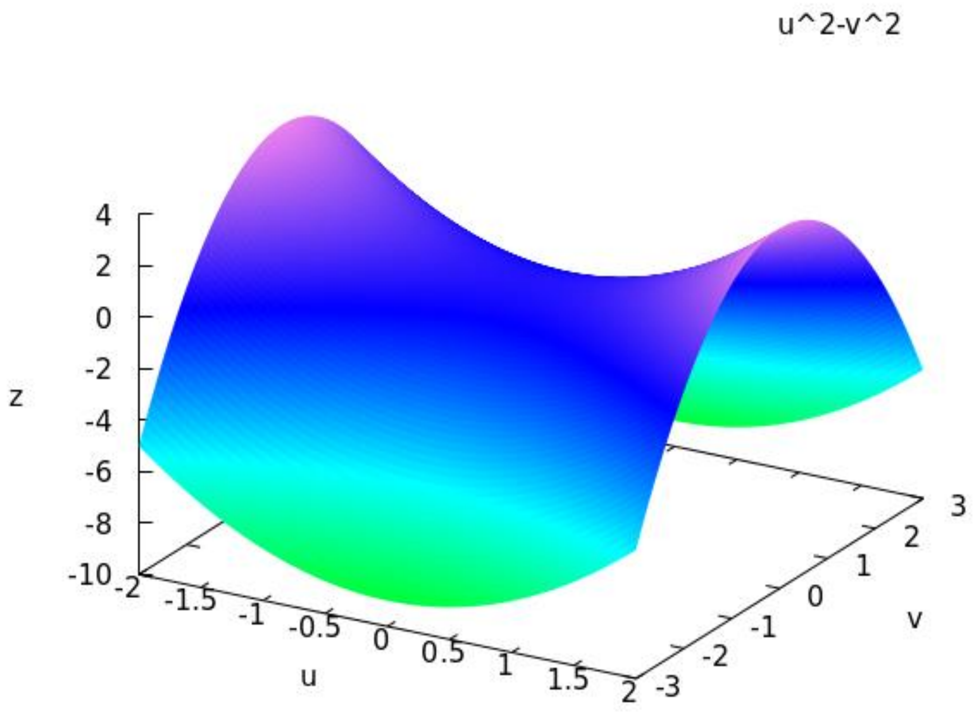
1. Explicit function.

```
(%i1) plot2d (sin(x), [x, -%pi, %pi])$
```



2. Plot of a function of two variables:

```
(%i1) plot3d (u^2 - v^2, [u, -2, 2], [v, -3, 3], [grid, 100, 100],
        nomesh_lines)$
```



REFERENCES

1. Zhitnikov V. Computers, mathematics and freedom. Computerra. No. 16 (636), April 2006.
2. Chichkarev E.A. Computer mathematics with Maxima: A guide for schoolchildren and students / E.A. Chichkarev - M. : ALT Linux, 2012. - 384 p.
3. http://www.pmtf.msiu.ru/chair31/students/spichkov/maxima2.pdf (Methodological guide for studying the Maxima mathematical package) Mathematical workshop using the Maxima package. (PDF).
4. Documentation for the current version of the package: http://maxima. sourceforge.net/docs/manual/en/maxima.html.
5. Version maxima-5.45.1/share/doc/wxm axima/wxmaxima.html.
6. Version maxima -Maxima 5.45.0.
7. https://maxima.sourceforge.io/docs/manual/maxima_singlepage.html#Mathematic al-Functions.

MSC 68D18

APPLICATION OF THE MAXIMA MATHEMATICAL PACKAGE IN TEACHING HIGHER MATHEMATICS PROBLEM SOLVING

Yunusov Sh.E. [1], Yunusov I.E. [2]
[1]*Kyrgyz National University named after Zhusup Balasagyn*
[2]*Kyrgyz State University I. Arabaev*

In the present article touches, upon the issues of using the Maxima package for solving some typical problems of higher mathematics.
Key words: package Maxima, matrix, differentiation, integration.

Бул макалада Maxima пакетин жогорку математиканын айрым негизги маселелерин чечуудө колдонуу каралган.
Урунттуу сөздөр: Maxima пакети, матрица, дифференцирлөө, интегралдоо.

В данной работе затрагиваются вопросы использования пакета Maxima для решения некоторых типовых задач высшей математики.
Ключевые слова: пакета Maxima, матрица, дифференцирование, интегрирование.

The Maxima is a descendant of Macsyma, the legendary computer algebra system developed in the late 1960s at the Massachusetts Institute of Technology. It is the only system based on that effort still publicly available and with an active user community, thanks to its open source nature. Macsyma was revolutionary in its day, and many later systems, such as Maple and Mathematica, were inspired by it.

The Maxima branch of Macsyma was maintained by William Schelter from 1982 until he passed away in 2001. In 1998 he obtained permission to release the source code under the GNU General Public License (GPL) [2]. It was his efforts and skill which have made the survival of Maxima possible, and we are very grateful to him for volunteering his time and expert knowledge to keep the original DOE Macsyma code alive and well. Since his death, a group of users and developers has formed to bring Maxima to a wider audience.

Maxima is updated very frequently, to fix bugs and improve the code and the documentation. Maxima is a system for the manipulation of symbolic and numerical expressions, including differentiation, integration, Taylor series, Laplace transforms, ordinary differential equations, systems of linear equations, polynomials, sets, lists, vectors, matrices and tensors. Maxima yields high precision numerical results by using exact fractions, arbitrary-precision integers and variable-precision floating-point numbers. Maxima can plot functions and data in two and three dimensions [1-4].

The Maxima source code can be compiled on many systems, including Windows, Linux, and MacOS X. The source code for all systems and precompiled binaries for Windows and Linux are available at the SourceForge file manager.

Consider the issues of using the Maxima package to solve some typical problems of higher mathematics.

Consider working with matrices in Maxima [2], [5].

Maxima defines rectangular matrices.The main way to create matrices is to use the function matrix. Call syntax: matrix(row1,... ,rown).

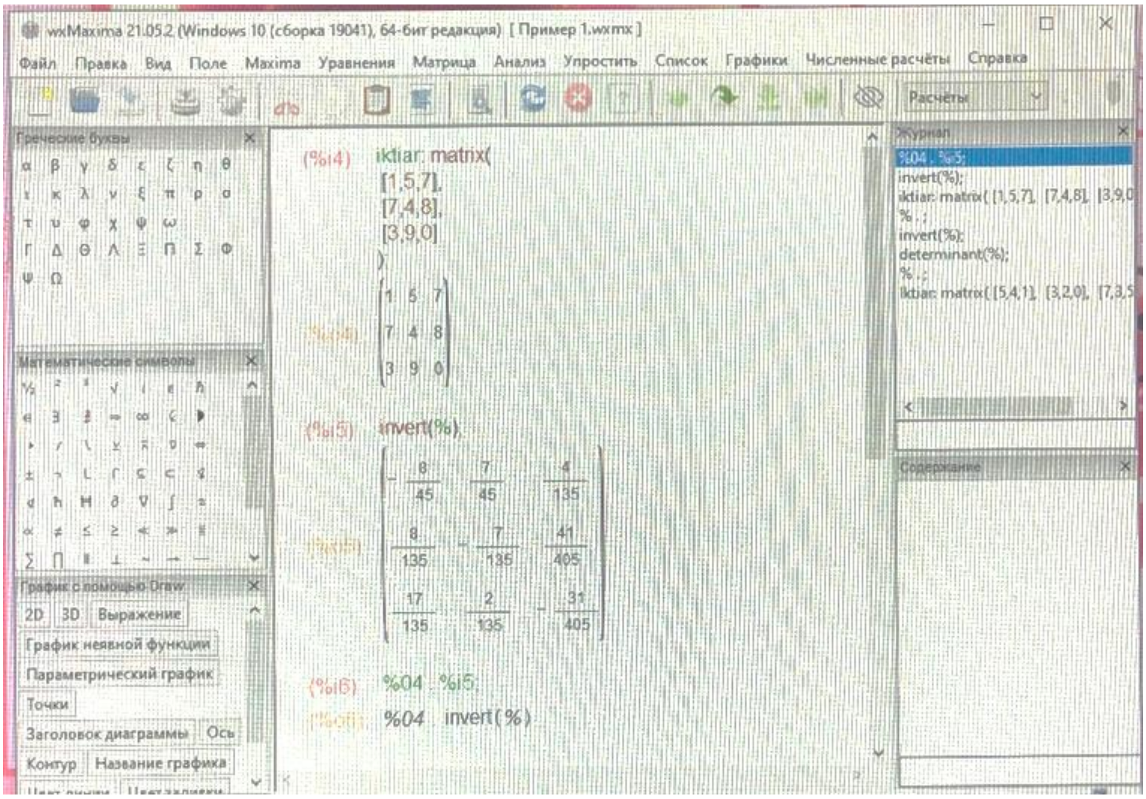


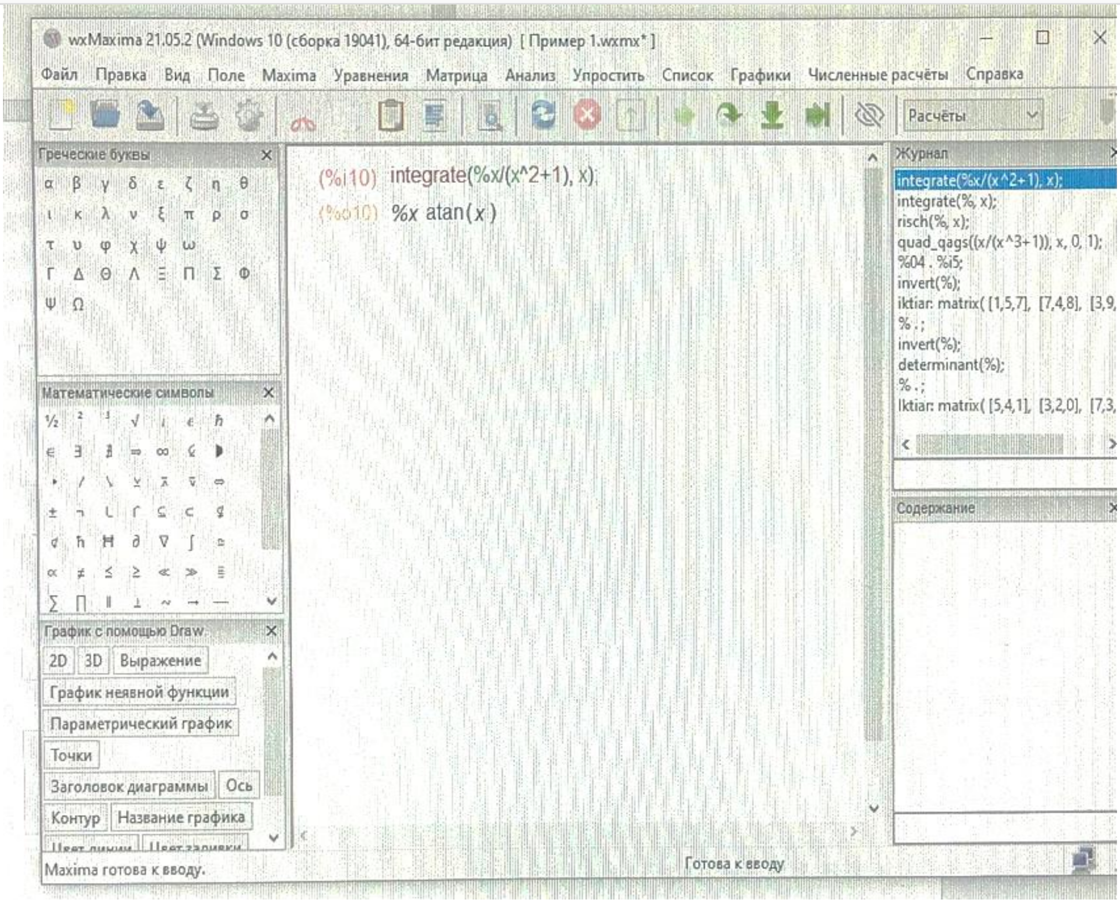
Figure 1. General view of the xMaxima working window when working with matrices

Each line is a list of expressions, all lines are the same length. On the set matrices define the operations of addition, subtraction, multiplication and division. These operations are performed element by element if the operands are two matrices, a scalar and a matrix, or a matrix and a scalar. Erection to degree is possible if one of the operands is a scalar. Multiplication matrices (generally a non-commutative operation) is denoted symbol “. ". The operation of multiplying a matrix by itself treated as exponentiation. Raising to the power of −1 - as inverses (if possible).

integrate(x/(x^3 + 1), x);→ $\frac{\log(x^2-x+1)}{6}+\frac{\arctan(\frac{2x-1}{\sqrt{3}})}{\sqrt{3}}-\frac{\log(x+1)}{3}$

diff(%, x); → $\frac{2}{3\left(\frac{(2x-1)^2}{3}+1\right)}+\frac{2x-1}{6(x^2-x+1)}-\frac{1}{3(x+1)}$

ratsimp(%); → $\frac{x}{x^3+1}$



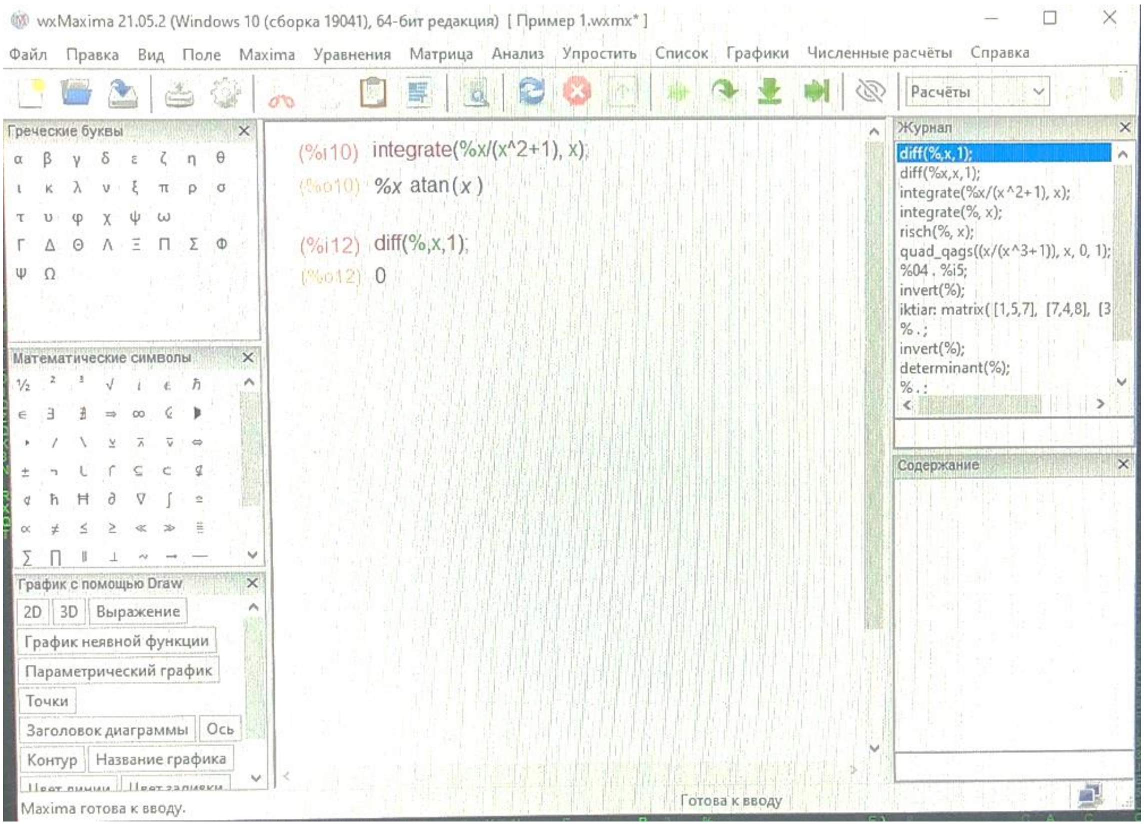


Figure 2. General view of the xMaxima working window when working with differentiation and integration

REFERENCES

1. Zhitnikov V. Computers, mathematics and freedom. Computerra. No. 16 (636), April 2006.
2. Chichkarev E.A. Computer mathematics with Maxima: A guide for schoolchildren and students / E.A. Chichkarev - M. : ALT Linux, 2012. - 384 p.
3. http://www.pmtf.msiu.ru/chair31/students/spichkov/maxima2.pdf (Methodological guide for studying the Maxima mathematical package) Mathematical workshop using the Maxima package. (PDF).
4. Documentation for the current version of the package: http://maxima. sourceforge.net/docs/manual/en/maxima.html.
5. Version maxima-5.45.1/share/doc/wxmaxima/wxmaxima.html.

CONTENS

Формат 60×48. Объем 10,5 п.л.
Подписано в печать 17.06.22г.
Тираж 100 экз.
Отпечатано в ОсОО “Ала-Тоо Полиграф Сервис”
Пр. Чуй 265а